ON CATASTROPHES AND CHAOS THEORY

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One of the areas of insurance in which insurers still struggle to address the risk adequately with a clear demonstration of solvency is cat risk. Cat risks as we know have very low occurrence probability, but once manifest the intensity of its impact is quite severe, even phenomenal. Since the risk is a matter of concern for large projects and other types of entrepreneurial endeavors requiring sinking of huge capital for long periods before any return begins to flow back, insurers cannot shy away from it.

Insurers do accept cat risks and hope to manage them by devious techniques. For one thing their strategy is to hold such risks with them for as little duration as possible before the elapse of which they manage to pass the buck to others. When every one plays the same trick, the very same risks they think they have transferred, come back to them through the tortuous routes of direct or indirect retrocession.

As in any other risk, the degree of insurability is the prime problem with cat risks only that this degree is bordering on unmanageability. Most other risks insurers accept are capable of being reasonably forecasted for their occurrence and intensity. This is because statistical modeling and averaging is possible. Furthermore, if the insurer has adequate reserves to fall back upon, adequate volume of the risk in his books plus an efficient model to rate the risks – all these together ensure a high degree of confidence in the insurer. But that is not the case with cat risks. Per se it is unmanageable for a single insurer or in some cases even a pool of insurers might find it hard to carry it. Nevertheless, it has to be managed. The business of risk is the raison d'étre of an insurer.

Is it a freak of Nature that we have to expect once in a way catastrophes? Do they follow a pattern? Do they have any law – in other words is their behavior deterministic? Are they purely random events following some arcane statistical distribution? To be pedantic, are they stochastic?

Meteorologists almost every time predict the weather wrong enough to be ludicrous, but people put faith on them for want of anything better. When they say it is going to be rains, Nature is determined to delay the rains. When they say it is going to be a dry spell, sardonic Nature defeats them with a sporadic or even plentiful precipitation. But rains are no catastrophes and so we may not be much bothered. But what about hurricanes, tornadoes, earthquakes, gigantic landslips?

In 1935 Charles Richter in association with Beno Gutenberg developed a scale for measuring the intensity of earthquakes based on their study of Californian earthquake records. The apparatus used is called a torsion seismometer the invention of which is credited to Wood and Anderson. This then can be regarded as the first serious steps to study and measure earthquakes leading to the search for the predictability of such catastrophes that vex humanity. Gutenberg and Richter have proposed an empirical law of earthquakes which is quite simple.

$$\text{Log } N = A - b^*M$$

M refers to the magnitude of the earthquake and N refers to the number of events expected that falls within the magnitude range defined by M in a given region and duration. A and b are constants determined by applying past records of earthquakes.

The magnitude of the quake is determined by the amplitude of the displacement of the needle against the torsion of the material on which it is fixed in the Wood-Anderson seismogram. It seems Richter took cues from Astronomy. Stellar brightness is one way of classifying stars. Hipparchus in circa 100 BC introduced a scale for measuring observed brightness of stars to classify them as first, second and so on up to 6th magnitude stars. The first magnitude constituted the very bright stars visible, the second less bright and so on until those dim stars that could be just viewed by naked eye graded as 6th magnitude. This is now recognized as a logarithmic scale with base 10 and still applied with modification. The magnitude scale adopted by Richter to measure earthquakes is similar.

The impact recorded at a local seismometer from a quake depends on the distance of the station from the epicenter of the quake. The greater the distance of the station, the less is the amplitude of the waves. Richter's original empirical law captures the relationship between epicenter distance and the magnitude of a quake as:

$$M = \log_{10}A - \log_{10} A_o\delta$$

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where A is a constant $A_o\delta$ = a constant dependent on epicenter distance of the station.

Since a logarithmic scale is in operation for measurement of earthquakes, a quake put at 4 on the Richter scale would be 10 time stronger than one that was 3 on the same scale. Logically, the destructive power of an earthquake will have to be in proportion to the energy released by the quake. Studies have shown that in terms of energy released and therefore the destructive power of a quake put at 4 on the Richter scale would be 31.6 times more than a quake which measured 3 on the scale.

More refined methods of measurement of quakes are now available. The Japanese use a formula different from the Gutenberg-Richter proposition for frequency of occurrence with a modified scale for measurement of intensity. Today we have thus a good lot of information on earthquakes and some strong credible relations that explain how frequency and intensity are connected. In spite of such extensive researches and findings, they do not provide a basis for prediction in a way suitable for rating the risk of earthquakes.

Some idea about the frequency of occurrences can be had from the following table.

Richter Magnitude	Nature	Effects felt	Annual frequency
Less than 2.0	INSIGNIFICANT	Never felt	2920000
2.0 – 2.9	MINOR	Never felt but recorded	365000
3.0 – 3.9	MINOR PLUS	Felt but rarely causing any damage	49000
4.0 – 4.9	MILD	Noticeable, shaking of doors, rattling noise –little damage	6200
5.0 – 5.9	MODERATELY SEVERE	Major damage to ill equipped buildings	800
6.0 – 6.9	SEVERE	Can be destructive over a radius of 100 miles in populated areas	120
7.0 – 7.9	STRONG	Serious damage over larger areas	18
8.0 – 8.9	VERY STRONG	Serious damage in areas several hundreds of miles across	1
9.0 – 9.9	GREAT	Devastating thousands of miles across	0.05

The frequency of occurrence and the destructive power though correlated with the Richter scale, for an insurer it gives only a very broad idea as to what would constitute a catastrophe. A Scale-6 tremor can be a catastrophe if the area hit is a metropolitan city. But a Scale-9 or even higher earthquake under the Pacific may pass off as just another event unless it produces a Tsunami like the one that devastated large populated areas following the Sumatra quake of 2007.

Today we have means to detect an asteroid straying into the gravitational field of the earth raising the potential danger of a collision. We become aware of this approaching catastrophe much before the event strikes and so have time enough to take defensive maneuvers. Perhaps a gentle nudge of the flying peril by a rocket fired towards it could deflect its course to avert the impending doom. But nearer earth, just a few thousand metres below the ground, we are not yet fully equipped to detect the emerging earthquake. We know that tremors of insignificant or mild scales strike the earth almost every hour, but which one is the harbinger of the holocaust in the making, where will it eventually strike, that we do not know.

The protagonists of Chaos Theory believe that seemingly insignificant and unnoticed quakes could be the cause of a major quake. Way back in 1960 a meteorologist Edward Lorenz was working with his weather prediction models in his computer. When a few initial inputs were given to his computer it produced a prediction pattern by applying the deterministic equations that work on these initials inputs. Lorenz got these patterns and numbers printed by the computer which used the results following the initial inputs, to produce a second set of new inputs. The resulting numbers now became the initial inputs for the next prediction and so on. One day he wanted to look at a particular pattern resulting from a given set of initial inputs. He fed these inputs into the computer expecting to see the same pattern that emerged from such inputs previously. Surprisingly, the pattern he now saw was entirely different, the inputs being very nearly the same how the pattern could be so wildly varying! An insignificant difference in the initial inputs producing such a widely different output is incredible – chaotic.

The equations on which the computer crunched the weather related numbers were deterministic and so the resulting graph should be expected to be identical if the inputs were the same. If the inputs were a shade different, the outputs must be similar tracing closely the previous outputs. But Lorenz noticed an entirely different pattern for a very slight, insignificant deviation in the initial input data to the computer. Why so?

The Chaos theory advanced by many scientists postulates that insignificant small changes seen now can produce violent upheavals in due time. The flapping of a butterfly's wings in one remote corner of the globe could produce a devastating tornado in another locale. Huge violent effects we experience in atmosphere in the shape of hurricanes or on ground in the form of Scale-7 or higher earthquakes could be the results from initial conditions. This is the basis of Chaos Theory, the sensitive initial conditions lead to unexpected large scale erratic experiences.

We can experiment with this phenomenon in a computer. Suppose the value of some phenomenon ranges between 0 and 1. Suppose that the value changes to some value in this range in successive identical time intervals, say a year. Let the change in the value in the next year is expressed by the equation:

$$X_{t+1} = k * X_t * (1 - X_t)$$

where X(t) represents current year value and X(t+1) that value in the next year and k is some constant defining the initial conditions of these changes. You can run the successive values following an initial value for X and k in an Excel Worksheet. It will be observed that for low values of k say less than 3, the sequence converges to some value, but once you raise the value beyond 3 the values jump between two different convergences. Further increase could produce 4 such convergences, 8, 16 and so on and when the k value is quite high, the worksheet produces values that have too many convergences and the situation looks chaotic as we are unable to separate these by inspection.

What does this signify? A population developing over time based on some deterministic law can break down into two populations of a similar nature when some initial aspect (in our above equation the constant k) takes a shade higher value. For still higher values the replication of the population breaks down into four similar populations and in such a geometric bifurcation of the population for higher values the entirety of the population becomes a mix of many populations. After a certain stage of the development of this population if we take successive values of the population these would look erratic and appear to be random. But values flowing from a deterministic equation cannot be regarded as random.

Perhaps many of the observations we are witnessing today on phenomena like earthquakes, hurricanes etc may be the result of certain definite initial conditions that existed long ago leading to these values under some as yet unraveled deterministic law. No wonder however much we try to fit a law for these phenomena based on immediate past data, the predictions based on such empirical laws proves futile as Nature has something else to say based on its deterministic law. On other hand if we presume that the immediate past data we have with us on a phenomenon are but random values resulting from some stochastic law, then also we fail, because no reasonably matching mathematical models could be found. No wonder either then even those probabilistic predictions based on such stochastic models therefore lack credibility.

But some advances have been made on the so-called Chaos theory. Concepts such as sensitive dependence on later events on even minor changes in initial conditions are gaining greater credibility. The rate of self multiplication into different populations out of initial conditions is believed to remain constant. Even though the rate is constant, the magnitude of the change occasioned at a future time may be quite high, but shape and structure of the phenomenon will be retained.

If Chaos theory advances throw light on what to expect from the current experience during a time in the near or distant future, what we call cat risk becomes better manageable by doing something now to change advantageously the current conditions that might either eliminate or at best postpone the calamity to a more distant time. One is reminded of the concept of "Expanding Funnel of Doubt" introduced by the celebrated British actuary, Frank Redington. Perhaps he had the suspicion that the small changes in the conditions existing now are not really insignificant as the future we look at is farther and farther still.

Birth Centenary of Shree K A Pandit

Shree Kantilal Anandray Pandit

[12- 8 - 1909 to 22 -1 - 1993]

IAI remembers Shree Kantilal Anandray Pandit, FIA 1940 as a founder member of ASI (now IAI) (1945) on his birth centenary on 12-08-2009