Reserve Variability and Ranges

By Debarshi Chatterjee

Abstract

For the last few years, one hotly-debated topic in General Insurance is: How to measure the uncertainty in the actuarial loss reserve estimates. This paper discusses the growing importance of measuring this uncertainty and coming up with confidence intervals around the reserve estimates. It also looks at the current guidelines in different countries on reporting of reserve uncertainty in the loss reserves. Finally it discusses some of the methods proposed for calculating the reserve variability and ranges.

Introduction

Some of the traditional methods used in loss reserving are the Chain-Ladder or Loss Development Method, the Expected Loss Ratio Method and the Bornhuetter-Ferguson Technique. These are all deterministic methods. This means that when they are applied to the loss data we obtain an estimate of the ultimate losses but we don't get an idea of how much our estimates can vary. A rough idea of the variability is obtained by looking at the results of the different methods. But this doesn't give us an idea of the variability in the sense of understanding the probabilities associated with various outcomes.

In order to look at probabilities we can consider using statistical methods of loss reserving. These would give us a confidence interval around our reserve estimates based on the distribution of losses and the error associated with the estimation methods. In other words, this will give us a statistical range of our reserve estimates.

Benefits

We noted above the primary reason behind the growing importance of statistical measures with respect to reserve ranges. These methods are becoming all the more important because:

a) International regulatory requirements and accounting standards are clearly moving toward requiring more information on the distribution around liability point estimates.

a) Methods based on objective statistical analysis may require less judgment than traditional methods

b) The correlation between different lines of business can be estimated giving an idea of the interrelationships between them.
c) The reserve estimate distribution can be used as an input to company financial models needed for risk and capital management decisions such as the appropriate reinsurance structure

Regulatory Guidelines

Let us look at the guidelines that are available in some of the major Insurance markets, and also at an international level.

United Kingdom

Relevant points from Guidance Note 12 General Insurance Business: Actuarial Reports [1]

8 Uncertainty

8.1 The report should normally indicate the degree and sources of uncertainty surrounding the point estimates that the member has made and sensitivities to key assumptions. Uncertainty for a particular point estimate would normally be quantified by providing a range of values around the point estimate together with an indication of the likelihood that the true value lies above, below or simply outside the range. This can be achieved by specifying quantiles or by using any other appropriate descriptive summary.

8.2 If there are specific features of the business that present potential concerns or significantly increase the uncertainty of the results, beyond that which an informed reader of the report would reasonably expect, then this fact must be clearly highlighted in the corresponding reservations, or limitation of scope, included in the report.

8.3 If there is a substantial probability of a material impact on the reported profit and loss or balance sheet strength resulting from adverse deviation from modelled results, the member should normally draw attention to this in the report.

GN50: General Insurance Principles and Practice [2]

2.6 Point estimates

2.6.1 When providing quantitative advice, the *member* should normally include a specific point estimate in the context of the purpose of the advice. Provision of a range of outcomes is often desirable (see 2.7.1) but the provision of a range of outcomes without a specific point estimate could be open to misinterpretation.

2.7 Communicating Uncertainty

2.7.1 The *member* must consider the uncertainty surrounding advice or opinions formed and communicate this appropriately. The need to communicate uncertainty will depend upon the audience and the degree and importance of the uncertainty in the context of the purpose of the work. The less likely the audience is to appreciate the importance or extent of this uncertainty, the greater the need is for the *member* to communicate it. If uncertainty is important in the context of the purpose of the work (for example if alternative advice could give rise to different decisions or conclusions) then it should be communicated.

2.7.2 In discussing uncertainty, the *member* must consider whether elaboration of the sources of uncertainty is appropriate and act accordingly.

United States of America

ASOP 36 [3] - Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves

- 3.6 Uncertainty: …The actuary should consider the implications of uncertainty in loss and loss adjustment expense reserve estimates in determining a range of reasonable reserve estimates…
- 3.6.4 Range of Reasonable Reserve Estimates—The actuary may determine a range of reasonable reserve estimates that reflects the uncertainties associated with analyzing the reserves. A *range of reasonable estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable. The actuary may include risk margins in a range of reasonable estimates, but is not required to do so, except as may be required by ASOP No. 20. A range of reasonable estimates, however, usually does not represent the range of all possible outcomes.

Australia

Professional Standard 300 [4] - Actuarial Reports And Advice On General Insurance Technical Liabilities

A **central** estimate of the liabilities is the expected value of the liabilities. In other words, if all the possible values of the liabilities are expressed as a statistical distribution, the central estimate is the mean of that distribution.

Institute of Australia – Guidance note on evaluation of central estimate of claims liabilities

The approved actuary undertaking a statutory valuation under APRA Prudential Standard GPS 210 is required to determine a central estimate of the liability and to recommend a valuation margin which, when added to the central estimate, gives a provision intended to secure a 75% probability of adequacy (but not less than half a standard deviation above the mean).

Prudential Standard GPS 210 [5] – Liability Valuation for General Insurers

9. The valuation of insurance liabilities for each class of business must comprise:
(a) a central estimate value of the Outstanding Claims Liabilities;
(b) a central estimate value of the Premiums Liabilities; and
(c) risk margins that relate to the inherent uncertainty in each of these central estimate values.

As we can see above, each of the guidelines talk of reporting on uncertainty. Some countries like Australia stress on the importance of quantiles. However, they leave it up to the actuary to decide on what method he/she wants to use. But the below points from the International Accounting Standards Board specifically talk about stochastic methods.

International Accounting Standard Board

The International Accounting Standards Committee (IASC) started a project on Insurance accounting in 1997. The Issues Paper was published in December 1999 with comments requested up to 31 May 2000. The project steering committee has considered the comments received in formulating a report to the International Accounting Standards Board (IASB), which replaced the IASC. The report was in the form of a Draft Statement of Principles (DSOP) [6]

Some of the relevant points from the DSOP are:-

- In order to meet the DSOPs requirements it would be necessary to run stochastic models separately for each unit of account. How is unit of account defined?

Advantages of stochastic models are:

- the reliability of the fitted model and likely magnitude of random variation of future payments can be estimated
- fewer parameters and more objective than many traditional reserving techniques
- input assumptions can be transparent and auditable
- statistical tests can verify the model assumptions made

- determine reserve variability and hence appropriate margins
- can provide greater understanding of underlying processes

Summarizing the different points in the above paragraphs, we can say that there is an impetus to know more about the loss distributions and to apply the knowledge to come up with the best estimate, a measure of the reserve uncertainty and a range of reserves.

Methodologies

From our discussions in the previous section, it can be seen that we would need to look at stochastic methods to measure the uncertainty. Let us look at some of the methods currently available.

The Casualty Actuarial Society had formed a Working Party on "Quantifying Variability in Reserve Estimates". Here are some of the salient points from their summary report [7].

The methods for evaluating reserve variability can be broadly divided into

1) Analytical evaluation of incremental data
2) Bootstrap simulations
3) Bayesian evaluation

1) Analytical evaluation

    a. Data: The variability of future payment estimates can be estimated from a data triangle of incremental payments. A distributional form is chosen for the incremental payments, which could be an overdispersed Poisson, negative binomial, gamma, or many others.
    b. Structural Form: The form for the expectation of the incremental payments could be either non-linear in the parameters or modeled in a generalized linear model.
    c. Estimation: Through maximum likelihood method.
    d. Variability: Measured through variance of distribution of future payments
        i. decomposed into process variance & parameter variance
        ii. should take into account correlation between predicted values for different development periods in the same accident year.
        iii. variance of total future payments = variances for each accident year future payment estimates + covariance between them

2) Bootstrap Simulation
    a. Sampling with replacement from the scaled Pearson residuals after fitting a model (generalized linear/non-linear)

b. Create large number N of pseudo past triangles.
c. For each pseudo triangle, future payments are estimated using maximum likelihood approach.
d. Mean & variance calculated from N future payment triangles.

Simplified Approach (England & Verrall):-

a. Use standard chain-ladder method to obtain future (lower) triangle as well as past (upper) triangle.
b. Use the fitted past and actual payment values to calculate the residuals.
c. Follow above procedure to get pseudo triangles.
d. Use chain-ladder method to get future triangle for each pseudo triangle.
e. Create the incremental future payments
f. Simulate a future payment using the above created future payments as the mean.
g. Mean & variance calculated from N future payment triangles

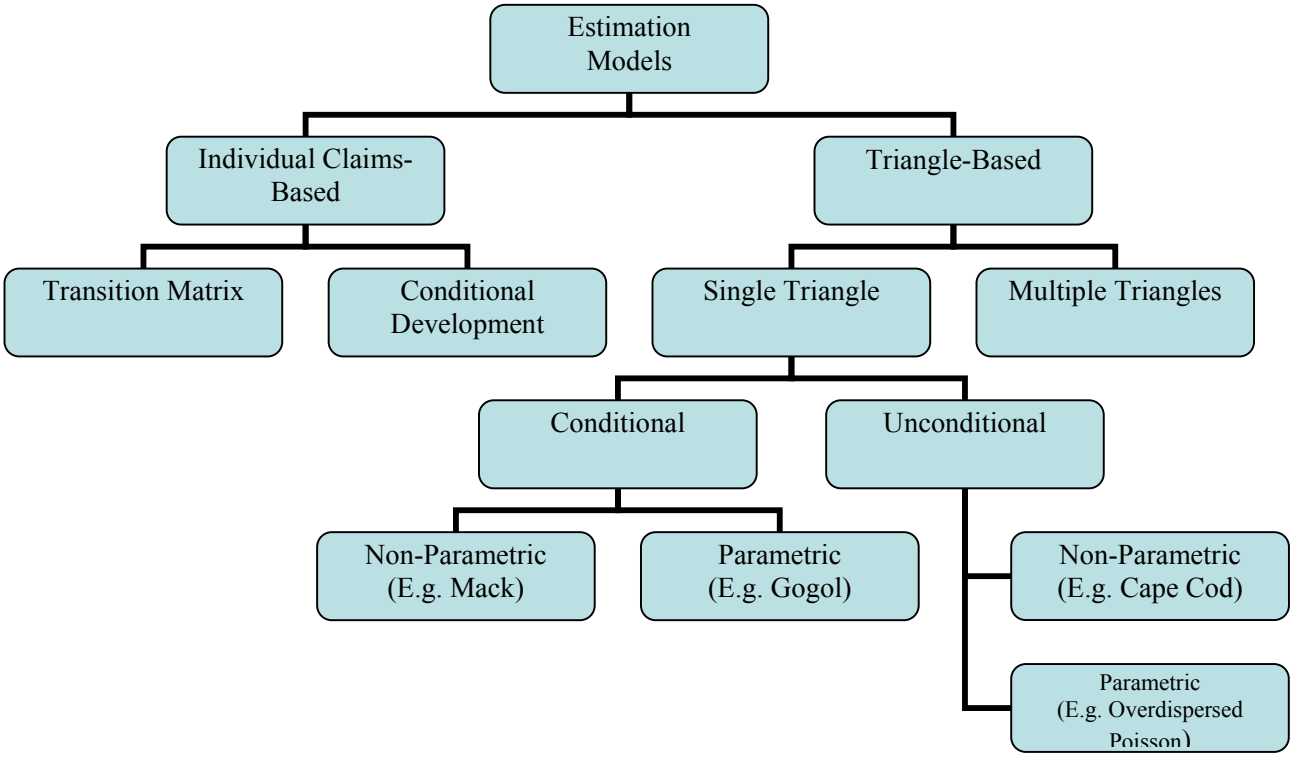
Advantage:

- Not only variance, but the whole distribution of future payments is obtained.
- Variance calculation is less complex.

3) Bayesian Evaluation:-

- Expands the analytical approach by treating the parameters of the fitted model as a further set of random variables.
- Markov Chain Monte Carlo can be used to generate an empirical posterior distribution of the model parameters.
- This distribution is then plugged into the original model to generate corresponding distribution of future payments.

In the above section we talked broadly about the three methods for evaluating reserve variability. Now, we look at the different models which can be fitted to the loss data in order to estimate the ultimate losses and come up with a prediction error. These can be classified as per the following chart:-



Model Selection and Evaluation

1. Criteria for Selecting an Appropriate Modeling Technique

    a. *Aims of the Analysis*.
    b. *Data Availability*
    c. *Non-Data Specific Model Evaluation*
    d. *Cost/Benefit Considerations*

2. Overall Model Reasonability Checks

    a. *Coefficient of Variation by Year:* should be the largest for the oldest (earliest) year and will, generally, get smaller for the more recent years.
    b. *Standard Error by Year:* should be the smallest for the oldest (earliest) year and will, generally, get larger for the more recent years.
    c. *Overall Coefficient of Variation*: should be smaller for all (accident, policy or report) years combined than for any individual year.
    d. *Overall Standard Error*: should be larger for all (accident, policy or report) years combined than for any individual year.
    e. *Correlated Standard Error & Coefficient of Variation*: The standard error should be smaller for all lines of business combined than the sum of the individual lines of business
    f. *Reasonability of Model Parameters and Development Patterns*.
    g. *Consistency of Simulated Data with Actual Data*
    h. *Model Completeness and Consistency*.

3. Model Goodness-of-Fit and Prediction Error Evaluation

    a. *Validity of Link Ratios:* link ratios are a form of regression and how they can be tested statistically
    b. *Standardization of Residuals: Normality checks – Q-Q plots, histograms*
    c. *Analysis of Residual Patterns*.: plot standardized residuals against the following x-dimensions:
        - Development period;
        - Accident period;
        - Calendar period; and
        - Fitted value.
    d. *Prediction Error and Out-of-Sample Data*.
    e. *Goodness-of-Fit Measures*.
    f. *Principle of Parsimony*.
    g. *Predictive Variability*
    h. *Model Validation*.: systematically remove the last several diagonals from the triangle and make the same forecast of ultimate values without the excluded data

The process of determining forecast distributions consists of a number of steps:

1. Choose a family of models that is suitable for your purpose and sufficiently flexible to model all the features in the data (criteria 1-4).
2. Identify the members of that family that provide an adequate fit to the data (criteria 14-15).
3. Select the "best" models. Are the models reasonable (criteria 5-8, 10)? Do they validate well (criteria 16, 20)? Are simulated datasets similar to the real data (criterion 11)? Are the models parsimonious (criteria 13, 17-18)?
4. Utilize any other information that would improve the model estimates (criterion 12).
5. Decide what assumptions are reasonable for the future, bearing in mind what the data says about the past (criterion 12).
6. Produce forecasts that incorporate model uncertainty, parameter uncertainty and process variability (criterion 19).

Case Study

Here we discuss two of the common approaches – Mack's method and England's Bootstrapping approach and test them on a sample data set.

We use the US Industry data for **Commercial Auto Liability** paid losses, published by AM Best, as our data set.

Here's the data:-

	12	24	36	48	60	72	84	96	108	120
1995	2,080,603	4,400,246	6,187,916	7,405,600	8,093,038	8,473,460	8,666,398	8,751,581	8,804,026	8,834,772
1996	2,298,885	4,670,475	6,642,118	7,957,591	8,793,728	9,194,385	9,407,989	9,471,266	9,530,205	
1997	2,320,272	4,824,652	6,916,389	8,357,273	9,204,483	9,655,188	9,847,382	9,935,205		
1998	2,334,087	4,942,768	7,062,712	8,581,110	9,478,203	9,912,058	10,104,520			
1999	2,486,782	5,329,469	7,656,997	9,300,178	10,192,608	10,523,105				
2000	2,652,433	5,540,718	7,840,405	9,376,418	10,278,541					
2001	2,617,064	5,367,231	7,606,847	9,121,804						
2002	2,292,688	4,790,710	7,061,036							
2003	2,281,043	4,854,306								
2004	2,429,841									

Mack's Method

This is based on the paper "*Measuring the variability in Chain ladder estimates"* by *Thomas Mack* [8]
The steps in this method are:-

- Determine the weighted average factors to be used in the analysis using the following formula

  $f_k = \sum C_{j,k+1} / \sum C_{j,k}$ (j = 1 to I – k, I = total no. of accident years , k denotes k'th development period)

- Determine the square of the distance between each age to age factors and the selected factor using the following formula

  $C_{i,k} * [C_{i,k+1} / C_{i.k} - f_k \text{ est.}]^2$

- Determine the variance of selected development factors using the following formula

  $\alpha_k^2 = (1/ (I – k – 1))* [\sum C_{i,k} * (C_{i,k+1} / C_{i.k} - f_k \text{ est.})^2]$

  For the most prior year we assume $\alpha_k^2 = \min(\alpha_{k-1}^4 / \alpha_{k-2}^2, \min(\alpha_{k-1}^2, \alpha_{k-2}^2))$

- The standard error of the reserves for i'th year are then evaluated as follows

$$(\text{s.e.}(C_{iI}))^2 = C^2_{iI} * \left[\sum_{k=I+1-i}^{I-1} (\alpha_k^2 / f_k^2) * (1/C_{ik} + 1/E_i) \right]$$

where $E_i = \sum_{j=1}^{I-k} C_{jk}$

- The standard error for the overall reserves is then determined based on the following formula

$$(\text{s.e.}(R))^2 = \sum_{i=2}^{I} [(\text{s.e.}(C_{iI}))^2 + C_{iI} * (G_{(i)}) * (H_{(i)})]$$

Where,

$$G_{(i)} = \sum_{j=i+1}^{I} C_{jI}$$

$$H_{(i)} = 2 * (\alpha_i^2 / f_i^2) / E_i$$

The results of using the Mack's method to come up with the Standard Errors for the reserves are given below:-

AY	Reserves	Standard Error	Standard Error %
1995	0	0	0%
1996	0	0	0%
1997	60,727	1,925	3%
1998	82,095	91,924	112%
1999	117,321	257,129	219%
2000	567,941	270,546	48%
2001	1,459,927	271,118	19%
2002	2,801,797	270,317	10%
2003	4,835,939	305,568	6%
2004	7,723,598	364,345	5%
Total	**17,649,344**	**1,000,757**	**6%**

Bootstrapping Method

This is based on the paper "*Stochastic Claims Reserving In General Insurance"* by P.D England & R.J. Verrall [9]
The steps in this method are:-

- Obtain the incremental triangle from cumulative data
- Obtain standard chain ladder development factors from cumulative data (f_k)
- Obtain fitted values from the past triangle based on selected factors and backwards recursion

$$C_{i,k-1} = C_{i,k} / f_k$$ ($C_{i,k}$ = cumulative losses for the ith accident period after k periods of development, I = total number of accident periods, k = development period)

- Obtain incremental fitted values from the fitted triangle by differencing
- Calculate unscaled fitted triangles from the original triangle
- Calculate Pearson scale parameter ϕ

  where $\phi = \sum r_p^2 / (n - p)$ (n is the total number of data points, p is the total number of parameters to be estimated)

  and $r_p = (C - m)/ \sqrt{m}$ (C = actual incremental losses, m = fitted incremental losses)

- Scale the Pearson residuals by adjusting for the degrees of freedom.

  Degrees of freedom = $\sqrt{n/(n-p)}$

- Begin iterative loop, to be repeated N times (N is defined by the user)
  - Resample residuals with replacement creating a new (pseudo) triangle of past residuals
  - For each cell determine the pseudo incremental loss data
  - Create associated set of pseudo cumulative data
  - Fit standard chain ladder model to the pseudo cumulative data
  - Project to form cumulative triangle of future payments
  - Obtain corresponding form of incremental future payments by differencing to be used as mean while simulating from process distribution
  - For each cell (i,j) in the future triangle simulate from a distribution with mean m(i,j) and variance $\phi m(i, j)$
  - Sum the simulated payments in the future triangle for each origin year and overall to give origin year and total reserve estimates separately
  - Store results and return to start of the iterative loop.

The results of using the Bootstrapping method to come up with the Standard Errors for the reserves are given below:-

Acc Yr	Reserves	Standard Error	Standard Error %
1995	0	0	0
1996	0	0	0
1997	88,709	126,586	143%
1998	128,828	156,576	122%
1999	181,148	180,195	99%
2000	702,010	351,185	50%
2001	1,700,674	538,939	32%
2002	3,188,362	818,010	26%
2003	5,345,012	1,245,932	23%
2004	8,520,404	2,434,368	29%
Total	**19,855,147**	**3,115,963**	**16%**

Appendix

1) Guidance Note 12: General Insurance Business: Actuarial Reports by the General Insurance Board, UK. Latest version: 01.09.99
2) Guidance Note 50: General Insurance Principles and Practice by the General Insurance Board, UK. Latest version: 01.04.06
3) Actuarial Standards of Practice 36 by the Actuarial Standards Board, USA. March 2000.
4) Professional Standard 300 [4] - Actuarial Reports And Advice On General Insurance Technical Liabilities by the Institute of Actuaries, Australia. 2006
5) Prudential Standard GPS 210 [5] – Liability Valuation for General Insurers by the Institute of Actuaries, Australia
6) The Implication Of Fair Value Accounting For General Insurance Companies – Clark, Hinton et al
7) The Analysis and Estimation of Loss & ALAE Variability: A Summary Report by the CAS Working Party on Quantifying Variability in Reserve Estimates, chaired by Roger Hayne & James Leise, CAS Forum, Fall 2005
8) Measuring The Variability In Chain Ladder Estimates by Thomas Mack 1994
9) Stochastic Claims Reserving In General Insurance by PD England & RJ Verrall, 2001

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Summary: More than six years of non-life actuarial experience in Europe and US markets. Main focus is on reserving with experience in reinsurance and loss modelling. Proficient in MS Excel & SAS.

Employment History:

Deloitte Consulting India Pvt. Ltd **03/05 – Present**

Manager

- Manage property-casualty actuarial operations in Hyderabad office of Deloitte's Actuarial and Insurance Solutions Practice.
- Perform duties such as reserving and audit support.
- Work on Research projects like developing reserve ranges or enhancing the Deloitte uniform reserving model
- Participate in recruiting, training, mentoring and development of department personnel in Hyderabad office.

GE Insurance Solutions, Gurgaon, India **06/00 – 03/05**

Assistant Manager

- Manage India reserving team.
- Perform regular reserve review of European reinsurance segments using ReservePro.
- Create profitability and actual vs. expected monitoring reports
- Global coordination on the establishment of reserve ranges
- Stochastic Claims Reserving
- Catastrophe Loss Modeling using Catrader

Education:

Diploma in Actuarial techniques from Institute of Actuaries, UK
Currently taking Associateship exams
Masters of Statistics, Indian Statistical Institute, Calcutta, India
Bachelors of Statistics (Hons) with Distinction, Indian Statistical Institute, Calcutta, India
Member of the Actuarial Society of India

Other Trainings:

On-job-training on reserving in Deloitte & Touche, Hartford, USA
Technical Training in Deloitte & Touche, Orlando, USA

Advanced Programming Techniques in SAS & SAS Macros in SAS Institute, Brussels, Belgium
On-job-training on reserving in GE Insurance Solutions, Munich, Germany
Training on Insurance and Actuarial Basics in GE Insurance Solutions, Kansas City, USA
Certified Green Belt in Quality DMAIC techniques.

Achievments:

Team award for Catastrophe Loss Modeling work in GE Insurance Solutions
Received prize for all-India highest score in Stochastic Modelling exam
Cleared all Core Technical Series exams on first attempt
Cleared CAS Exam I with the highest score of 10