Eckler Ltd.
Consultants and Actuaries

M/S. K.A. Pandit
Consultants and Actuaries

Use of Stochastic Techniques to Estimate the Cost of Minimum Interest Guarantees

A Case Study

Category: Life Insurance

A Short Paper Presented at the

9th Global Conference of Actuaries

12th-13th February 2007

Mumbai, India

by

Sylvain Goulet, fcia, fsa, maaa

Affiliate Member of the (British) Institute of Actuaries

and

Stéphane Lévesque, fcia, fsa

Table of Contents

1. Overview

There are essentially two important aspects of dealing with a product in which the insurer guarantees a minimum level of investment performance. The first aspect is to understand the risk, to determine its potential cost, and essentially determine the sensitivity of the various market forces and how the policyholder reacts to them. The second aspect is to mitigate these risks and how to deal with them, avoid them, control their cost, use hedging techniques, etc.

In this paper, we are principally concerned with the first aspect: we want to quantify and understand the risks and want to identify potential pitfalls.

The risk associated with investment fund guarantees is characterized by low frequency and potentially high severity costs. Over the last few years, much research has been done to better understand and evaluate this unique and significant risk. In Canada, the use of stochastic techniques is recommended as appropriate actuarial practice to measure the obligations created by products with investment guarantees.

To illustrate the process, we present a case study using an actual investment product, which we will call the Guaranteed Investment Plan or "GIP". The types of guarantee being offered include minimum guaranteed maturity benefit ("MGMB") and minimum guaranteed death benefit ("MGDB").

The potential risk created by investment guarantees offered are evaluated and specific recommendations made to mitigate the risk for the Company without causing prejudice to policyholders. We will use stochastic modeling techniques on the basis outlined in the *CIA Task Force on Segregated Fund Investment Guarantees, March 2002 ("CIA Report")* with some adjustments to take into consideration the specific characteristics of the product and investment market. This paper deals exclusively with policy liabilities without consideration for minimum capital requirements, which is of course important but beyond the scope of this paper.

2. Stochastic Projections

Policy liabilities may be projected by the deterministic method or the stochastic method. For the more traditional life insurance products being offered today, the traditional deterministic approach of setting assumptions and calculating policy liabilities is still used.

However, one could argue that even for traditional products, actuaries should be determining costs using stochastic techniques rather than deterministic techniques. The advantages of using deterministic techniques are speed, relative simplicity of the method, predictability of and smoothness of results, easily verifiable calculations, and so on. The scope of this paper does not include elaboration of these points.

The stochastic approach requires much more work. Its objective is to generate policy liabilities that will be adequate to represent the benefits in a majority of cases or for a certain percentage of the cases.

2.1 Why Is the Stochastic Approach Appropriate for an Investment Guarantee Product?

Under a traditional product structure, benefits are usually eventually paid, whether it is a death benefit, a surrender benefit, or some other (no benefit being considered a benefit of $0). Under term policies, there is usually no surrender benefit. However, mortality rates being relatively predictable, the expected death benefits can be accurately projected if there are a sufficient number of policyholders covered.

Under a product with investment fund guarantees, if investment returns decline to a point where the fund is below the guaranteed or reset amount, then in all likelihood a death, surrender or maturity benefit will be paid. However the fund could easily keep above the minimum guarantees, and no benefits at all might ever be paid out.

2.2 Why Is the Deterministic Approach Not Appropriate for an Investment Guarantee Product?

Mortality rates are fairly predictable, and lapse experience is usually easily determined as well. Using such averages is an accepted method for traditional products because it is expected that over a large number of insureds, lapses and deaths will occur as predicted. The average return of virtually any fund over a long period of time has usually been at least greater than the Management Expense Ratio ("MER") charged to the fund and consequently if we were to use such averages there will be absolutely no cost at all for most guarantees, which of course is not appropriate.

This type of risk is much like a catastrophe risk, like the risk of a hurricane or an earthquake. Damages caused by a hurricane could be $0 in one year and $100 million the next year. A traditional average cost will not work.

That is why the stochastic approach is a more appropriate method to determine not only the potential cost but also the distribution of the costs. Having determined under the stochastic method that the average cost is, say, 50 basis points, it is important to know at what level is the 95th percentile for example. If it is at 55 basis points, then there is no great variation. If it is at 200 basis, then the risk profile is much more significant. Hence the volatility is also an important measure of the risk involved.

3. Product Description

3.1 In Force Business

The Guaranteed Investment Plan was sold from 1990 to 2005, including three different product generations with revisions in 1995 and 2002.

As of December 2005, there were approximately 1,600 GIP policies with a total fund value of $7.3 million. The following table shows the in force business distribution by product generation:

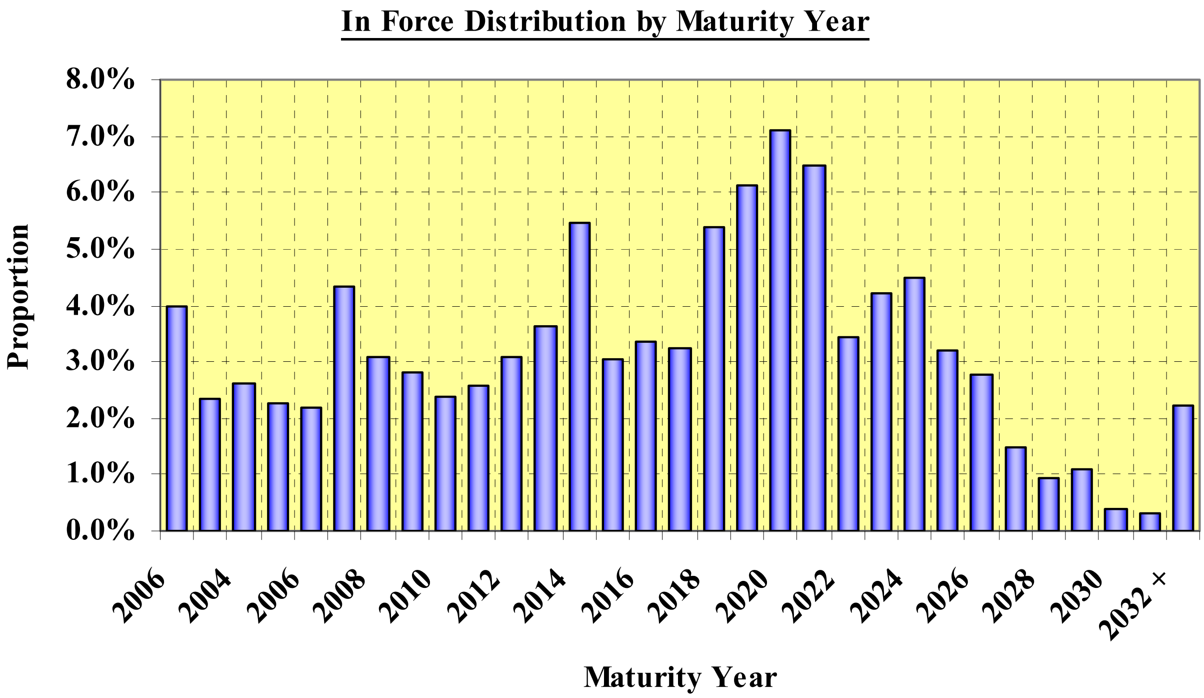
GIP Generation	Period	Number of Policies (#)	Number of Policies (%)
"A" Series	1990 – 1995	688	42.4%
"B" Series	1995 – 2002	626	38.6%
"C" Series	2002 – 2005	307	19.0%
Total		**1,621**	**100.0%**

The GIP provides investment guarantees comparable to segregated fund products offered by insurance companies in Canada and the United States; however, some features of the plan are materially different. Although sold as an investment plan, the GIP is essentially designed and priced like a life insurance product. We will review in this section the specific characteristics of the GIP.

3.2 Policy Maturity

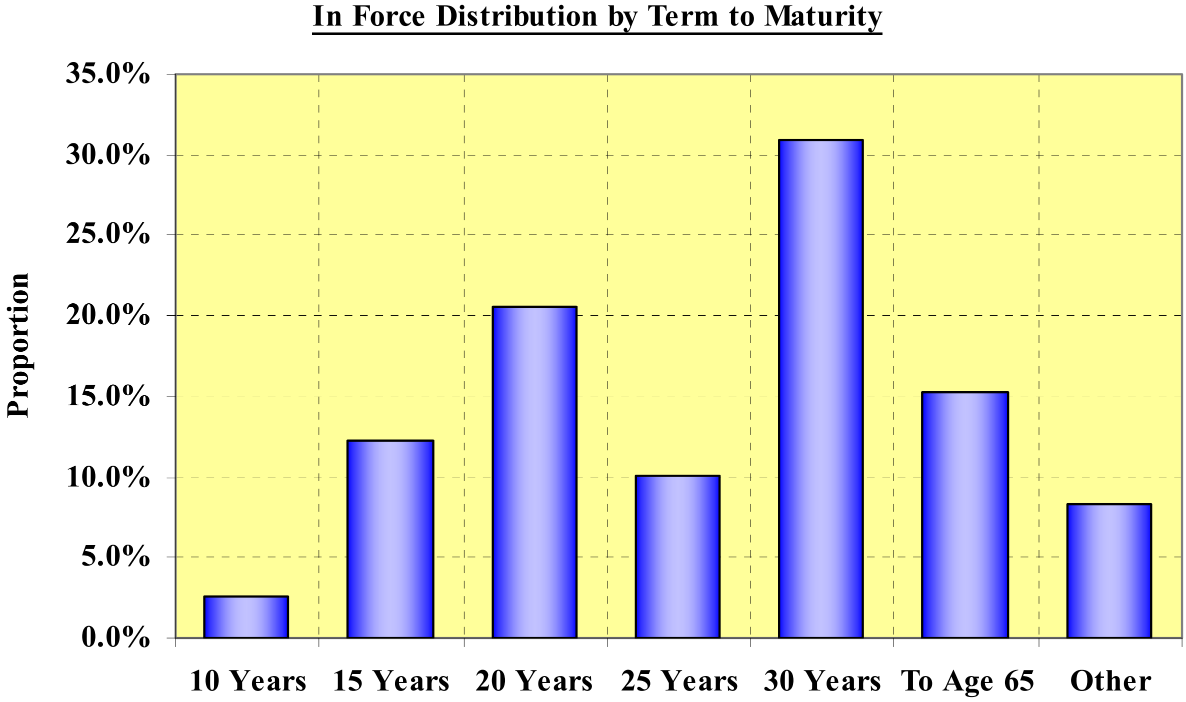
Unlike most of the segregated funds in the market, the GIP maturity date is fixed and determined at inception of the policy and there are no reset features allowing the policyholder to change the maturity date. Maturity dates range from 10 to 30 years or to age 65.

The graph below shows the distribution of the GIP fund by maturity year.



About 30 percent of the business will mature within the next 10 years, 45 percent between 11 to 20 years and 25 percent after more than 20 years.

The following graph shows the distribution of the GIP policies by original term to maturity.



Most of the policies have a term to maturity of 20 years or more, and less than 3 percent have a term to maturity of 10 years. This distribution should be favorable in terms of the cost associated with investment guarantees payable at maturity.

3.3 Investment Premiums

The GIP premiums are fixed and payable for the term of the policy, which ranges from 10 to 30 years or to age 65. The portion of premium applied to purchase units of the GIP fund is equal to the gross premium less front-end loadings. The table below illustrates the premium loadings by GIP generation and policy year:

GIP Generation	Premium Loadings by Policy Year			
	1st Year	2nd Year	3rd Year	Total
"A" Series	80.0%	75.0%	25.0%	180.0%
"B" Series	55.0%	45.0%	20.0%	120.0%
"C" Series	50.0%	0.0%	0.0%	50.0%

During the first three years, premium loadings were high on the "A" and "B" GIP series, primarily to cover high commissions paid on the plan. As shown in the table below, the GIP commission structure is similar to a life insurance product, which is very unusual and expensive for an investment plan.

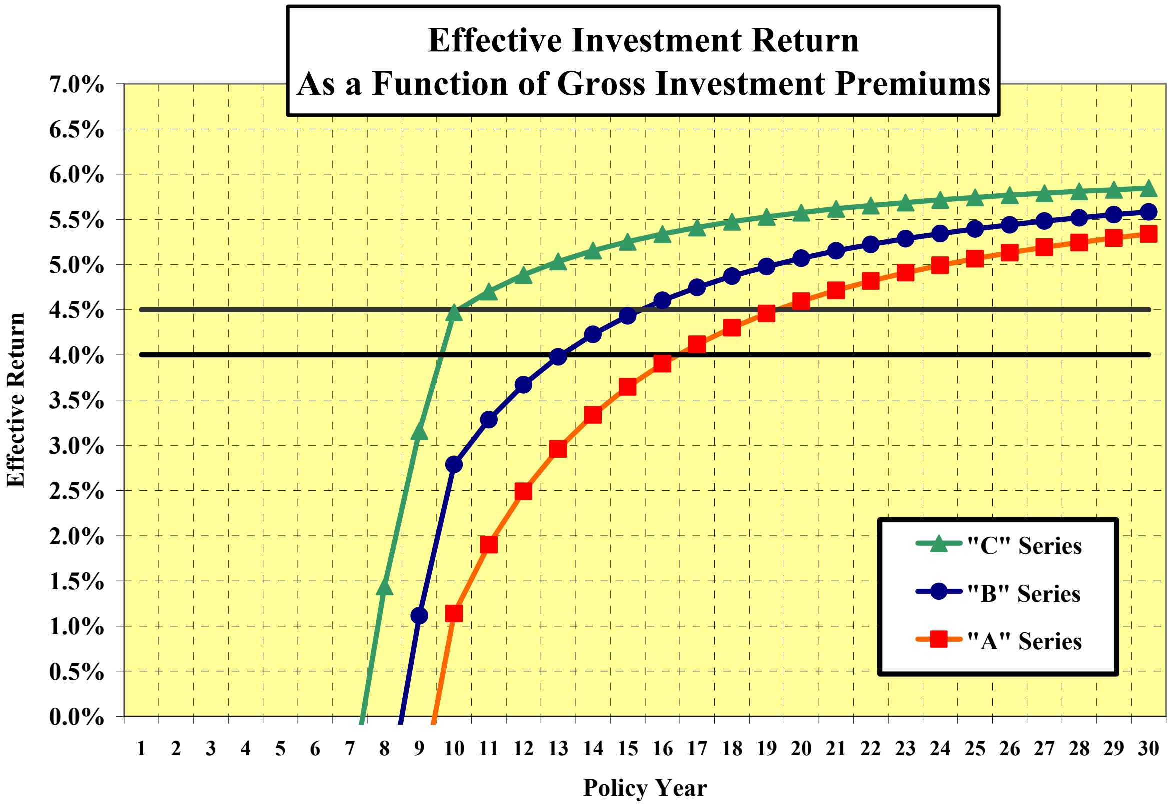
GIP	Commission Percentage by Policy Years			
Generation	1st Year	2nd Year	3rd Year	Total
"A" Series	60.0%	20.0%	10.0%	90.0%
"B" Series	30.0%	10.0%	5.0%	45.0%
"C" Series	30.0%	0.0%	0.0%	30.0%

The product has been re-priced twice to make it more competitive by reducing both the premium loadings and commissions. However, the "A" and "B" GIP series still represent 70 percent of current in force business. On voluntary surrender, or contract maturity, policyholders are complaining that the investment return realized on their gross premiums paid is extremely low.

The problem created by high premium loadings is worsened by the fact that the GIP fund did not perform exceptionally well over the past years, averaging 6.20 percent annual return, which is not sufficient to overcome the premium loadings.

To illustrate the impact of premium loadings, we calculate the effective investment return realized as a function of gross premiums paid. The return is calculated using an average increase of the GIP unit values of 6.20 percent per annum.

The graph below shows the results by GIP generations:



During the first 10 years, the return is negative for the most part because of the surrender charges and the short period of time to overcome the premium loadings. At the 10th policy anniversary, average return varies from 1.1 to 4.5 percent. It takes between 15 to 20 years for the "A" and "B" GIP series respectively to reach the guaranteed return of 4.5 percent, even if the GIP unit values increase at 6.20 percent over the same period. It takes approximately 10 years for the "C" GIP series to reach the guaranteed return of 4 percent.

3.4 Investment Fund

The net investment premiums are used to purchase units of the GIP fund and the return is directly linked to the fund performance. The GIP investment fund is not segregated from the company's assets; unit values are calculated using a notional fund. This characteristic of the GIP is very important because it gives the Company an opportunity to modify the asset composition without any restrictions.

As of December 2005, the GIP fund was composed of the following assets:

Assets	Amount	(%)
Cash and Fixed Deposit	1,279,882	17.6%
Government Bonds	654,485	9.0%
Residential Mortgage	189,073	2.6%
Stocks / Mutual Funds	1,745,293	24.0%
Policy Loans	1,039,904	14.3%
Inter Company Note	2,363,417	32.5%
Total GIP fund	**7,272,054**	**100.0%**

Historically, fixed income assets have represented between 60 to 80 percent of the assets allocated to the GIP fund.

3.5 Unit Price

At the end of each month, the Company calculates the value of the GIP bid price by dividing the total value of the GIP investment fund by the number of units outstanding. All the benefits provided under the GIP policy are calculated using the bid price.

For the purpose of allocating new units to GIP policies, the Company determines the GIP offer price, which is equal to the bid price multiplied by an adjustment factor of 100/97. The price adjustment factor is equivalent to charge a 3 percent front-end load on the gross premiums. For the "A" and "B" GIP plans, the offer price adjustment is defined in the contract at 100/95, but the Company is currently using 100/97 on the entire portfolio. For the "C" GIP policies, the Company has the right to change the rate as long as it falls within 100/95 to 100/100.

3.6 Investment Guarantees

The GIP offers minimum investment guarantees payable upon maturity or policyholder death. On maturity, the Company guarantees a minimum rate of return of 4.5 percent per annum for the "A" and "B" GIP generations, and 4 percent for the "C" GIP generation. On policyholder death, the Company disburses the greater of the GIP fund value and the total gross investment premiums paid.

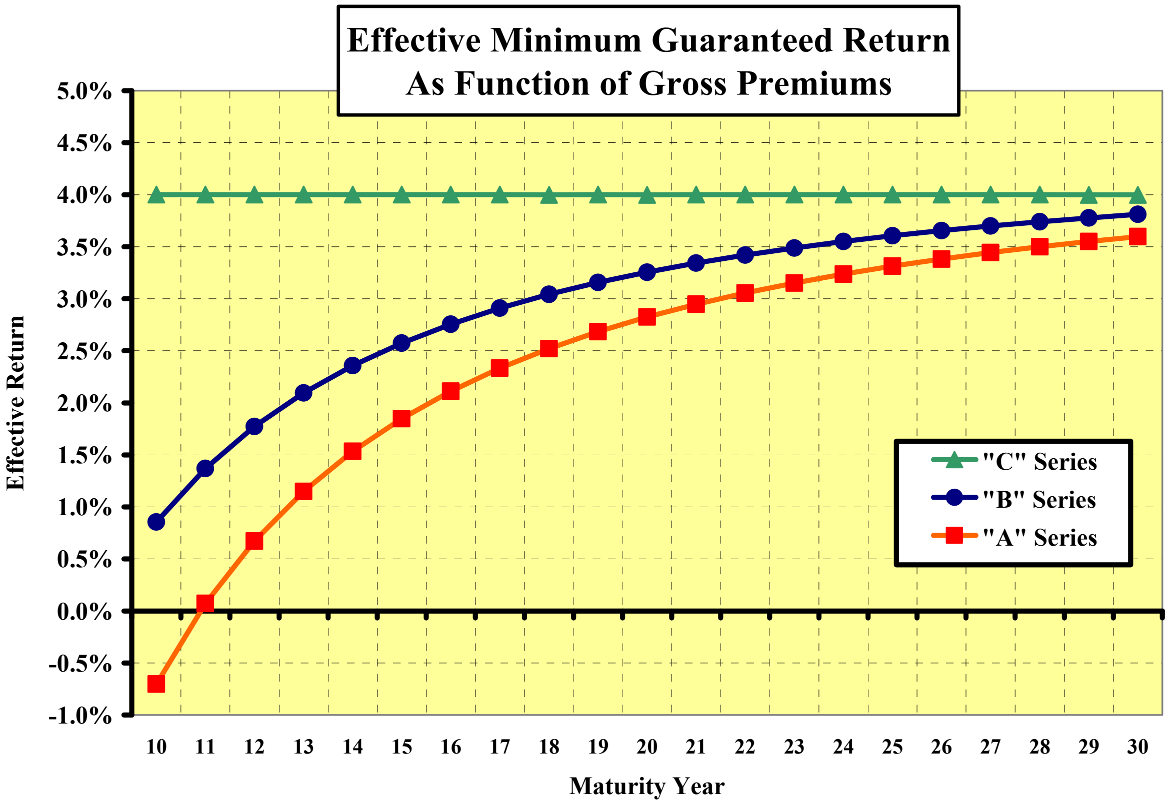
The critical issue in setting the reserve for the cost associated with the GIP maturity guarantee is to determine the premium amount that should be used to calculate the minimum guaranteed value and when it should apply, which could have a significant impact on the policy liabilities. In order to determine the appropriate base, it is important to review the GIP contract, illustrated guaranteed values, past marketing practices and current administrative procedures.

3.6.1 GIP Contract

For the first two GIP generations ("A" and "B" series), the guaranteed amount payable at maturity is shown on the policy specification page. However, the minimum guaranteed interest rate of 4.5 percent and the calculation method are not disclosed in the contract. Based on the illustrated guaranteed values, minimum maturity benefit is calculated using net investment premiums, after premium loadings, and unit allocation based on GIP offer price with adjustment factor of 100/95. Also, nothing in the contract suggests that the guaranteed amount is payable before maturity.

For the last GIP generation introduced in 2002 ("C" Series), the objective of the product review was to ensure that each policyholder receives at least 4 percent return on the gross premiums paid should they remain in the plan for 10 years or more. To achieve this goal, guaranteed values are calculated using the gross premiums, without premium loading and offer price adjustment. The contract wording was also reviewed with the objective to provide for a guaranteed value on surrenders occurring after the tenth policy anniversary, irrespective of the contract maturity date.

The graph below illustrates the effective minimum guaranteed return as a function of the gross premiums for terms to maturity between 10 to 30 years:



The effective minimum guaranteed returns as a function of gross investment premiums are significantly lower than 4.5 percent for the “A” and “B” GIP series. This shortfall is caused by the premium loadings and offer price adjustment factor. Because of that, the effective minimum guaranteed returns never reach 4.5 percent.

3.6.2 Current Administrative Practice

After the 2002 revision of the GIP product, the Company implemented a new administrative procedure in regard to payment of minimum guaranteed values. It was decided at that time that the Company shall pay the minimum guaranteed amount on voluntary termination after the tenth anniversary, irrespective of the maturity date. Additionally, the guaranteed value was calculated using the gross investment premiums, without premium loadings and offer price adjustment factor, independent of the guaranteed value shown on the GIP contract.

The decision was made in response to an increasing number of dissatisfied policyholders complaining that the earnings on their policies were very low and, in many cases, benefits received were lower than the total gross premiums paid. As previously indicated, this was due to high premium loadings applied to the gross premiums and insufficient amount of time to build up the fund to overcome the charges.

3.7 Surrender Value

An additional source of dissatisfaction with the GIP is the high surrender charges applicable on voluntary termination. The policy does not have any surrender value during the first three years and the following surrender charges are applicable on the net fund value after the third policy anniversary:

Policy Year	Surrender Charges	Policy Year	Surrender Charges
1st Year	100.0%	6th Year	25.0%
2nd Year	100.0%	7th Year	20.0%
3rd Year	100.0%	8th Year	15.0%
4th Year	35.0%	9th Year	10.0%
5th Year	30.0%	10th Year	5.0%

The surrender charges are not included as a source of revenue in the stochastic model, taking the conservative approach that these charges are used to recover past acquisition expenses. Therefore the surrender charges do not have any impact on the cost of investment guarantees.

4. Stochastic Modeling

As mentioned earlier, traditional actuarial valuation methods are inappropriate to estimate the policy liabilities for the risk associated with products with investment guarantees. The stochastic modeling technique is used to properly estimate the cost. In this section, we describe the various steps in developing a stochastic model.

4.1 Random Number Generator

The first step in stochastic modeling is to generate a sequence of random numbers. There are many algorithms available to simulate pseudo-random numbers. For this paper, we have developed a random number generator using the *linear congruential method*. Appendix A.1 provides a detailed description and analysis of the generator.

Before using the generated sequence of random numbers, we must make sure that the series is adequate for the stochastic model. Some requirements that need to be met:

4.1.1 Periodicity of the Generator

The random number generator must have sufficiently high periodicity, which is defined as the number of values that can be produced by a generator before the sequence repeats. The periodicity depends heavily on the choice of the parameters and seed value used with the generator, so careful considerations must be given in choosing them. There are many widely used and well-tested generators that provide very good results with high periodicity. Our generator produces a sequence of random numbers with a periodicity of $2^{31} - 2$, which is substantially higher than the number of random deviates needed for this paper.

4.1.2 Results Should Be Reproducible

The numbers generated by our algorithm are called pseudo-random because they are not truly random. If the generator is run with the same parameters and seed value, it should always generate the same sequence of random numbers.

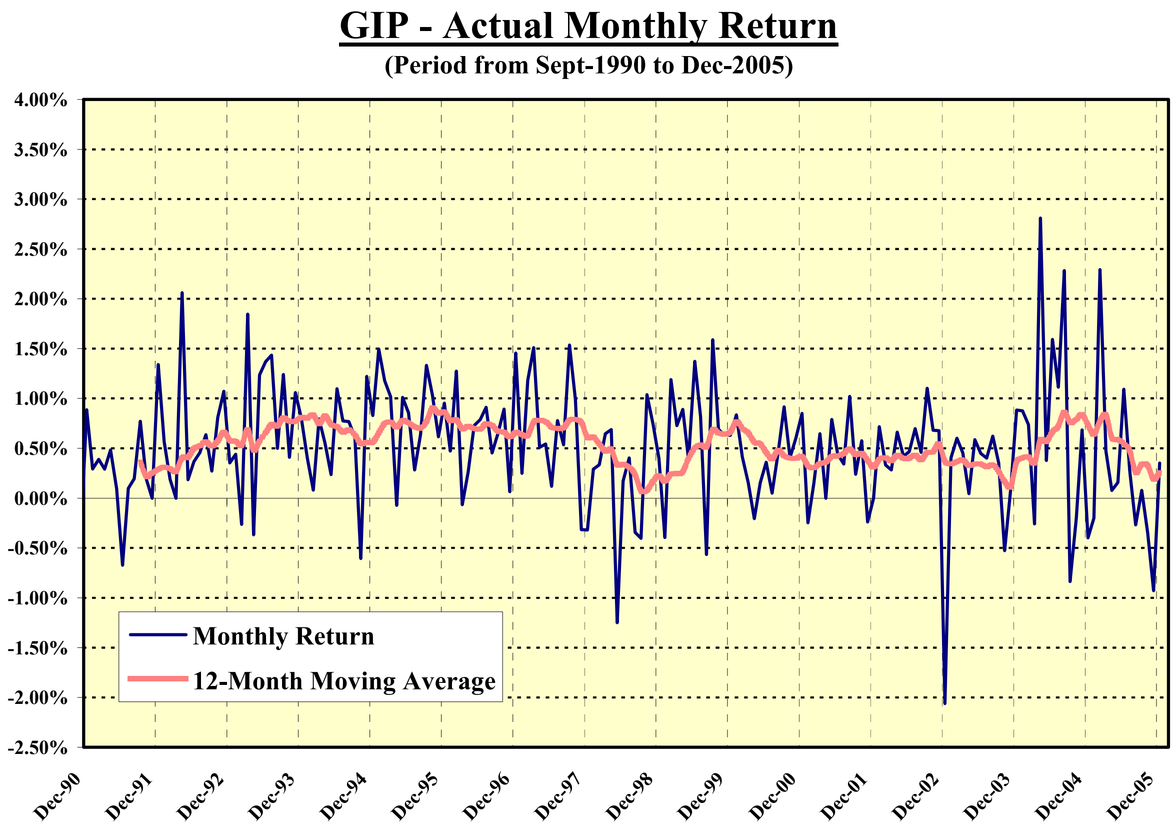
4.1.3 Generator Should Not Exhibit Any Bias

The generator must produce a sequence of random numbers that follow the assumed distribution; in this paper, the objective is to have a sequence of random numbers with a Normal distribution. Statistical testing can be done to validate the distribution. Appendix A.1 illustrates the statistical testing done to validate the random number generator.

4.2 Selecting an Appropriate Proxy

The next step is to build a theoretical proxy replicating the specific characteristics of the GIP fund in term of asset mix, return and volatility. The benchmark index should normally be a combination of recognized market indices, rather than the specific fund performance. In the case of the GIP fund, it is very difficult to build a benchmark index because historical market performances are limited or not readily available.

As a practical alternative, we decide to estimate the investment model parameters using the GIP fund historical performance. We believe that this approach produces reasonable results. The historical monthly closing prices are available from September 1990 to December 2005, and the graph below illustrates the historical monthly return of the GIP fund and the 12-month moving average over that period.



4.3 Investment Return Model

A key component of any stochastic simulation is the model used to generate investment return scenarios over the projection horizon. The number and diversity of investment return models available are impressive, but no model provides a perfect fit to historical data. Since the objective of this paper is not to determine which model better reproduces the type of distribution describing the market return, we used the lognormal model which is a widely used model in the financial markets. The model is very simple, easy to implement and reasonably fits historical financial market data.

The lognormal model is based on the assumption that a normal distribution of the market returns is equivalent to a lognormal distribution of market prices. If the market return over the period (Δt) is denoted r_t, then:

$$\text{Market Return } (r_t) = \text{Log}\,(S_{t+1} / S_t)$$

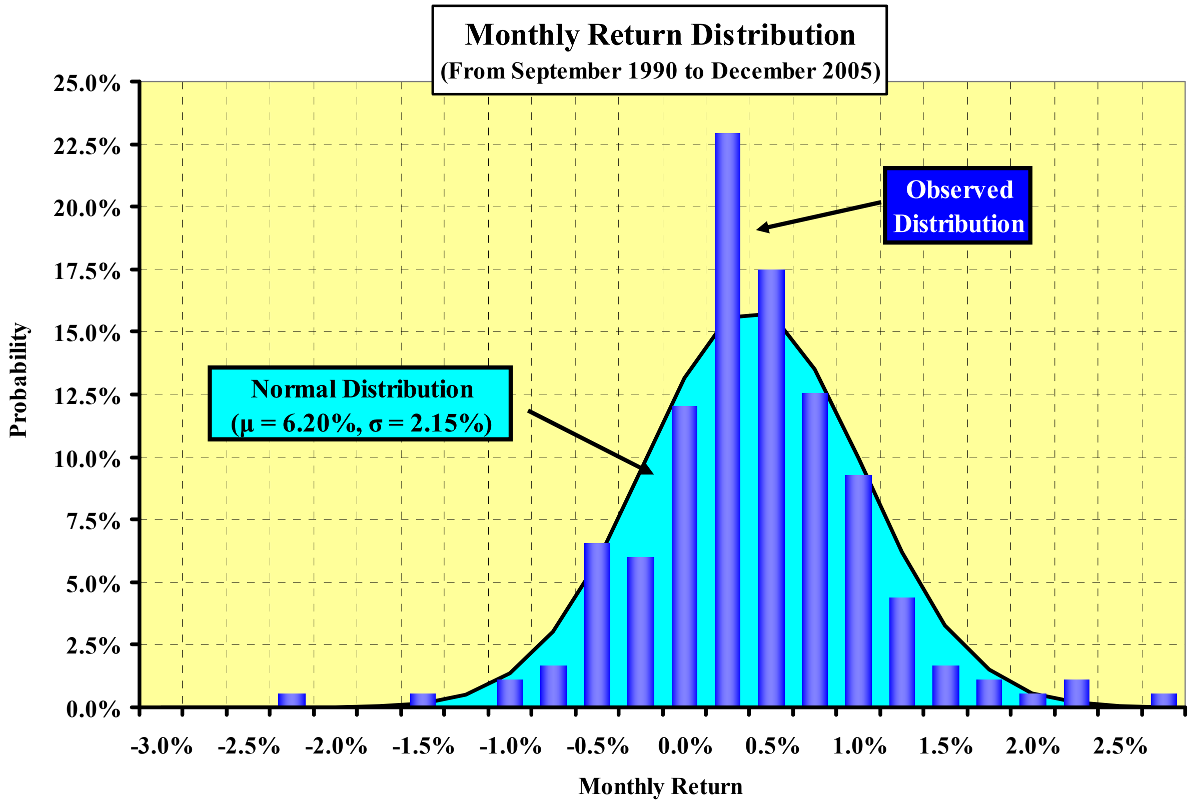
and,

$$\text{Market Return } (r_t) \sim \text{Normal}\,[\,\mu - \tfrac{1}{2}\sigma^2(\Delta t),\ \sigma^2(\Delta t)\,]$$

where,	r_t	is the market return
	S_t	is the market price
	μ	is the expected return, also called the drift
	σ	is the standard deviation, also called the volatility
	Δt	is the time period

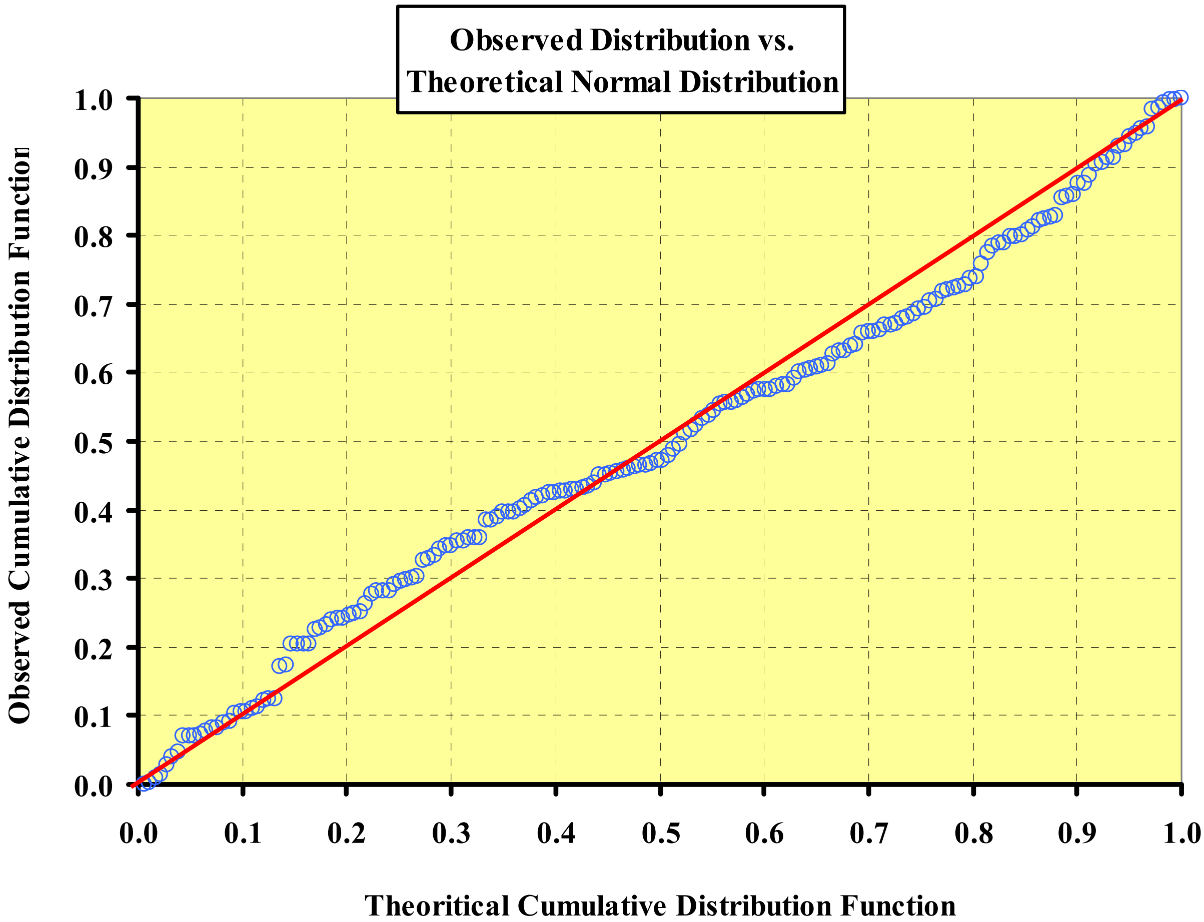
The investment model parameters μ and σ are estimated using historical return and volatility of the asset class being modeled.

The histogram below illustrates the distribution of the GIP monthly returns over the period of September 1990 to December 2005. A Normal distribution is superimposed using the observed mean and volatility of the data.



The histogram is an effective graphical technique to characterize the distribution of a dataset. A quick look of the above graph indicates that there is a reasonable fit between the observed dataset and a normal distribution. Also, we can see that the investment returns are well concentrated around the mean of the distribution.

Another simple visual test to check for the fit of a dataset distribution is to plot the observed cumulative distribution function against the theoretical cumulative normal distribution. If the theoretical cumulative distribution approximates the observed distribution well, then most of the points should fall onto the diagonal line as shown on the graph below:



The fit is not perfect but it is close enough to support our assumption that the GIP monthly returns are normally distributed. Detailed statistical analysis is required to measure precisely the degree of fitness between a dataset and a theoretical distribution, which is beyond the scope of this project.

4.4 Model Parameters Estimation

The next consideration in constructing the investment return model is to estimate the model parameters μ and σ based on historical data of the selected proxy. Using the GIP monthly returns, we compute the sample mean and sample standard deviation:

- Sample mean: $\hat{r} = \mathbf{\textit{0.5149\%}}$
- Sample standard deviation: $\hat{\sigma} = \mathbf{\textit{0.6208\%}}$

Next, we convert these values to annual return (μ) and volatility (σ) by applying the following formulas:

- $\sigma = \hat{\sigma}\sqrt{12} =$ ***2.1506%***
- $\mu = (\hat{r} * 12) + (\frac{1}{2}\sigma^2) =$ ***6.2025%***

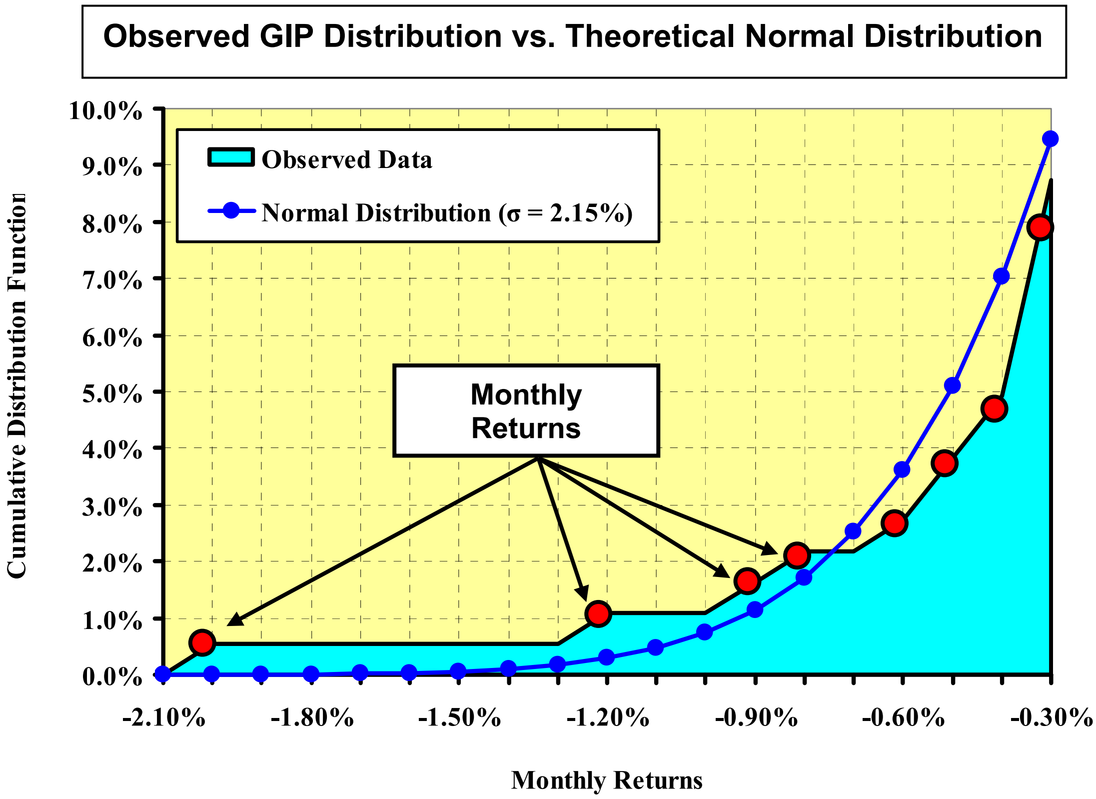
Before adjustment for calibration, our investment return model has an annual return of 6.20 percent and a volatility of 2.15 percent. The following table shows the GIP return (μ) and volatility (σ) over different periods based on monthly historical closing prices:

Period	Mean (μ)	Volatility (σ)
Sept. 1990 – Dec. 1995	7.42%	1.78%
Jan. 1996 – Dec. 2000	6.16%	1.92%
Jan. 2001 – Dec. 2005	4.86%	2.56%
Sept. 1990 – Dec. 2005	**6.20%**	**2.15%**

It is interesting to observe that the volatility of the GIP fund increased since 1991, while the average return over the same period declined. It is difficult to explain such a pattern. However, the model parameters should be based on historical data as opposed to the recent market performance.

4.5 Investment Return Model Calibration

Earlier in this section, we noted that the normal model fit the GIP historical returns relatively well. Since the risk of investment guarantees is concentrated in the left tail of the distribution, fitting of that portion is more important and has more impact than the general shape of the distribution. A closer look at the left tail of the distribution indicates that the theoretical model fails to reproduce the fatness of the observed distribution as shown in the graph below:



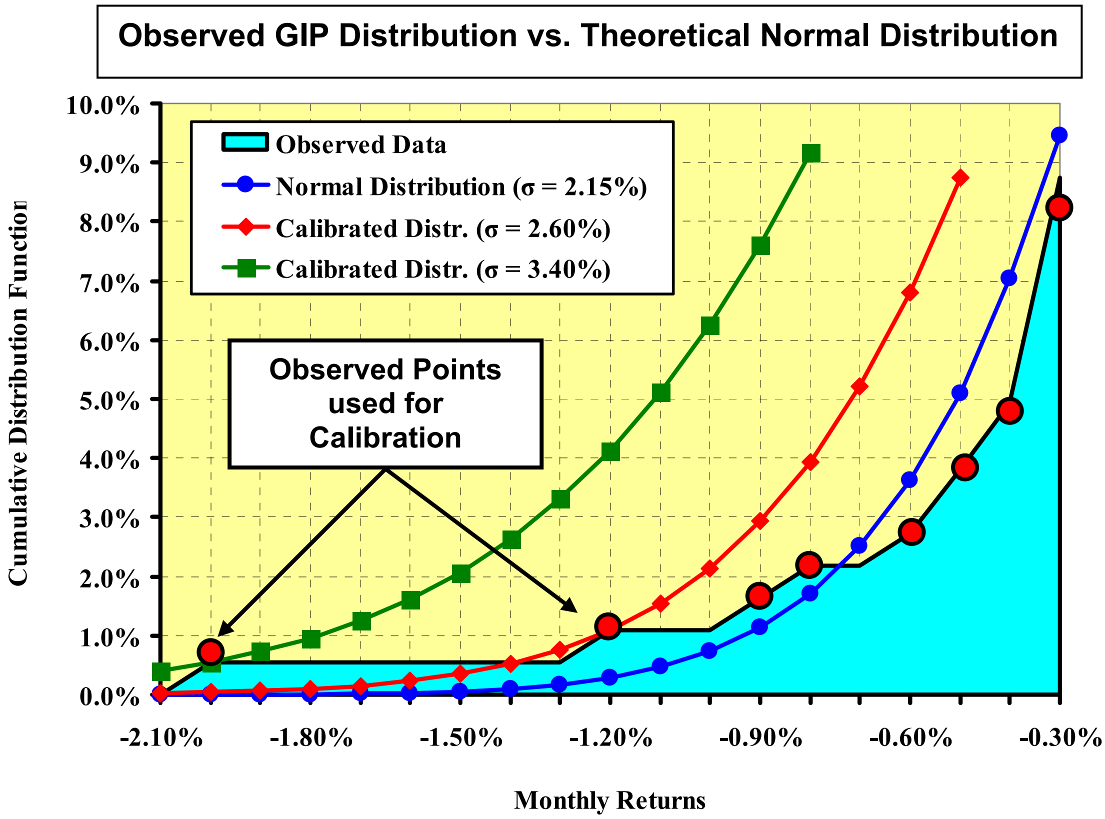
We can see on the graph that the normal model produces lower probability compared to the observed distribution for the first four observations. The model should therefore be adjusted to better fit the left tail fatness of the observed distribution.

The calibration ensures that the model generates scenarios that take into account the pattern of the left tail observed in historical data. Development of calibration criteria specific to the GIP fund is beyond the scope of this paper, thus we will use other simple techniques to improve the fitting between the left tail of the model and the observed data.

To calibrate the model, we change the volatility parameter (σ) to increase the probability in the left tail. The calibrated parameters are established to produce the same cumulative probability at two different levels:

- We increase the volatility (σ) from 2.15 to 2.60 percent to have the same cumulative probability at the second observed monthly return;
- We increase the volatility (σ) from 2.15 to 3.40 percent to have the same cumulative probability at the first observed monthly return.

The graph below superposes the two calibrated curves with the historical data and the original distribution:



The second calibrated model with volatility (σ) at 3.40 percent produces an overly conservative distribution. I used the first calibration with volatility (σ) at 2.60 percent, which fits reasonably well the left tail of the observed distribution with some degree of conservatism.

5. Policy Liabilities

5.1 Methodology

Policy liabilities associated with the investment guarantees are calculated using stochastic techniques as described in the *CIA Report*. Under this technique, the stochastic model is used to generate multiple investment return scenarios and estimate the liability by projecting the costs and revenues for each generated scenario. All other contingencies, such as mortality and surrender, are set on a deterministic basis using the best-estimate assumption with margin for adverse deviations ("MfADs"). The policy liabilities are computed on a contract-by-contract basis.

To determine the appropriate reserve amount we determine the *Conditional Tail Expectation ("CTE").* The CTE is the statistical measure which corresponds to the average of outcomes generated by the stochastic model which are above a specified level, when results are ordered from the lowest to the highest net cost. For example, CTE(80%) represents the average of the worst 20 percent of the outcomes.

The provision for adverse deviations ("PfADs") is established by requiring the policy liability to cover a range of stochastic results based on the CTE measure. An acceptable range for the CTE is between CTE(60%) and CTE(80%). By measure of conservatism, the policy liabilities are set at the highest level, CTE(80%), primarily because of the following factors:

Newly developed random number generator and stochastic model;

Imperfect fit between the lognormal model and the GIP historical data;

Model parameters estimation using the GIP fund as opposed to market index;

5.2 Valuation Assumptions

All non-scenario tested valuation assumptions, such as mortality and surrenders, are set using the best-estimate assumption with explicit margins for adverse deviations. The margins should normally fall within the standard range of 5 to 20 percent, which is consistent with the standards described in the CIA Standards of Practice for Valuation of Policy Liabilities of Life Insurers.

5.2.1 Interest Rate for Assets Supporting the Liability

The assumption for interest rates on assets supporting policy liabilities for investment guarantees is set at 6 percent flat for valuation and 7.5 percent for best-estimate.

5.2.2 Expenses

For the purpose of calculating the policy liabilities, we assume no expenses or commissions and exclude the corresponding revenues such as policy fees, MER, offer price adjustment and surrender charges. We use the conservative assumption that these revenues would not be available to cover the cost of investment guarantees, but fully used to pay for administrative expenses and recovery past acquisition costs.

5.2.3 Mortality

The best-estimate mortality assumption used to calculate the policy liabilities is 105% of the CIA 86-92, age last birthday, sex distinct, aggregate mortality table. The ultimate mortality rates are used because the GIP policies were not underwritten.

The mortality "MfADs" is set at 20% of the best-estimate assumption. The high margin level is used on the mortality assumption to take into consideration the limited mortality experience and the fact that the policies were not medically underwritten.

5.2.4 Surrenders

The best-estimate assumption is based on the most recent GIP lapse study. Appendix A.4 illustrates the results of the study. The table below shows the lapse rate assumption used by duration:

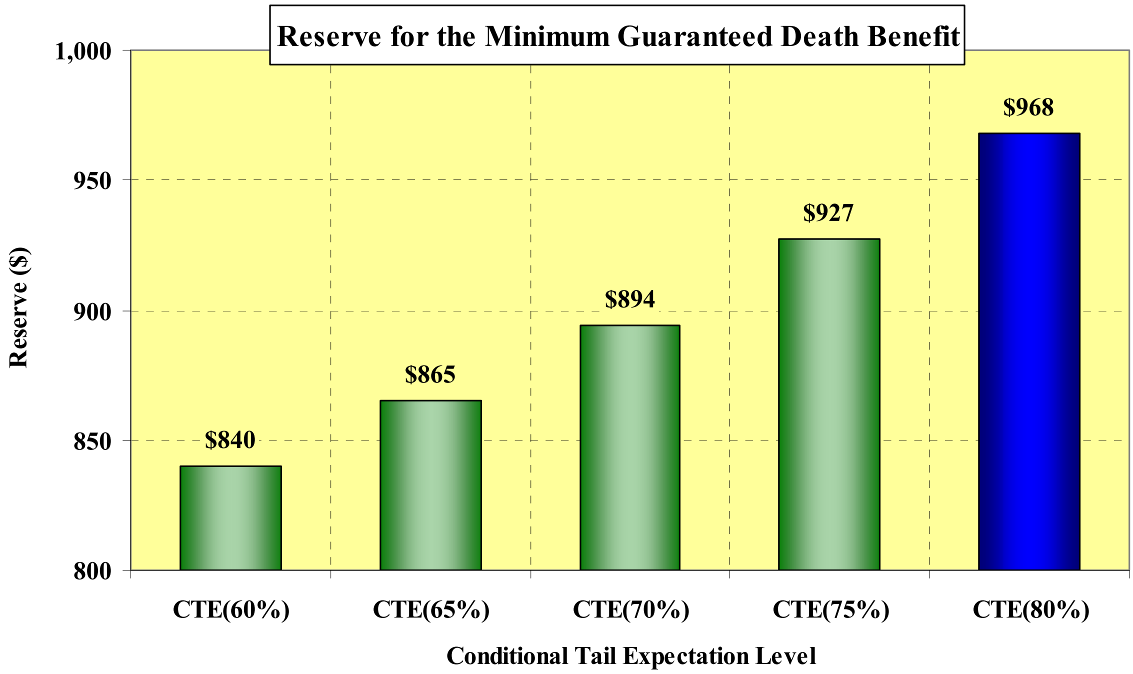
Policy Year	Lapse Rates	Policy Year	Lapse Rates
1	20.0%	7	6.0%
2	15.0%	8	6.0%
3	8.0%	9	6.0%
4	7.0%	10	6.0%
5	6.0%	11	10.0%
6	6.0%	12 +	5.0%

The "MfADs" on the lapse assumption is set at 10 percent of the best-estimate rates. Testing was performed to determine the sign of the margins to ensure that it results in an increase of the policy liabilities.

6. Results Summary

6.1 Minimum Guaranteed Death Benefit

The cost associated with the minimum guaranteed death benefit is negligible with reserve amount ranging between $840 to $968 for all policies depending on the CTE level as shown on the graph below:



The small cost for the guaranteed death benefit can be explained by the very young profile of the GIP portfolio, with an average issue age of 26, an average attained age of 34 and a projected age to maturity of only 51.

6.2 Minimum Guaranteed Maturity Benefit

As discussed earlier in this report, the minimum guaranteed value at maturity can be calculated on different bases depending on the investment premium amount (gross versus net), the GIP prices (bid price vs. offer price), and payment timing (on surrender vs. maturity) used to calculate the guaranteed values. For the purpose of this paper, the cost of investment guarantees is estimated using the following three bases:

6.2.1 Contractual base

On a strict contractual base, the GIP investment guarantee for the “A” and “B” GIP policies should apply at the contract maturity only. Additionally, the guaranteed amount should be calculated using net investment premiums after premium loadings, and unit allocation based on the GIP offer price with adjustment factor of 100/95.

For the “C” GIP series, guaranteed values should be payable on voluntary surrenders after the tenth policy anniversary using gross investment premiums.

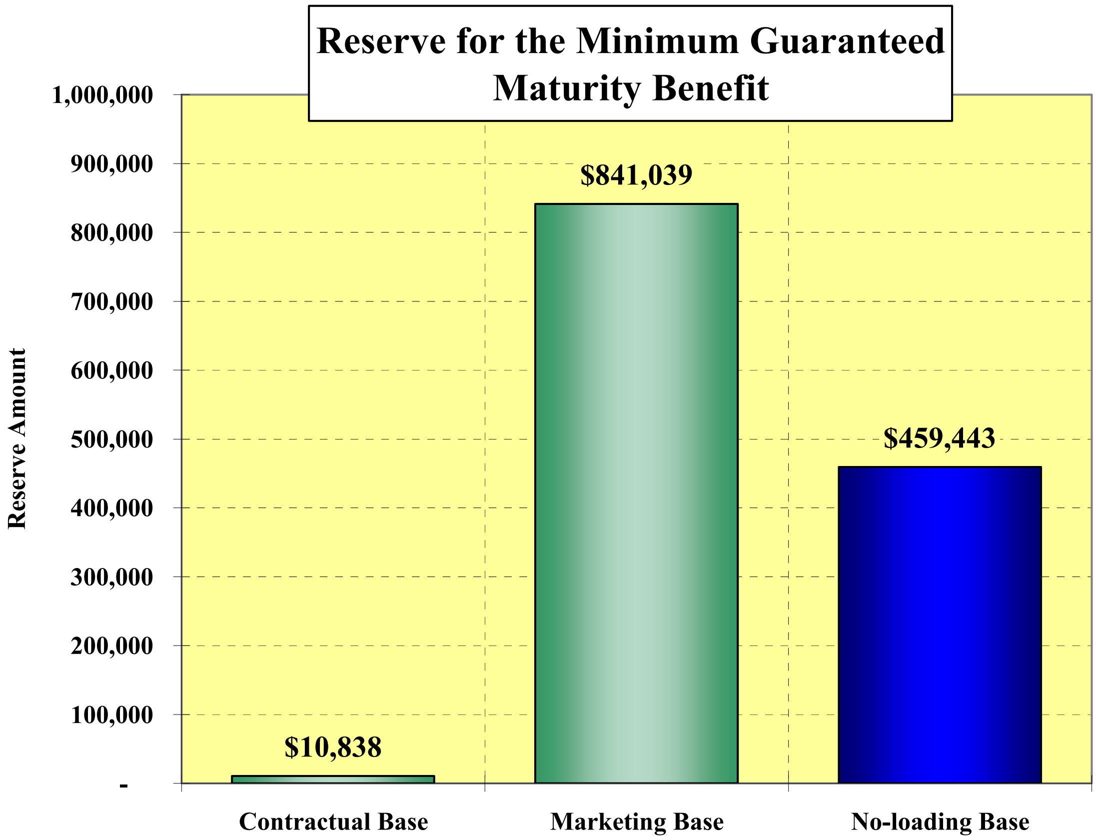
6.2.2 Marketing base

The reserve calculated with this method is consistent with past marketing practices and current administrative procedures, assuming that the guaranteed values are payable on surrender occurring after the tenth policy anniversary, irrespective of the maturity date. The value is calculated using gross investment premiums (without premium loading) and the GIP bid price (without offer price adjustment factor).

6.2.3 No-loading base

The no-loading base is similar to the marketing base, except that the guaranteed value is calculated using the GIP offer price, as opposed to the bid price. The offer price adjustment is clearly divulged in the GIP contract and a price adjustment factor of 100/95 represents in my opinion a reasonable premium loading. This base is a reasonable and equitable compromise between the contractual and the marketing bases.

The graph below summarizes the reserve for all policies depending on the different calculation bases:

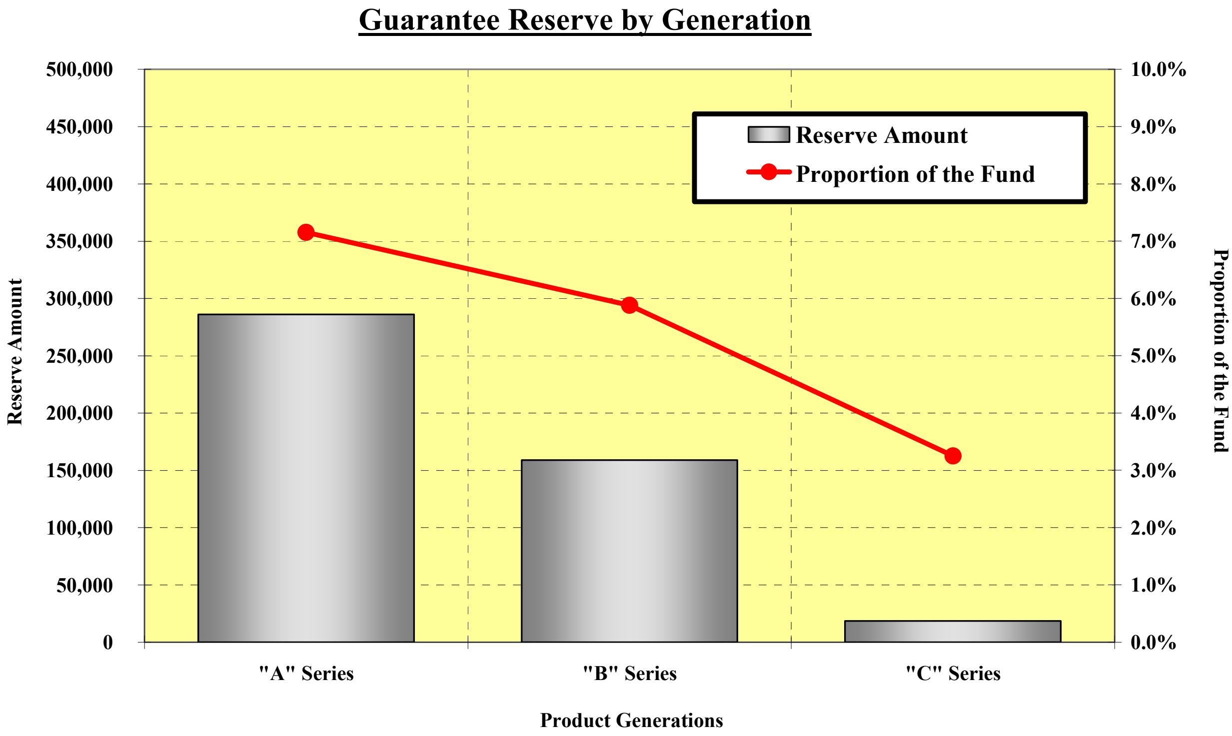


As expected, the lowest reserve is generated by the investment guarantee payable at maturity, with a cost of $10,838. Although the method is consistent with the "A" and "B" GIP contracts, the Company is exposed to potential market conduct problems because of past marketing practices and inappropriate disclosure of the front-end premium loadings in the contract.

The marketing base generates the highest cost, with a reserve amount of $841,039. This method is consistent with current administrative practices and the "C" GIP contract.

The no-loading base produces reasonable results, which is a fair compromise for both the Company and the policyholders and is the recommended method. The detailed results presented in the remainder of this report are based on this method.

The graph below shows the reserve by GIP generations, using the no-loading base with guarantee payable on termination after the tenth policy anniversary, with gross investment premiums and unit allocation based on the GIP offer price:



The cost as a function of the fund is much higher for the older GIP generations, starting at 7.2 percent for the “A” GIP series, decreasing to 5.9 percent for the ”B” series and “3.3 percent for the latest generation. Again, this pattern is mainly caused by the premium loadings and offer price adjustment.

6.3 Provisions for Adverse Deviations

The reserve is also calculated using the best-estimate assumptions and the non-calibrated investment model. The detailed PfADs for each assumption, along with the impact of the investment model calibration, are summarized in the table below:

Description	Parameters		Policy	PfADs	
	Best-Est.	Valuation	Liability	($)	(%)
Best-Estimate Non Calibrated			**265,309**		
Model Calibration	σ = 2.15%	σ = 2.60%	316,551	51,242	16.2%
Interest MfADs	7.5%	6.0%	336,218	19,667	5.8%
Mortality MfADs	105%	85%	337,334	1,116	0.3%
Lapse MfADs	Varies	±10.0%	357,439	20,105	5.6%
CTE(%)	CTE(60%)	CTE(80%)	459,443	102,004	22.2%
Total			**459,443**	**194,134**	**42.2%**

The numbers in the above table suggest that the proposed policy reserve is relatively conservative, with a total provision for adverse deviations of more than $194,000, or 42 percent of the total non-calibrated reserve.

7. Key Recommendations

On the basis of the work accomplished in this paper, the following recommendations are suggested:

7.1 Reserve for the Minimum Guaranteed Death Benefit

The cost associated with the minimum guaranteed death benefit is immaterial; therefore, the minimum guaranteed death benefit may be ignored in setting the reserve for the GIP investment guarantees on ground of materiality.

7.2 Reserve for the Minimum Guaranteed Maturity Benefit

The payment of guaranteed values upon maturity and voluntary surrender after the tenth policy anniversary is recommended using gross investment premiums and the GIP offer prices. The proposed base produces reasonable results and represents a fair compromise for the Company and the policyholders. The main reasons justifying this recommendation are as follow:

- The premium loading is not properly disclosed in the GIP contract,
- The GIP contract has been sold and administered with the belief that investment guarantees apply after the tenth anniversary.
- The offer price adjustment is clearly divulged in the GIP contract and a price adjustment factor of 100/95 represents a reasonable premium loading for an investment product.

The recommendation would eliminate the risk of market conduct problems. It is also important to note that the GIP fund level is currently over-stated because of the advance GIP unit allocation method; consequently sufficient margins are available to fully cover the cost of investment guarantees for the proposed base.

7.3 Asset Portfolio Composition

The investment return volatility has a great impact on the cost of the investment guarantees. In order to minimize the cost associated with the investment guarantees, GIP assets should be invested in diversified fixed income securities with very little equity investments. The strategy is to create a diversified portfolio with minimal volatility and reasonable investment return between 6 to 8 percent.

The table below shows the actual GIP asset mix along with the recommended target distribution:

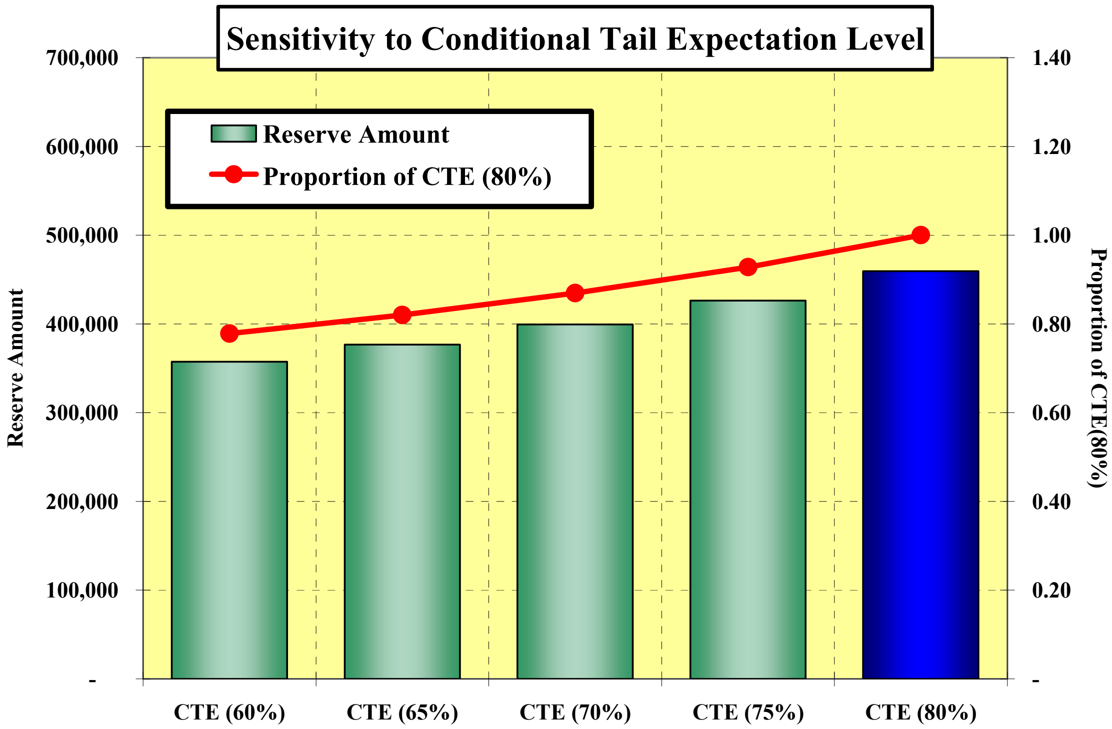
Asset Types	Actual (%)	Target (%)
Cash and Fixed Deposits	17.6%	10.0% to 20.0%
Money Market Funds	0.0%	10.0% to 20.0%
Government Bonds	9.0%	20.0% to 50.0%
Residential Mortgages	2.6%	20.0% to 40.0%
Local Equity	2.5%	0.0%
US$ Mutual Funds	21.5%	0.0%
Policy Loan	14.3%	5.0% to 15.0%
Inter Company Note	32.5%	0.0%

Additionally, Investment Policy Guidelines should be adopted for the GIP fund to ensure complete compliance to the investment strategy.

8. Sensitivity Testing

8.1 Conditional Tail Expectation ("CTE") Level

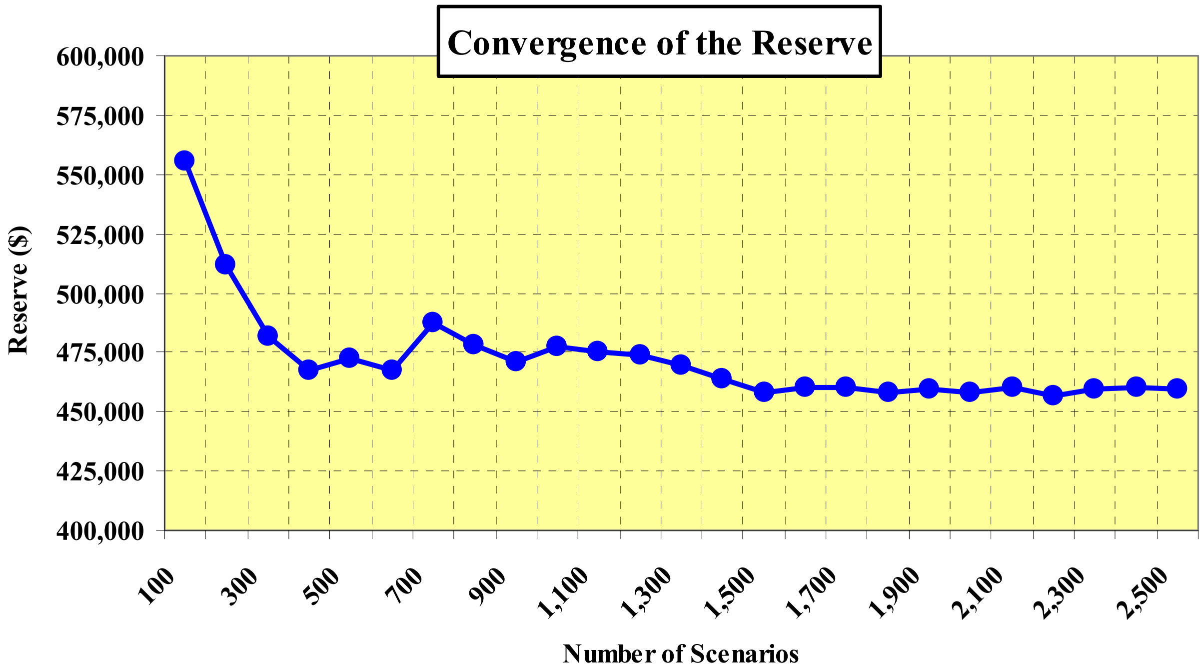
In setting the policy liabilities, the reserve is calculated conservatively using the CTE (80%) level. The following graph shows the reserve at different CTE levels and as a function of the CTE (80%) reserve.



A reserve at CTE (70%) will be more appropriate once a comprehensive audit of the GIP in force data and fund value has been completed. The difference between CTE (70%) and CTE (80%) reserves is approximately 13 percent, or $60,000.

8.2 Number of Scenarios

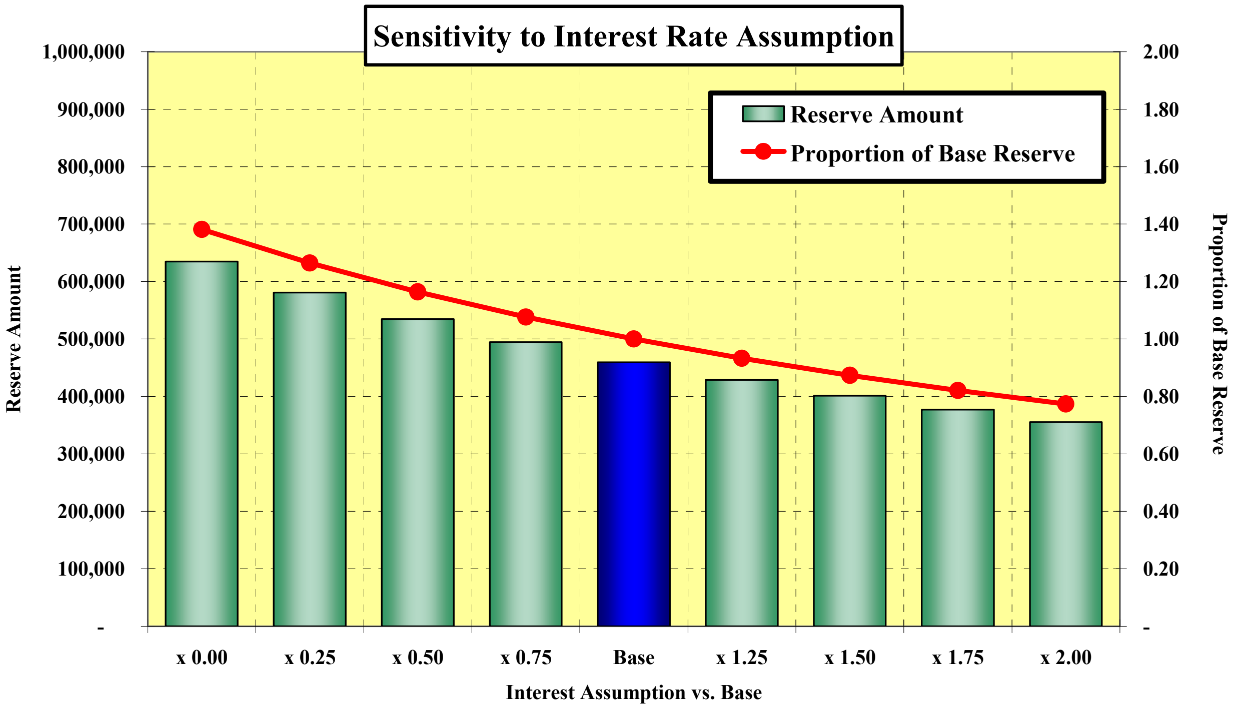
The CIA Report gives some guidance as to the number of scenarios that need to be generated; it is suggested to produce a minimum of 1,000 stochastic scenarios. The graph below shows the reserve as a function of the number of stochastic scenarios:



The reserve converges quickly after 1,500 simulations. In the policy liability model, 2,500 stochastic scenarios are used to provide a greater degree of accuracy.

8.3 Interest Rate Assumption

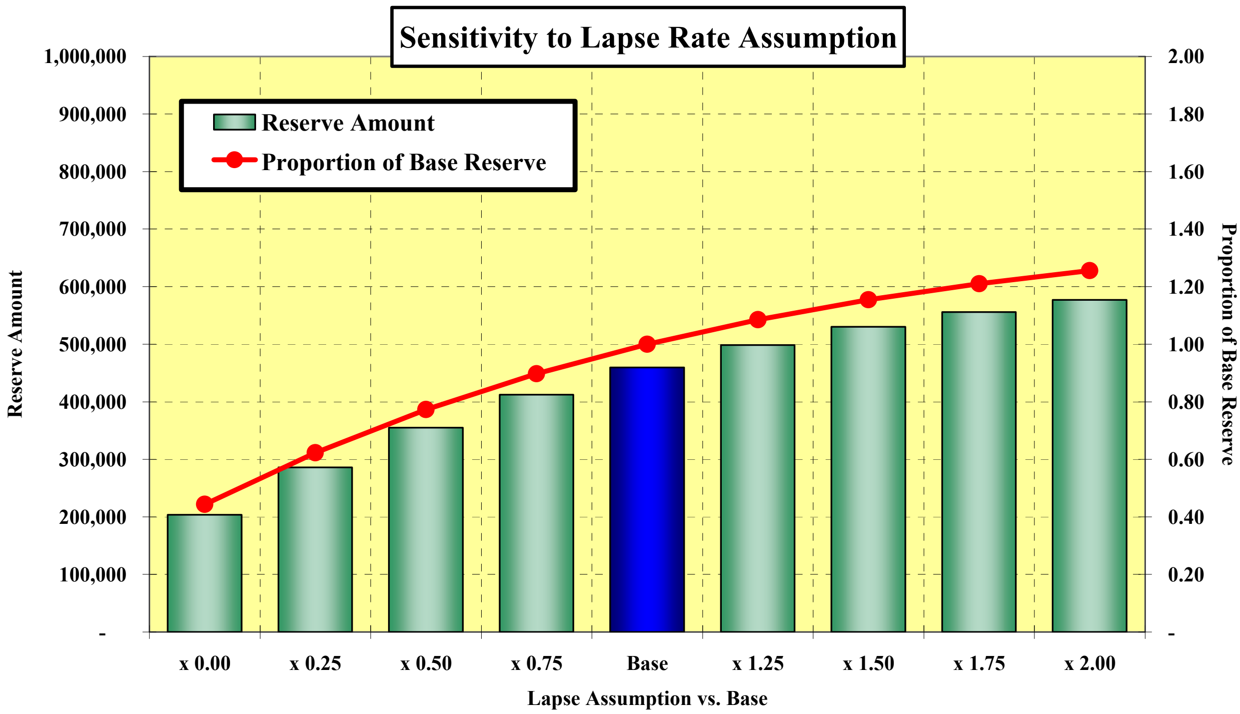
The graph below shows the reserve amount using different interest rate assumptions as a function of the 6 percent base assumption:



The minimum guaranteed maturity reserve is very sensitive to the interest rate assumption used to discount the liability cash flows. This can be explained by the long duration of the liabilities, with an average remaining term to maturity of about 17 years. As a function of the base reserve, the cost varies between 77 to 138 percent, with interest rate assumption decreasing from 12 to 0 percent. Obviously, a zero percent interest rate is not a realistic assumption, but it is used for illustration only. The negative impact of a decline in interest rates is greater than the impact of an increase in interest rates.

8.4 Lapse Rate Assumption

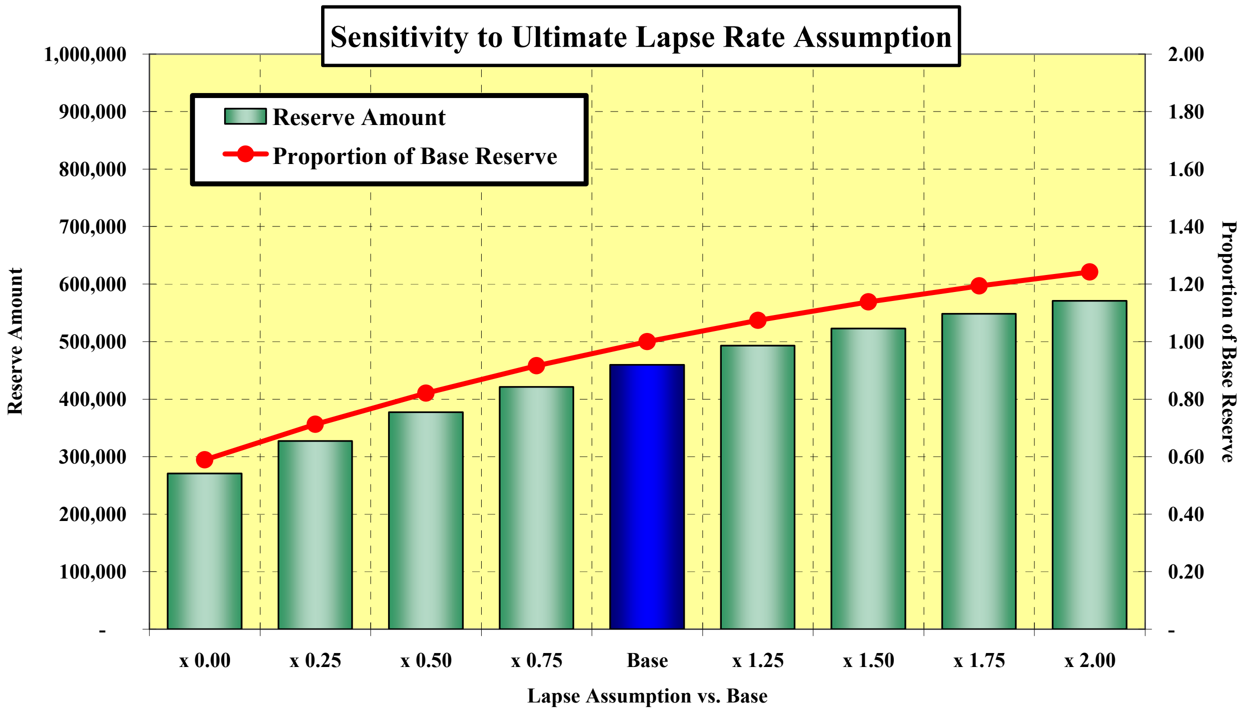
The graph below shows the reserve amount using different lapse rate assumptions as a function of the base assumption:



For minimum guaranteed value payable on voluntary surrenders after the 10th policy anniversary, the reserve is very sensitive to the lapse rate assumption with cost varying between 44 and 126 percent of the base reserve. The negative impact of an increase in lapse rate is much smaller than the impact of a decline in the rates. It can also be observed that the GIP in not lapse-supported on the proposed investment guarantee base.

8.5 Ultimate Lapse Rate Assumption

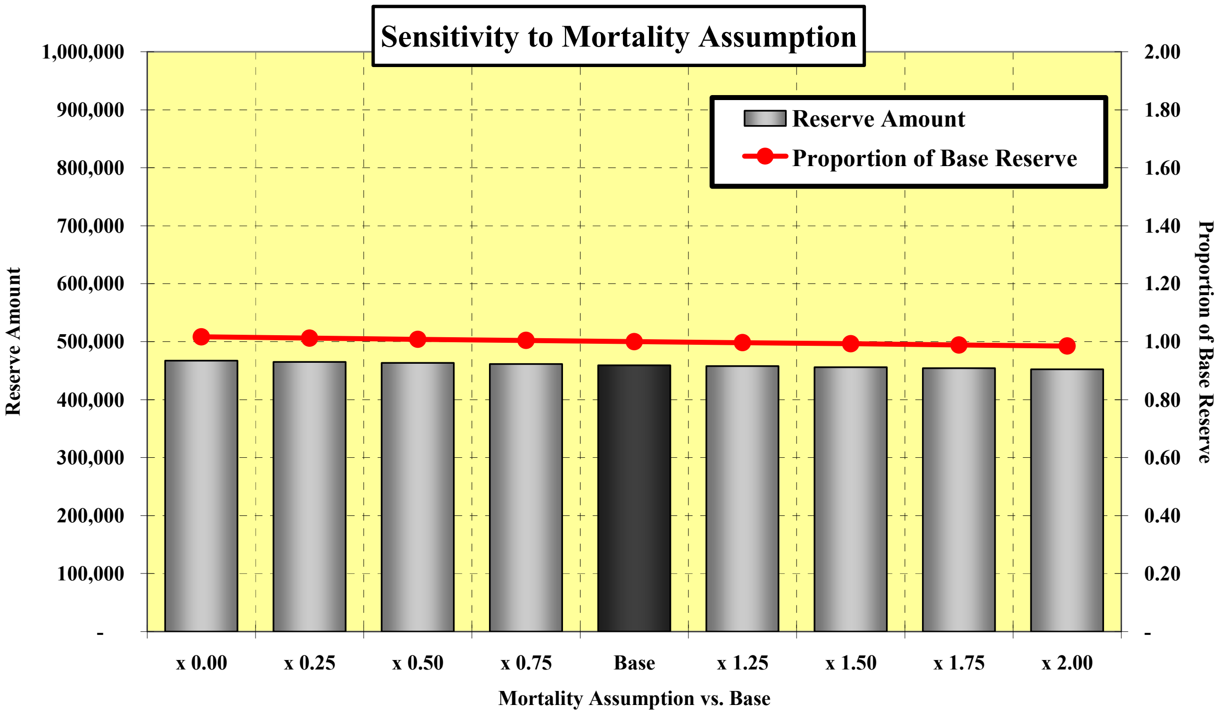
Since limited experience is available to determine the ultimate lapse rate assumption, it is interesting to look at the sensibility of the reserve to different rates as a function of the base assumption of 5 percent:



As a function of the base reserve, the cost varies between 59 and 124 percent, for corresponding ultimate lapse rate assumption ranging from 0 to 10 percent.

8.6 Mortality Rate Assumption

The following graph shows the reserve amount using different mortality assumptions as a function of the base assumption of 105% of the CIA table.



The reserve is not sensitive to the mortality assumption, with variations of less than 2 percent for mortality variation of 0 to 200 percent of the base assumption. This can be explained primarily by the very young profile of the GIP policyholders. Obviously, the mortality assumption at 0 percent is not realistic but it is used for illustration only.

It is also interesting to note that an increase in the mortality level reduces the guaranteed maturity reserve, primarily because a smaller proportion of the policyholders reach their maturity date.

Appendix A.1: Random Number Generator

A.1.1 Generating Random Numbers from the Uniform Distribution

The sequence of random numbers is generated iteratively using the *linear congruential generator* formula:

$$X_n = (a X_{n-1} + b) \text{ modulus } m$$

We first start by selecting the model parameters *a*, *b* and *m*, the first number X_0, also called the seed value. The following parameters provide very good results with high periodicity:

Description	Variable	Criteria	Value
Seed value	X_0	$0 < X_0 < m$	1×10^9
Multiplier	a	0 < a < m	48,271
Increment	b	0 < b < m	0
Modulus	m	m > 0	1×2^{31}

The parameters determine the characteristics of the generator and the seed value determines the particular sequence generated. The formula generates iteratively a sequence of random integers $X_1, \ldots, X_n$ over the interval 0 to 2^{31}. The example below illustrates the formula:

X_0 = 1,000,000,000

X_1 = (48,271 x 1,000,000,000 + 0) modulus 2^{31} = 2,010,066,381

X_2 = (48,271 x 2,010,066,381 + 0) modulus 2^{31} = 308,138,487

X_3 = (48,271 x 308,138,487 + 0) modulus 2^{31} = 681,649,565

This procedure is repeated to generate the number of random values required for the stochastic model projection. Then, by dividing each generated random number X_n by *m*, we obtain a new sequence of random numbers U_n from a standard Uniform distribution over the unit interval 0 to 1.

A.1.2 Generating Random Numbers from a Normal Distribution

To generate random numbers with a Normal distribution, the initial random sequence is transformed by using the polar form of the Box-Mueller mathematical transformation. This numerical algorithm transforms uniformly distributed random variables to a new set of random variables with a Normal distribution with zero mean and a standard deviation of one.

A.1.3 Validating the Random Numbers Generator

There are many statistical tests that can be used to ensure that the sequence of random numbers is adequate for the stochastic model. One tool to validate that the generator produces random numbers that follow the assumed distribution is to compare the moments of the theoretical distribution and the distribution of the random numbers generated by the model. The tables below show the results of the tests.

Statistical tests on the Uniform Distribution:

	U (0,1)	Random Numbers			
Nb. of observations		1,000	10,000	100,000	1,000,000
Mean (μ)	0.5000	0.5121	0.5044	0.5001	0.5002
Std. Deviation (σ)	0.2887	0.2931	0.2862	0.2890	0.2886
Skewness	0.0000	(0.0120)	(0.0074)	0.0013	0.0000

Statistical tests on the Normal Distribution:

	N (0,1)	Random Numbers			
Nb. of observations		1,000	10,000	100,000	1,000,000
Mean (μ)	0.0000	(0.0128)	(0.0012)	(0.0015)	0.0005
Std. Deviation (σ)	1.0000	0.9958	1.0021	1.0002	1.0001
Skewness	0.0000	(0.0971)	(0.0193)	0.0004	0.0012

A quick observation of the above results demonstrates that the random number generator produces the assumed distribution. We can see that the various moments converge to their theoretical values.

Appendix A.2: GIP Historical Unit Value

Monthly Closing Offer Prices:

	Months											
Year	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
1990									10.00	10.19	10.15	10.24
1991	10.27	10.31	10.34	10.39	10.40	10.33	10.34	10.36	10.44	10.46	10.46	10.60
1992	10.66	10.68	10.68	10.90	10.92	10.96	11.01	11.08	11.11	11.20	11.32	11.36
1993	11.41	11.38	11.59	11.55	11.69	11.85	12.02	12.08	12.23	12.28	12.41	12.51
1994	12.56	12.57	12.67	12.74	12.77	12.91	13.01	13.11	13.19	13.27	13.38	13.52
1995	13.58	13.74	13.88	13.87	14.01	14.13	14.17	14.26	14.45	14.60	14.69	14.83
1996	14.90	15.09	15.08	15.12	15.23	15.35	15.49	15.56	15.66	15.80	15.81	16.04
1997	16.08	16.27	16.52	16.60	16.69	16.71	16.84	16.93	17.19	17.36	17.31	17.25
1998	17.30	17.36	17.47	17.59	17.37	17.40	17.47	17.41	17.34	17.52	17.65	17.72
1999	17.65	17.86	17.99	18.15	18.22	18.47	18.62	18.52	18.81	18.94	19.06	19.18
2000	19.34	19.42	19.45	19.41	19.44	19.51	19.52	19.61	19.79	19.87	19.99	20.16
2001	20.11	20.14	20.27	20.27	20.43	20.52	20.59	20.80	20.85	20.97	20.92	20.92
2002	21.07	21.14	21.20	21.34	21.43	21.53	21.68	21.78	22.02	22.17	22.32	21.86
2003	21.95	22.08	22.18	22.19	22.32	22.42	22.51	22.65	22.73	22.61	22.63	22.83
2004	23.03	23.20	23.14	23.79	23.88	24.26	24.53	25.09	24.88	24.83	25.00	24.90
2005	24.85	25.42	25.54	25.56	25.60	25.88	25.95	25.88	25.90	25.81	25.57	25.66

Appendix A.3: GIP Historical Monthly Return

Monthly Returns:

	Months											
Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1990										1.90%	-0.39%	0.89%
1991	0.29%	0.39%	0.29%	0.48%	0.10%	-0.67%	0.10%	0.19%	0.77%	0.19%	0.00%	1.34%
1992	0.57%	0.19%	0.00%	2.06%	0.18%	0.37%	0.46%	0.64%	0.27%	0.81%	1.07%	0.35%
1993	0.44%	-0.26%	1.85%	-0.37%	1.24%	1.37%	1.43%	0.50%	1.24%	0.41%	1.06%	0.81%
1994	0.40%	0.08%	0.80%	0.55%	0.24%	1.10%	0.77%	0.77%	0.61%	0.61%	0.83%	1.05%
1995	0.44%	1.18%	1.02%	-0.07%	1.01%	0.86%	0.28%	0.64%	1.33%	1.04%	0.62%	0.95%
1996	0.47%	1.28%	-0.07%	0.27%	0.73%	0.79%	0.91%	0.45%	0.64%	0.89%	0.06%	1.45%
1997	0.25%	1.18%	1.51%	0.51%	0.54%	0.12%	0.78%	0.53%	1.54%	0.99%	-0.32%	-0.32%
1998	0.29%	0.33%	0.65%	0.69%	-1.25%	0.17%	0.40%	-0.34%	-0.40%	1.04%	0.74%	0.40%
1999	-0.40%	1.19%	0.73%	0.89%	0.39%	1.37%	0.82%	-0.57%	1.59%	0.69%	0.63%	0.63%
2000	0.83%	0.41%	0.15%	-0.21%	0.15%	0.36%	0.05%	0.46%	0.92%	0.41%	0.60%	0.85%
2001	-0.25%	0.15%	0.65%	0.00%	0.79%	0.44%	0.34%	1.02%	0.24%	0.58%	-0.24%	0.00%
2002	0.72%	0.33%	0.28%	0.66%	0.42%	0.47%	0.70%	0.46%	1.10%	0.68%	0.68%	-2.06%
2003	0.40%	0.60%	0.45%	0.05%	0.59%	0.45%	0.40%	0.62%	0.35%	-0.53%	0.09%	0.88%
2004	0.88%	0.74%	-0.26%	2.81%	0.38%	1.59%	1.11%	2.28%	-0.84%	-0.20%	0.68%	-0.40%
2005	-0.20%	2.29%	0.47%	0.08%	0.16%	1.09%	0.27%	-0.27%	0.08%	-0.35%	-0.93%	0.35%

Appendix A.4: GIP Lapse Study

Lapse Study by Number of Policy:

Policy	Lapse Study Year					5-Year	Lapse
Year	2004-05	2003-04	2002-03	2001-02	2000-01	Average	Assumption
1	22.6%	20.8%	28.8%	16.2%	9.8%	21.0%	**20.0%**
2	21.8%	18.4%	9.6%	11.9%	9.2%	14.9%	**15.0%**
3	9.9%	8.0%	4.5%	5.6%	7.5%	7.2%	**8.0%**
4	11.6%	4.7%	4.9%	9.8%	14.4%	9.1%	**7.0%**
5	6.2%	10.3%	5.4%	5.6%	5.4%	6.6%	**6.0%**
6 to 10	7.4%	4.5%	6.3%	5.6%	4.7%	5.6%	**6.0%**
11	14.4%	9.9%	10.8%	10.1%	9.9%	11.0%	**10.0%**
12 +	7.7%	7.6%	5.9%%	7.8%	N/A	7.4%	**5.0%**

Appendix A.5:CIA86-92 Aggregate Mortality Table-Ultimate Rates

Mortality rates x 1,000:

Age	Female	Male	Age	Female	Male	Age	Female	Male
1	0.60	0.67	36	0.75	1.25	71	17.47	30.07
2	0.35	0.40	37	0.80	1.25	72	19.36	33.11
3	0.23	0.30	38	0.88	1.25	73	21.46	36.43
4	0.17	0.23	39	0.98	1.28	74	23.80	40.07
5	0.15	0.18	40	1.11	1.34	75	26.41	44.05
6	0.14	0.16	41	1.24	1.43	76	29.32	48.39
7	0.14	0.15	42	1.36	1.56	77	32.56	53.14
8	0.12	0.15	43	1.47	1.70	78	36.16	58.31
9	0.11	0.15	44	1.60	1.87	79	40.17	63.96
10	0.10	0.15	45	1.74	2.06	80	44.64	70.11
11	0.10	0.17	46	1.87	2.28	81	49.60	76.81
12	0.11	0.20	47	2.01	2.52	82	55.12	84.10
13	0.14	0.27	48	2.17	2.80	83	61.25	92.03
14	0.19	0.35	49	2.34	3.11	84	68.05	100.64
15	0.24	0.46	50	2.54	3.46	85	75.60	109.99
16	0.27	0.59	51	2.74	3.85	86	83.97	120.12
17	0.30	0.71	52	2.98	4.29	87	93.24	131.10
18	0.32	0.82	53	3.23	4.78	88	103.49	142.97
19	0.34	0.90	54	3.51	5.31	89	114.81	155.80
20	0.36	0.96	55	3.82	5.91	90	127.31	169.64
21	0.38	0.98	56	4.17	6.58	91	141.07	184.54
22	0.37	0.99	57	4.55	7.31	92	156.21	200.58
23	0.36	0.97	58	4.97	8.12	93	172.83	217.78
24	0.35	0.96	59	5.44	9.02	94	191.03	236.22
25	0.33	0.96	60	5.96	10.00	95	210.92	255.93
26	0.34	0.97	61	6.54	11.09	96	232.59	276.94
27	0.38	0.99	62	7.18	12.29	97	256.14	299.29
28	0.43	1.02	63	7.90	13.61	98	281.63	323.00
29	0.49	1.04	64	8.70	15.06	99	309.13	348.07
30	0.54	1.07	65	9.58	16.66	100	339.76	375.57
31	0.57	1.11	66	10.57	18.41	101	380.13	412.00
32	0.59	1.16	67	11.67	20.33	102	442.40	469.32
33	0.62	1.21	68	12.89	22.44	103	540.42	561.17
34	0.65	1.25	69	14.26	24.75	104	690.89	703.94
35	0.70	1.26	70	15.78	27.29	105	1000.00	1000.00