

Investment Guarantees: Issues and Considerations for Life Insurers

By Sanchit Maini & Varun Gupta

Abstract

The paper considers the various types of investment guarantees prevalent in the Indian life insurance market. The paper discusses various available approaches to quantify and manage such guarantees. We subsequently illustrate the use and significance of two commonly used risk measures using a non-participating and a unit-linked product with guarantees.

Keywords

Quantile Risk Measure; Conditional Tail Expectation; Investment guarantees.

1. Introduction

1.1. The insurance industry performs two core functions, underwriting of risk and investment of the monies received for assuming the risk. Risk management has historically focused on the insurance risks (such as mortality and morbidity) with investment not being considered a major source of risk. Traditional insurance products meant insurers assuming the entire investment risk but such contracts typically involved guaranteed benefits being matched with bond portfolios using immunization techniques. The advent of unit-linked products saw life insurers passing the investment risk to the policyholder. Many such products incorporate guaranteed benefits payable at death or maturity and thus contain explicit investment guarantees.

1.2. India has seen the launch of unit-linked products in the last five years and many of these products have introduced investment guarantees. This shift towards providing investment guarantees has been particularly pronounced in the last year with the stock markets crossing historic all time highs. Another possible reason for the introduction of guarantees may be the efforts by many private players to widen the customer base of such products.

1.3. Section 2 contains some comments on the nature of investment guarantees prevalent in India. In Section 3 we comment on the nature of investment risk and contrast it with diversifiable risk. Various methods of guarantee provision are discussed in Section 4. Risk measures are discussed in Section 5. Sections 6 and 7 contain practical examples of modelling investment guarantees illustrating the financial impact through two risk measures.

1.4. All views expressed in this paper are those of the authors and not necessarily those of their employer.

2. Investment Guarantees in India

2.1. Investment guarantees have existed in India in traditional products in the form of guaranteed maturity values and guaranteed surrender values in non-participating (“non-par”) products and a certain guaranteed component in participating products as well. With the low interest rate environment prevailing in the last few years there has been a steady decline in the investment guarantees being offered by several players including the LIC. This trend towards lowering the guarantees has also been encouraged by the IRDA.

2.2. With the increased penetration of unit-linked products in the Indian markets, the majority of the investment risk was passed on to the policyholder. The higher volatility seen in the Indian stock markets and also the all time high levels that we have witnessed during the past year have made many potential customers wary of taking on the investment risk. A natural response by many players has been to offer some investment guarantees to protect the downside risk whilst providing the opportunity to gain from returns linked to the market.

2.3. Apart from the basic implicit guarantee available in unit-linked contracts with respect to a minimum sum assured payable on death, several players offer guarantees on death or maturity which are more onerous and hence have to be adequately priced and reserved for.

Primary Investment Guarantees in India	
Guarantee	Event
Capital (Allocated fund net of charges)	Maturity
Capital (Return of total premium paid till date)	Maturity
Capital with growth (Allocated fund net of charges grows at a specified minimum rate every year)	Maturity
Capital (Allocated fund net of charges along with interest credited every year)	Maturity, Death

3. Nature of Risk

3.1. Insurance risks such as mortality can be effectively managed by diversification. This follows from the statistical theory whereby the uncertainty in estimating a quantity (such as the average claim cost) reduces with the size of claims. Thus a large portfolio of insurance policies provides a good basis for estimating the loss with a high degree of certainty. Investment risk is a non-diversifiable or systemic risk in which there is only a limited benefit of diversification. For example, an investment guarantee on a unit-linked portfolio may affect the entire portfolio if the assets backing them fall in value.

3.2. Diversifiable risks have been managed by life insurers using deterministic actuarial techniques with loss estimates based on expected values. Such values typically involve a margin over the best estimate to increase the degree of certainty in meeting the liabilities. The nature of non-diversifiable risks does not lend themselves to being managed in a similar manner. Incorporating a margin in the best estimate will typically underestimate the magnitude of non-diversifiable risks. Stochastic techniques, which have been used for a number of years in the general insurance industry, are increasingly being accepted as the standard practice for managing such risks in life insurance.

4. Guarantee Provision

4.1. In order to provide for the investment guarantees in insurance contracts there are broadly four options available:

- Reinsurance
- Dynamic Hedging
- Actuarial approach
- Ad hoc approach

Reinsurance

4.2. Insurers can provide for the liabilities arising out of the investment guarantees by buying equivalent options sold by other risk institutions. This is equivalent to reinsuring the entire amount of investment guarantee risk. Some reinsurers have been involved in such transactions. Along with passing the risk to the reinsurer, the insurer will also pass off most of the profits as well. Even when the entire amount of risk has been passed off in this manner a counterparty risk will remain with the insurer.

Dynamic Hedging

4.3. This is the financial economic approach to managing investment guarantees. Under this approach, a replicating portfolio of assets is constructed using for example the Black-Scholes framework. This framework gives the amount of assets to be invested in equities – the equity backing ratio – (“EBR”), which varies over time. The portfolio will thus require constant rebalancing (in practice at discrete intervals say weekly). The Black-Scholes framework does assume strong assumptions but nevertheless provides a powerful tool for hedging the liability. This approach assumes that the amount of assets set aside to provide for the guarantee is invested in the replicating portfolio. One of the basic criterion for this approach to be valid is presence of Capital markets that offer both depth and breadth to the participants. Currently in India this is not the case with limited availability of derivatives.

Actuarial Approach

4.4. Under the actuarial approach, a distribution of the guarantee is found using stochastic simulation. This distribution is converted into a capital requirement by using a quantile risk measure at a particular level say 99.5% (see Section 5.9 for details). This amount is discounted at the risk-free rate of interest.

Ad hoc Approach

4.5. The ad hoc approach uses judgment to provide for the liability. It has been used in the past where low frequency type options existed and often led to little or no provision for such guarantees.

5. Risk Measures

5.1. In order to model any investment guarantee a decision first needs to be made whether to use deterministic or stochastic techniques to model the return on assets. The scope of this paper does not include a comparison of the two approaches. As discussed in section 3.2 stochastic techniques are becoming the norm for non-diversifiable risks such as investment guarantees. We discuss the application of stochastic techniques assuming an appropriate asset model is available. This is not to suggest that the choice of such techniques or the asset model is by any

means a trivial task. Indeed, a discussion on this would be the subject of a separate paper by itself.

5.2. Using the output of the stochastic asset model, the reduction in the present value of the surplus can be found for each scenario. For pricing investment guarantees insurers may want to charge for the guarantees in order to restore the present value of surplus with a given level of probability. In order to view such output in graphical form, the simulated density function can be plotted. We then need appropriate risk measures in order to convert the distribution into a single value. A risk measure is defined as “a method of encapsulating the riskiness of a distribution in a single number or in a real-valued function” 1.

5.3. There are plenty of risk measures available with no unanimity of a single measure that outscores others in all aspects. It is said that, “Risk, like beauty, is in the eye of the beholder”. This echoes the different risk appetite of players in the market.

5.4. For readers interested in a detailed background on risk measures, we refer them to 1 “*A General Theory of Investment Risk*”. As pointed out in the paper, any realistic measure of investment risk should be:

- Asymmetric
- Relative to a benchmark
- Investor specific
- Non-linear

5.5. The asymmetric nature of risk arises from the fact that most investors are only concerned with the downside risk and thus a risk measure should treat upside and downside risks differently. Note that the standard deviation, which is often used as a risk proxy, is an example of a symmetric risk measure.

5.6. Risk is typically relative, for example a loss of capital invested. Thus it must be measured with respect to a benchmark. The benchmark may be a liability driven value, such that the risk measure looks at the probability of the assets falling short of the projected value of liabilities. In case the liability value is not known a proxy benchmark may be used. Other examples for the benchmark may be the budgeted return, risk-free rate of return, or inflation. The range of benchmarks makes it clear that risk is investor specific.

5.7. Risk is non-linear. This is illustrated in many surveys where two scenarios with the same expected loss are contrasted, one which has a large number of small losses; the other which has a small probability of a single large loss. Most investors view the latter scenario as far more risky. Indeed this is observed in the purchase behaviour of insurance where people hardly insure events that have a high probability of small losses.

5.8. We give below two risk measures, which are illustrated through practical examples in Sections 6 and 7.

Quantile Risk Measure

- 5.9. The quantile risk measure for a variable G is defined as the 100α percentile of the distribution of G. Mathematically, for a parameter α, 0 ≤ α ≤ 1, it is defined as:

$$V_{\alpha} = \inf \{V: \Pr [G \leq V] \geq \alpha\}$$

- 5.10. In words V_{α} is the 100α percentile of the loss distribution. Thus, V_{α} is the smallest sum to hold in risk-free assets (assuming that the assets are invested in risk-free assets) such that at maturity the probability of meeting the guarantee cost is at least α. The quantile risk measure is the basis of the *Value-At-Risk* calculation commonly used in banking.
- 5.11. The quantile risk measure does not capture the shape of the distribution on either side of the 100α percentile and in particular the right tail beyond the threshold.
- 5.12. The quantile risk measure is illustrated with some examples in the next section.

Conditional Tail Expectation

- 5.13. An alternative, and increasingly popular, risk measure is the conditional tail expectation (“CTE”). The CTE is also determined with respect to a parameter whose values lies between 0 and 1. For a given value the CTE is defined as the expected value of the loss, given that the loss falls in the upper (1-α) tail of the distribution. In order to relate the two measures, the CTE may be expressed mathematically as:

$$CTE_{\alpha}(G) = E[G | G > V_{\alpha}]$$

- 5.14. Thus the CTE is the expected value of the tail of the guarantee distribution. The CTE measure takes into account the exact shape of the distribution beyond the quantile. For example, two guarantee distributions may have the same 95 percent quantile but one of them may have a fatter tail than the other. The quantile risk measure will ignore this difference in the two measures, whereas the CTE will fully take this into account.
- 5.15. The quantile risk measure is an order-statistic, as it takes a single observation from the ordered distribution of guarantee liability. The CTE measure takes the average of a set of the largest outcomes. This makes the CTE measure more robust and hence less sensitive to sampling error. The CTE will always be more onerous than the equivalent quantile risk measure with the two measures converging as the value of alpha increases to 1.
- 5.16. The CTE measure is becoming prevalent with insurance regulators. For example in Canada, reserves for unit-linked products with guarantees are set at the CTE with around 80 percent and total solvency capital, including the reserve, is set at the CTE with around 95 percent 2. The Actuarial Society of Hong Kong’s guidance note on reserving for investment guarantees AGN8 also mentions the CTE measure at an alpha of 96 percent 3.
- 5.17. The ASI’s draft guidance note on investment guarantees (GN22) also made reference to the CTE as a measure for determining reserves.

6. Investment Guarantee – Non-Participating Product

6.1. In this section we first set out the methodology adopted in modelling an embedded investment guarantee on a traditional non-par product.

Methodology

6.2. As already mentioned, the paper illustrates the interpretation of results in a stochastic framework and the asset model used here does not form a part of the scope of this paper. However for the sake of completeness and reference, we must mention that we have used an autoregressive asset model with a mean reversion level of interest rates equal to 5% for projection of yield curves.

6.3. We consider a limited pay endowment assurance product. The product is a typical traditional non-par product sold in India. Further details and modelling assumptions for this sample product are set out in Appendix 1.

6.4. The premium rate has been set to target a given internal rate return. In order to illustrate the impact of investment guarantees we have illustrated the results by pricing the product using three sets of investment return assumption 3.5%, 4% and 5%. Thus, under the deterministic pricing at these investment returns, the premium rate has been set to achieve a zero present value of the surplus.

6.5. The determination of surplus requires reserves to be calculated for each policy duration. This is done using an iterative process explained in the following paragraphs.

6.6. In order to calculate the earned interest rate over a time period we have to consider the rates at which the current period cash flows get invested as well as the investment return locked in for cash flows that occurred in the past. A series of cash flows excluding reserves, solvency margin and investment income are invested in bonds of chosen duration, which are assumed to be held to maturity. Such cash flows will be subject to reinvestment risk in case the bonds mature before the policy duration. Thus a first-cut earned interest rate can be derived using the formula:

$$\text{Earned rate (t)} = \text{Investment income (t)} / (\text{Book Value (t)})$$

6.7. Appropriate margins for adverse deviations can then be applied to these earned interest rates and using the discounted cash flow approach we can obtain the first-cut reserves and solvency margin. This process can be termed as the first iteration.

6.8. The exercise described above is then repeated with the cash flows now including the first-cut reserves and solvency margin. These would result in a revised estimate of the earned interest rates, the so-called second-cut values. These second-cut earned rates together with MADs will lead to the second-cut reserves and solvency margin. This completes a second iteration. These iterations are repeated till an acceptable level of convergence (set to 0.01% in this instance) is observed in the earned interest rates.

6.9. Investment income for the time period is calculated in two parts given below:

- The first part by investing the cash flows occurring in the time period at the short rate for the period;

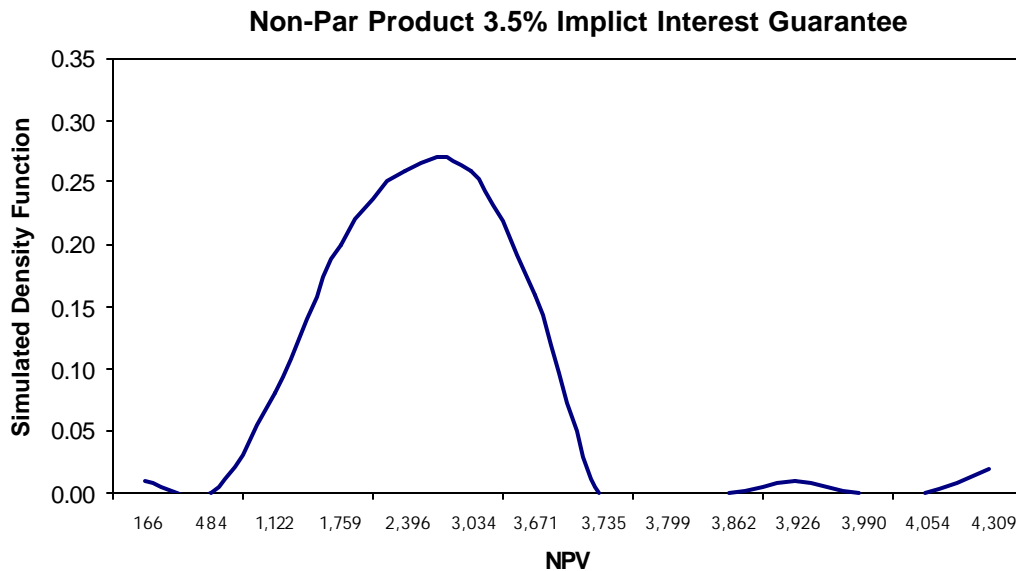
- The second part on the back of the reserves and solvency margin from the previous period earning the calculated earned interest rate for the period.

6.10. The surplus arising for the time period can then be calculated using the cash flows, investment income and the increase in reserves and solvency margin. The present value of surplus is determined for each scenario using output from a stochastic asset model as mentioned above. The aim of this exercise is to illustrate the range of possible surplus (positive/negative) scenarios that may occur.

Results

6.11. This part of the section contains sample output, including a simulated density function of the surplus scenarios and the two risk measures – the quantile risk measure and the CTE.

6.12. The surplus shown under the stochastic scenarios can be positive or negative. Since the deterministic scenarios anchor at zero, any negative surplus scenarios indicate that the deterministic pricing is inadequate whereas a positive surplus scenario indicates excess profits over the deterministic surplus estimate. Since the insurer takes on all of the investment risk under such a non-par product, any deviations from the embedded investment return flow through as a profit or loss.



6.13. The graph above shows the density function, that is, the range of surplus scenarios, with the associated probability to illustrate the likelihood of achieving the particular values. Note that the probability for each scenario is simply determined as the inverse of the number of simulations performed (each scenario is equally likely). The graphs shown in this section are based on 100 scenarios and hence may be subject to some simulation error.

6.14. The reason for the surplus scenarios exceeding zero is that the mean reversion level assumed for projecting the yield curve is 5% compared to the implicit interest guarantee of 3.5%. It can be observed that most of the values are concentrated around a central value of around 2,300. However, the distribution clearly illustrates the asymmetric nature of the range of values with the tail on the right being ‘fatter’ than the left tail. This occurs due to the nature of an auto-regressive interest rate model in which interest rates are ensured to be positive. There is not a

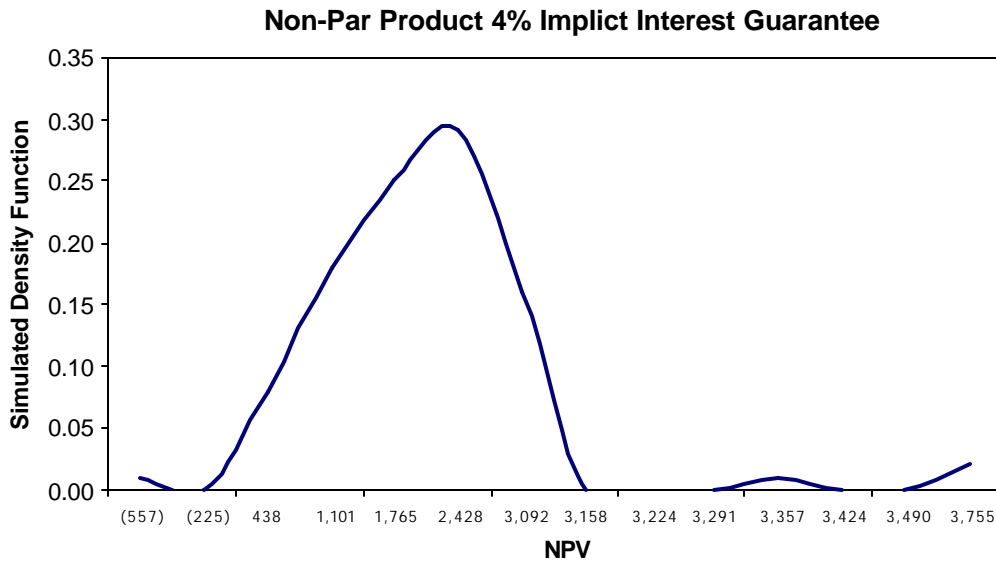
single scenario that leads to a negative surplus illustrating that a 3.5% interest guarantee is not onerous when viewed along with an assumption of an auto-regressive interest rate model with a mean reversion of 5%.

6.15. The table below shows the two risk measures at three quantiles, 90 percent, 95 percent and 99 percent. These measures can be used to set an acceptable level of minimum surplus with an associated probability.

	Quantile (Alpha)		
Risk Measure	0.9	0.95	0.99
Quantile Risk Measure	1,179.23	921.56	519.44
CTE	757.11	494.16	165.64

6.16. Since a positive surplus in the graph above shows profits (and none of the scenarios show a loss) we are interested in observing the left tail of such a distribution. Thus the CTE, in this case, is below the equivalent quantile risk measure (indicating lower profits). To illustrate, the CTE at alpha of 99% implies that with 99% probability the expected surplus will be 165.64.

6.17. The impact of an increase in the embedded investment guarantee, that is a reduction in the premium, is shown by using 4% and 5% rates of return for pricing the product.

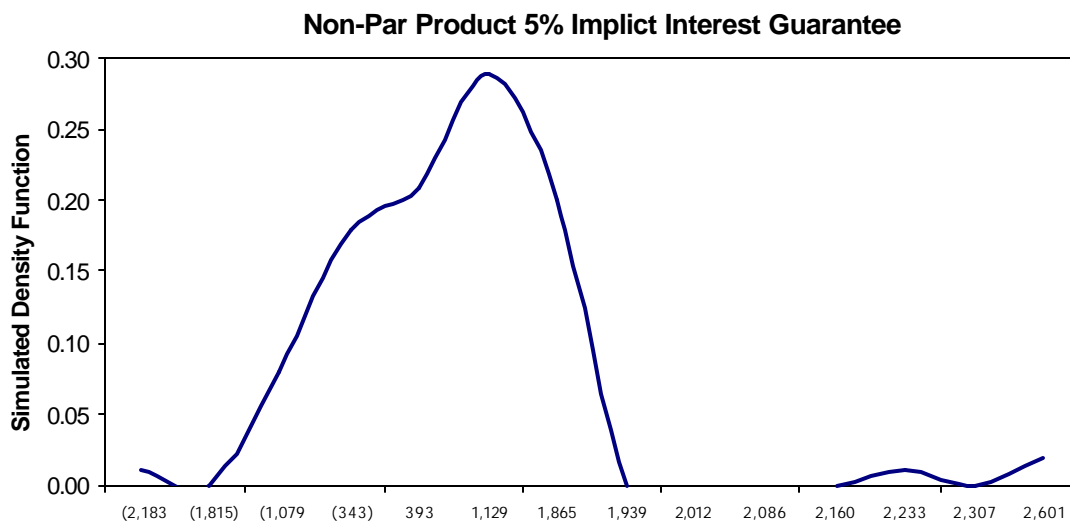


6.18. A modest increase in the guarantee from 3.5% to 4% (keeping the mean reversion level the same at 5%) gives rise to 5% scenarios in which the surplus turns negative. Another way of looking at this is that the quantile risk measure is zero at an alpha of 95%.

	Quantile (Alpha)		
Risk Measure	0.9	0.95	0.99
Quantile Risk Measure	486.75	0	(211.52)
CTE	53.48	(228.87)	(557.19)

6.19. As mentioned in Section 5.5, the CTE is a more onerous measure (in this case a negative figures implies a loss) than the equivalent quantile risk measure. We can solve for the alpha that equates to a CTE of zero; this comes out to 91%. Effectively the probability of achieving a zero quantile risk measure is higher than achieving a zero CTE.

6.20. Although it can be argued that the 95 quantile is too high for pricing it may not be so for reserving. Thus it is possible that there may be reserving implications by offering a 4% interest guarantee without charging the policyholder for such a guarantee leading to additional capital requirement by the shareholder.



6.21. Increasing the investment guarantee to 5% shifts the distribution significantly as can be observed from the graph above.

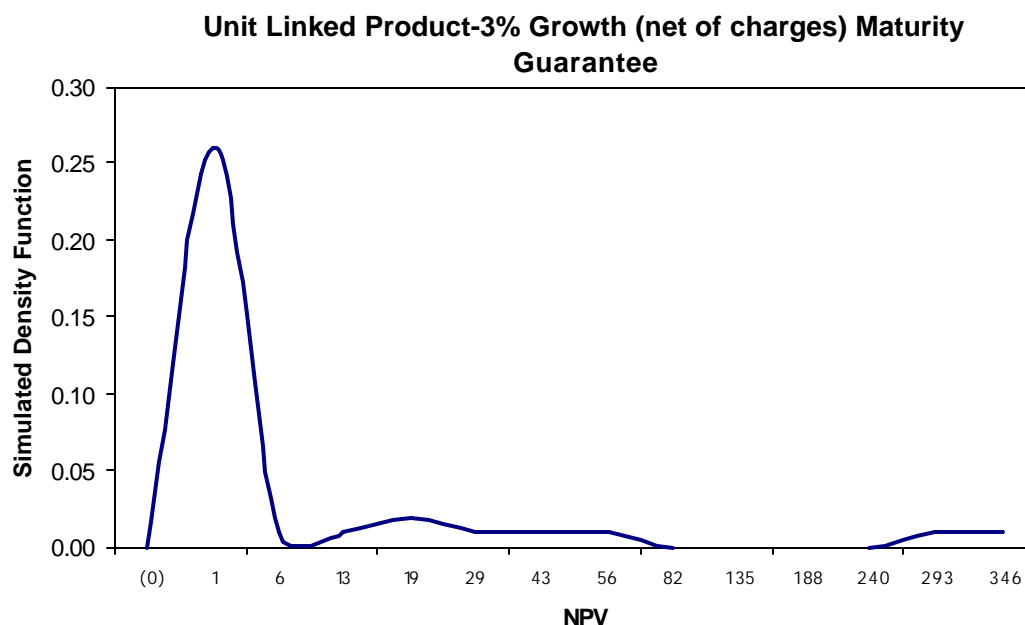
	Quantile (Alpha)		
Risk Measure	0.9	0.95	0.99
Quantile Risk Measure	(1,030.56)	(1,325.37)	(1,808.30)
CTE	(1,496.46)	(1,816.81)	(2,182.95)

6.22. The probability of the surplus being negative is 35%, while the 90th quantile surplus is (1,036.56). This can be interpreted in words as: the maximum loss with 90% probability is 1,036.56. The CTE of (1,496.46) gives the expected loss given the 10% extreme scenarios. Solving for the alpha that yields a zero risk measure we get 65% for quantile risk measure (since 35% cases result in negative surplus) and 21% for the CTE. It can be readily observed that as the guarantee increases the difference in alpha between the two risk measures increases rapidly.

6.23. The results must be viewed in light of the assumptions for calibrating the stochastic asset model, in particular the mean reversion level of the interest rates. They do however illustrate that implicit guarantees, which may be viewed as modest in a deterministic framework, do become material when viewed in a stochastic framework. The advantage of such a framework is that, given the risk appetite of an insurer, a more informed decision regarding the financial implications of an investment guarantee may be made.

7. Investment Guarantee - Unit-Linked Product

7.1. In this section we consider a typical unit-linked product being sold in India. The product details are set out in Appendix 2. We have modelled a guaranteed maturity benefit which is illustrated at guaranteed interest rates, net of charges, of 3%, 4% and 5%. In order to assess the impact of the cost of guarantee the surplus is first determined without the guarantee and then the maturity guarantee is introduced. The reduction in the surplus represents the cost of guarantee at maturity. The present value of the cost of guarantee is illustrated in the paragraphs below. Note that the cost is floored at zero; such scenarios indicate that the guarantee does not bite. The asset allocation assumes 25% invested in equities. For the unit-linked product, the graphs below show the cost of guarantee with a positive amount representing a cost, i.e. reduction in surplus due to the guarantee. The risk measures are therefore shown for the right tail, as we are concerned with the rise in the cost of guarantee. The CTE measure would thus lie to the right of the quantile risk measure.



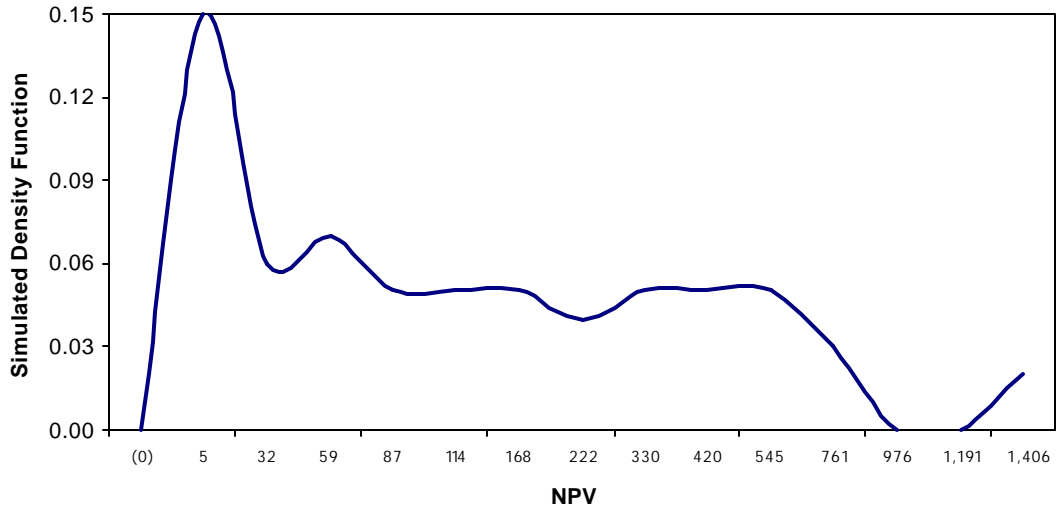
7.2. The probability of the 3% interest guarantee biting is 35%. The scenarios as distributed are shown above, with a mode of 0. Most of the scenarios show a modest cost of guarantee. The asymmetric nature of the distribution is again evident from the graph.

	Quantile (Alpha)		
Risk Measure	0.9	0.95	0.99
Quantile Risk Measure	0	20.13	271.06
CTE	79.94	147.14	345.84

7.3. As mentioned in Section 5.17 the CTE can be used to reserve for the investment guarantee by choosing an appropriate level of alpha. For example, at the 95th quantile the table above suggests a provision of 147.14. At the 99th quantile the provision more than doubles.

7.4. The graph below illustrates the increase in the cost of guarantee when it is set at 4%. The probability of the guarantee biting increases to 67%. Note the fat tail.

Unit Linked Product-4% Growth (net of charges) Maturity Guarantee

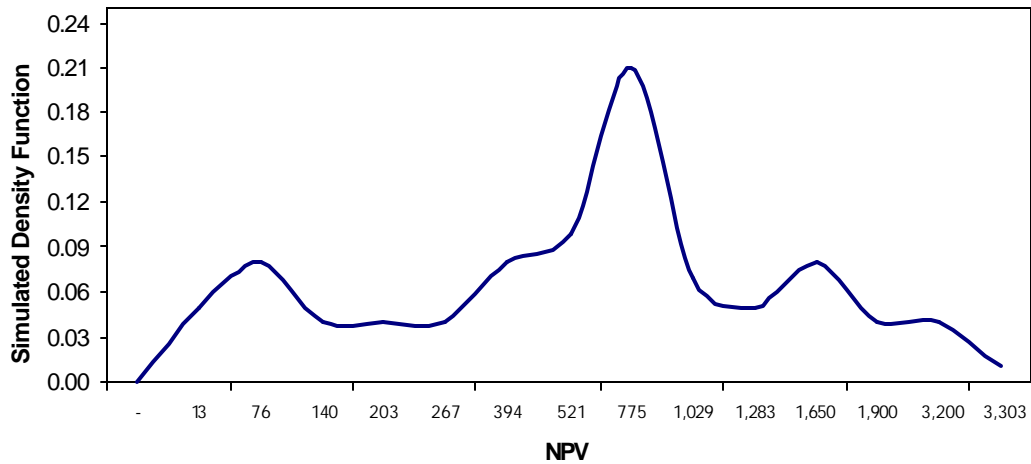


7.5. The table below shows how quickly the cost of guarantee rises.

Risk Measure	Quantile (Alpha)		
	0.9	0.95	0.99
Quantile Risk Measure	413.93	538.03	1,236.85
CTE	681.89	878.88	1,406.35

7.6. Finally, results for 5% interest guarantee are set out below.

Unit Linked Product-5% Growth (net of charges) Maturity Guarantee



7.7. The guarantee bites in 93% of the cases. This graph shows how a high level of guarantee shifts the mode of the cost of guarantee, similar to the non-par example.

Risk Measure	Quantile (Alpha)		
	0.9	0.95	0.99
Quantile Risk Measure	1,604.90	1,824.63	3,078.49
CTE	2,112.34	2,486.07	3,302.82

7.8. The table illustrates the convergence of the two measures as the value of alpha increases.

8. Conclusion

8.1. With the low interest rate environment prevailing in the last few years there has been a steady decline in the investment guarantees being offered by several players including the LIC on traditional products. At the same time unit-linked products have witnessed strong growth. With many investors becoming wary of entering the equity markets at record high levels several players have introduced guarantees on UL products to broaden their appeal.

8.2. The paper considers the financial implications of offering investment guarantees on a non-par and a UL product. It illustrates the use of stochastic modelling through two risk measures, the quantile risk measure and the CTE. The quantile risk measure has long been used in the banking industry for managing risk while the CTE measure is increasingly being used in the insurance industry.

8.3. The stochastic techniques discussed and the resulting analysis may also be used for resolving other product related issues like an optimal investment asset backing strategy, charging structure and effective risk management.

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Appendix 1

Sample Non-Participating Policy

Policy Details	
Age	30 Yrs
Policy Term	20 Yrs
Premium Payment Term	10 Yrs
Sum assured	100,000
Death Benefit	100,000
Maturity Benefit	100,000

Modelling assumptions

- The asset model assumed is an autoregressive one with a mean interest rate reversion level of 5%.
- Interest rates MADs for reserving purposes are assumed to be 10% of earned rates.
- The model assumes investment in zero-coupon bonds whereas assuming coupon-paying bonds would have better captured the reinvestment risks.
- The bonds purchased are assumed to be of constant duration whereas a more realistic approach would be to chase the liability duration.

Appendix 2

Sample Unit-linked Policy

Policy Details	
Age	30 Yrs
Policy Term	20 Yrs
Premium Payment Term	20 Yrs
Annual Premium	Rs20,000
Sum Assured	Rs200,000
Death Benefit	Sum Assured plus Value of Units
Guarantee	Allocated fund net of charges grows at a predefined (3%, 4% and 5% rate)
Maturity	Maximum of Value of units or the Guaranteed Value

Charging Structure	
Initial allocation load	25%
Subsequent Premium Load	2%
Administration charge (Monthly)	Rs100
Fund management charge (same across funds)	1%
Set up charge (First 5 Years)	Rs2,000
Mortality Charge	In accordance with Indian Assured Lives Mortality Table

Modelling assumptions

- The asset model assumes an autoregressive interest rate process with a mean reversion level of 5%.
- Return on equity is modelled as a function of the short rate with an appropriate risk premium on top of the short rate.
- The Premiums net of charges are assumed to be invested in a fund with allocation of
 - Money Market/Cash 20%
 - Debt 55%
 - Equity 25%
- The asset allocation has been assumed to be fixed for modelling simplifications, whereas a more realistic and optimal asset strategy would involve dynamic allocation varying with policy duration.

About the Authors

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Sanchit joined Max New York Life in 2005 as an Assistant Vice President and is responsible for product pricing of all lines of business.

Prior to this he worked as an Actuarial Consultant with Watson Wyatt in many countries including the UK, India and South East Asia. His experience at Watson Wyatt included product development, valuation, market entry and business planning for life and non-life insurers. He also worked on stochastic asset liability modelling and market consistent asset modelling projects for clients in the UK.

Before joining Watson Wyatt he was working for GE Capital, both in the US and India. At GE Capital, he was working in product development for the universal life and fixed annuity lines of business.

Sanchit graduated in Statistics from Delhi University in 1997 and later completed a postgraduate degree in Actuarial Science from Melbourne University in 1999. He qualified as a Fellow of the Institute of Actuaries of Australia in 2004. He is also a Fellow of the Actuarial Society of India.

Sanchit is a member of the Financial Risk Management and Education Boards of the ASI.

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Varun started his actuarial career when he joined Max New York Life in 2004 as a part of the Valuation team. He has been working in areas like US GAAP valuation, asset liability studies, stochastic modelling and business planning. He is a member of the ASI and the Institute of Actuaries, UK and has completed the CT series examinations.

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