Shapes of Yield Curve: Principal Component Analysis & Vector Auto Regressive approach

By Subhash Chandra

Abstract

Most economists agree that two major factors affect the shape of the yield curve: investors' expectations for future interest rates and certain "risk premiums" that investors require holding long-term bonds. Because the yield curve can reflect both investors' expectations for interest rates and the impact of risk premiums for longer-term bonds, interpreting the yield curve can be complicated. Economists and fixed-income portfolio managers put great effort into trying to understand exactly what forces are driving yields at any given time and at any given point on the yield curve. The way in which these forces simultaneously work to shape the yield curve can be understood. The main objective of this paper is to throw some light on the shape and cause of shapes of yield using Principal Component Analysis. Three factors have been identified, which are almost 99% responsible for the change and shift in the shape of yield curve. A Vector Auto Regressive approach has been applied to those factors, which explains and estimates the shape of yield curve*.

1. Introduction

Managing portfolios of financial instruments is in essence managing the tradeoff between risk and return. Optimization is a well suited and frequently used tool to manage this tradeoff. Financial risks arise due to the stochastic nature of some underlying market parameters such as interest rates. So, it is necessary to include stochastic parameters in optimization for portfolio managing, turning portfolio optimization in to stochastic optimization or stochastic programming. A vital part of stochastic programming in portfolio management is scenario generation. Monetary policy makers and observers pay special attention to the shape of the yield curve as an indicator of the impact of current and future monetary policy on the economy. However, drawing inferences from the yield curve is much like reading tea leaves if one does not have the proper tools for yield-curve analysis. The objective is therefore to construct a model capable of capturing the interest rates in order to generate interest rate scenarios.

1.1 What is yield? Yield refers to the annual return on an investment. The yield on a bond is based on both the purchase price of the bond and the interest, or coupon, payments received. There are two ways of looking at bond yields: current yield and yield to maturity.

- *Current yield* is the annual return earned on the price paid for a bond. It is calculated by dividing the bond's annual coupon interest payments by its purchase price. For example, if an investor bought a bond with a coupon rate of 6% at par, and full face value of Rs.1,000, the interest payment over a year would be Rs.60. That would produce a current yield of 6%. When a bond is purchased at full face value, the current yield is the same as the coupon rate. However, if the same bond were purchased at less than face value, or at a discount price, of Rs.900, the current yield would be higher at 6.6%).
- *Yield to maturity* reflects the total return an investor receives by holding the bond until it matures. A bond's yield to maturity reflects all of the interest payments from the time of

* It is important to emphasise that the purpose of the model is not to produce superior yield curve predictions, i.e. predictions that in any sense are assumed to out-perform the market and thereby may serve as a basis for tactical investment decisions aimed at outperforming a given benchmark strategy. Rather it is a tool, which supports the investment process related to strategic asset allocation decisions.

purchase until maturity, including interest on interest. Equally important, it also includes any appreciation or depreciation in the price of the bond. Yield to call is calculated the same way as yield to maturity, but assumes that a bond will be called, or repurchased by the issuer before its maturity date, and that the investor will be paid face value on the call date. Because yield to maturity (or yield to call) reflects the total return on a bond from purchase to maturity (or the call date), it is generally more meaningful for investors than current yield. By examining yields to maturity, investors can compare bonds with varying characteristics, such as different maturities, coupon rates or credit quality.

1.2 What is yield curve? The simplest kind of bond is called a zero-coupon bond. A zero-coupon bond (also known as a discount bond) makes a single payment on its maturity date. By contrast, a coupon bond makes periodic interest payments (called coupon payments) prior to its maturity when it also makes a final payment that represents repayment of principal. A coupon bond may be thought of as a portfolio of zero-coupon bonds. The yield curve is a line graph that plots the relationship between yields to maturity and time to maturity for bonds of the same asset class and credit quality. The line begins with the spot interest rate, which is the rate for the shortest maturity, and extends out in time, say, to 30 years. Investors use the yield curve as a reference point for forecasting interest rates, pricing bonds and creating strategies for boosting total returns. The yield curve has also become a reliable leading indicator of economic activity. *Fig.1* shows two yield curves for two different dates.

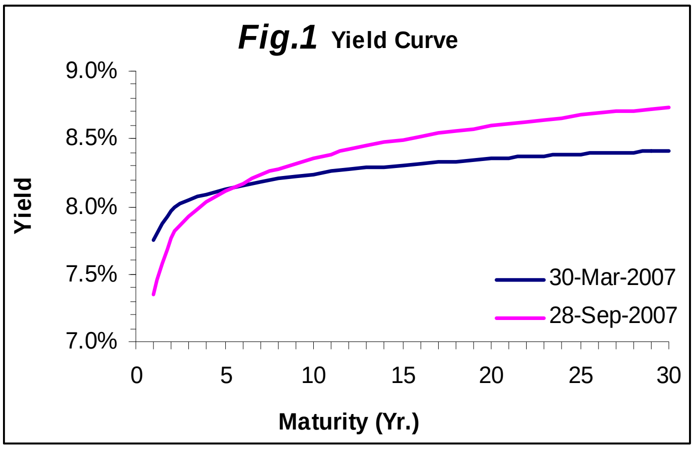


Fig.1 **Yield Curve**

1.3 Various shapes of yield curve?

Yield curves can have various characteristics depending on economic circumstances at a given point in time. An upward sloping curve with increasing but marginally diminishing increases in the level of rates, for increasing maturities, is commonly referred to as a *normal* shaped yield curve. The reason for this naming is due to the fact that this is the shape of a yield curve considered to be normal for economically balanced conditions. Other types of yield curves include a *flat* yield curve where the yields are constant for all maturities. A *humped* shaped yield curve has short and long term yields of almost equal magnitude, different from the medium term yields which are consequently either higher or lower. An *inverted* yield curve is converted invert normal shaped curve, i.e., a downward sloping yield curve with decreasing but marginally diminishing decreases in yields. In *Fig.2* all four types of yield curve have been shown as an example.

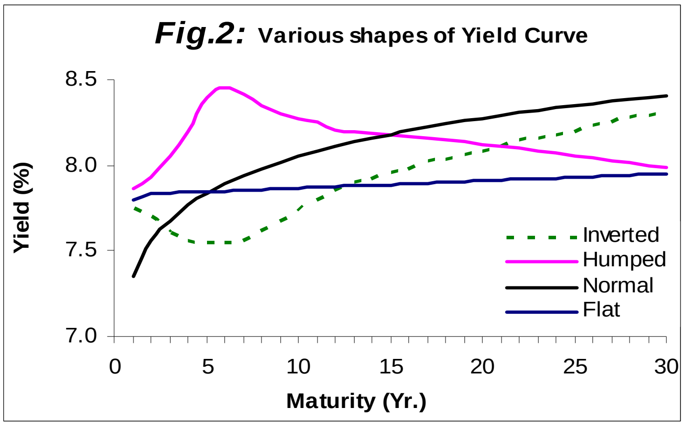


Fig.2: **Various shapes of Yield Curve**

1.4 What determines the shape of yield curve? Most economists agree that two major factors affect the slope of the yield curve: investors' expectations for future interest rates and certain "risk premiums" that investors require holding long-term bonds.

Three widely followed theories have evolved that attempt to explain these factors in detail:

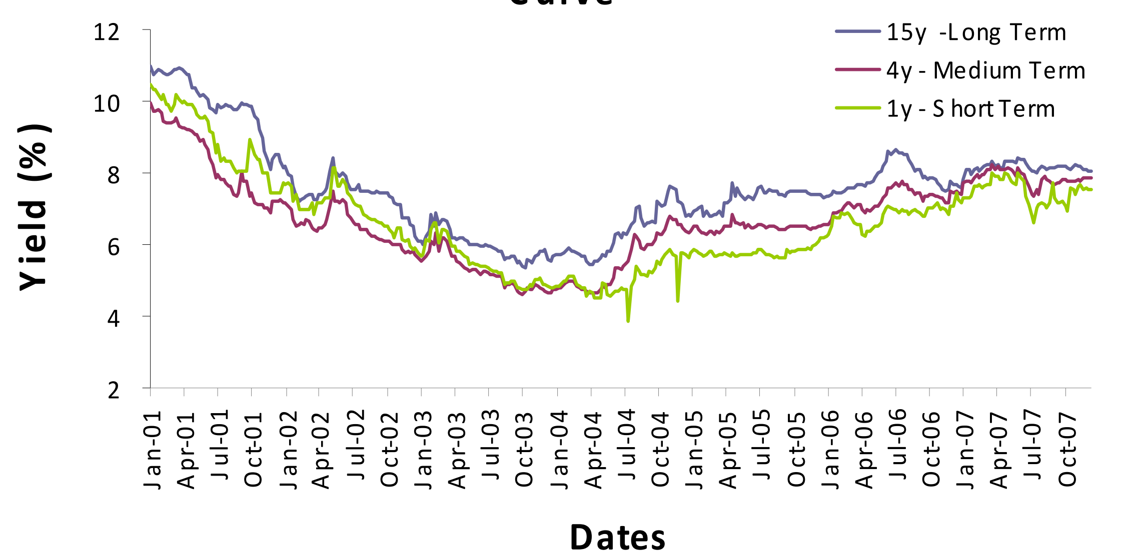
- The *Pure Expectations Theory* holds that the slope of the yield curve reflects only investors' expectations for future short-term interest rates. Much of the time, investors expect interest rates to rise in the future, which accounts for the usual upward slope of the yield curve.
- *The Liquidity Preference Theory*, an offshoot of the Pure Expectations Theory, asserts that long-term interest rates not only reflect investors' assumptions about future interest rates but also include a premium for holding long-term bonds, called the term premium or the liquidity premium. This premium compensates investors for the added risk of having their money tied up for a longer period, including the greater price uncertainty. Because of the term premium, long-term bond yields tend to be higher than short-term yields, and the yield curve slopes upward.
- Another variation on the Pure Expectations Theory, the *Preferred Habitat Theory* states that in addition to interest rate expectations, investors have distinct investment horizons and require a meaningful premium to buy bonds with maturities outside their "preferred" maturity, or habitat. Proponents of this theory believe that short-term investors are more prevalent in the fixed-income market and therefore, longer-term rates tend to be higher than short-term rates.

Because the yield curve can reflect both investors' expectations for interest rates and the impact of risk premiums for longer-term bonds, interpreting the yield curve can be complicated. Economists and fixed-income portfolio managers put great effort into trying to understand exactly what forces are driving yields at any given time and at any given point on the yield curve.

2. Available Data

Historical data for Indian G-security returns has been taken for all the analysis. Time period chosen is from January 2001 to December 2007. Though daily data was available but here weekly observation has been taken because of non availability of rates for some of the maturity years. Also, data was not available for all the maturity years from January 2001. Some adjustment has been done in that respect. Some proxy observations have been taken. As for example, say, for 3 year maturity observation for one 21st February 2001 is not available. So, as a proxy, observation from nearest maturity year rate has been taken. The data set covers 363 dates with 1 year to 15 year maturity. Because of

Fig.2.1: Short, Medium & Long term Yield Curve



unavailability of observations for more than 15 year maturity rates, the highest maturity year taken is 15 year. However, the same analysis can be done for higher maturity years too provided data is available. *Fig.2.1* displays the yield to maturity for three of the maturity years, viz. 1 year, 4 year and 15 year. In terms of maturities, 1 year maturity rate has been considered as Short-term rate, 4 year maturity as Medium-term and 15 year as Long-term rate. It can be observed that movements of the rates on various dates are most same for the three types of yield to maturity with some differences.

3. Factors, which decides the Shape of Yield Curve

3.1 Level, Slope & Curvature- Researchers in finance have studied the yield curve statistically and have found that shifts or changes in the shape of the yield curve are attributable to a few unobservable. Specifically, empirical studies reveal that more than 95% of the movements of various bond yields are captured by three factors, which are often called "level," "slope," and "curvature". The names describe how the yield curve shifts or changes shape in response to a shock. As an example, *Fig.3* illustrates the influence of a shock to the "level" factor on the yield curve. The solid line is the original yield curve, and the dashed line is the yield curve after the shock. A "level" shock changes the interest rates of all maturities by almost identical amounts, inducing a parallel shift that changes the level of the whole yield curve. *Fig.4* shows the influence of the "slope" factor on yield curve. The shock to the "slope" factor increases short-term interest rates by much larger amounts than the long-term interest rates, so that the yield curve becomes less steep and its slope decreases. *Fig.5* shows the response of the yield curve to a shock to the "curvature" factor. The main effects of the shock focus on medium-term interest rates, and consequently the yield curve becomes more "hump-shaped" than before. Various models have been developed and estimated to characterize the movement of these unobservable factors and thereby that of the yield curve by financial economists and bond traders in asset-pricing exercises. Few of these models, however, provide any insight about what these factors are, about the identification of the underlying forces that drive their movements, or about their responses to macroeconomic variables.

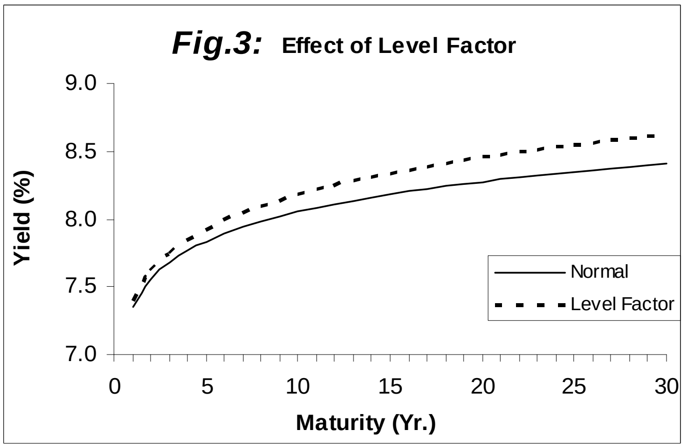


Fig.3: **Effect of Level Factor**

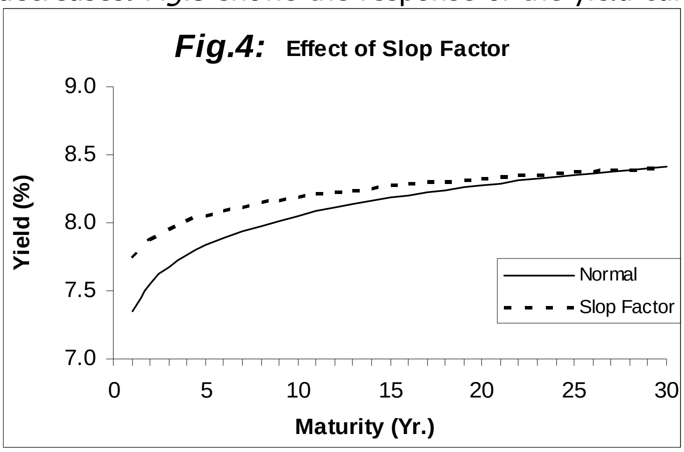


Fig.4: **Effect of Slop Factor**

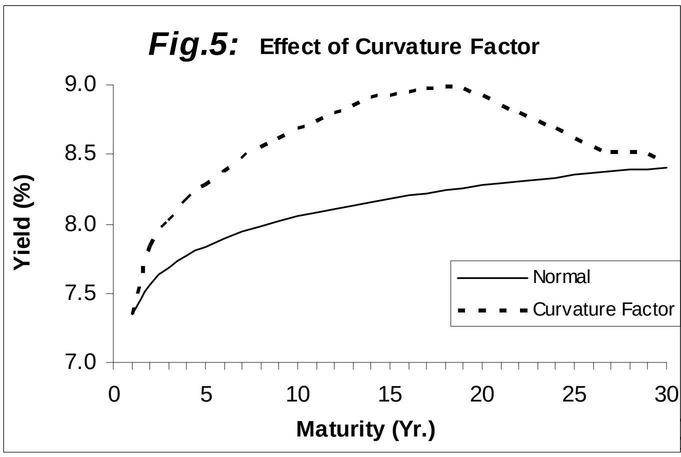


Fig.5: **Effect of Curvature Factor**

3.2 Identifying the factors- The aim of factor analysis is, as said before, to account for the variance of observed data in terms of much smaller number of variables or factors. To perform the factor analysis i.e. to recognize the factors we apply a related method called principal component analysis (PCA). The PCA is simply a way to re-express a set of variables, possibly resulting in more convenient representation.

PCA is essentially an orthogonal linear transformation of *n* individuals sets of *p* observed variables; x_{ij} , i = 1, 2, . . . , n and j = 1, 2, . . . , p, into an equal number of new sets of variables; $y_{ij} = y_1, y_2, \ldots, y_p$ along with coefficients a_{ij} , where i and j are indexes for n and p respectively. In this paper the historical yield curves are the n individual sets, containing p variables of different maturities each. Note the following relationships:
Each y is a linear combination of the x's i.e. $y_i = a_{i1}x_1+a_{i2}x_2+\cdots+a_{ip}x_p$
The sum of the squares of the coefficients a_{ij} is unity.
Of all possible linear combinations uncorrelated with y_1, y_2 has the greatest variance. Similarly y_3 has the greatest variance of all linear combinations of x_i uncorrelated with y_1 and y_2, etc.

Ind. Sampl. [I]	Variables [V]			
	V_1	V_2	$\ldots$	V_p
I_1	x_{11}	x_{12}	$\ldots$	x_{1p}
I_2	x_{21}	x_{22}	$\ldots$	x_{2p}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
I_n	x_{n1}	x_{n2}	$\ldots$	x_{np}

The new combinations y_i express the variances in a decreasing order so consequently the PCA can be used to recognize the most significant factors i.e. the factors describing the highest ratios of the variance. The method is perfectly general and the only assumption necessary to make is that the variables which the PCA is applied on are relevant to the analysis being conducted. Furthermore it should be noticed that the PCA use no underlying model and henceforth it is not possible to test any hypothesis about the outcome.

Principal Component Method has been implemented on data set of various time periods (as mentioned in section 2) in order to recognize the key factors for Indian Yield curve. In Table1, eigen values (when time period selected was 2001-2005) along with

Table1:	**PC 1**	**PC 2**	**PC 3**	**PC 4**	**PC 5**	**PC 6**	**PC 7**	**PC 8**
Eigenvalue	28.807	0.264	0.198	0.042	0.018	0.009	0.007	0.007
% of Var.	98.076	0.898	0.673	0.144	0.061	0.032	0.025	0.025
Cum. %	98.076	98.974	99.647	99.791	99.853	99.885	99.910	99.934

percentage of explanation are shown. Cumulative percentage is also included in Table1. It can be noticed that PC1 (factor1) alone is able to explain more than 98% variation and all the three factors together are explaining 99.6% variations. Annexure A1.1 illustrates

***Table2:* Coefficients (2001-2005)**

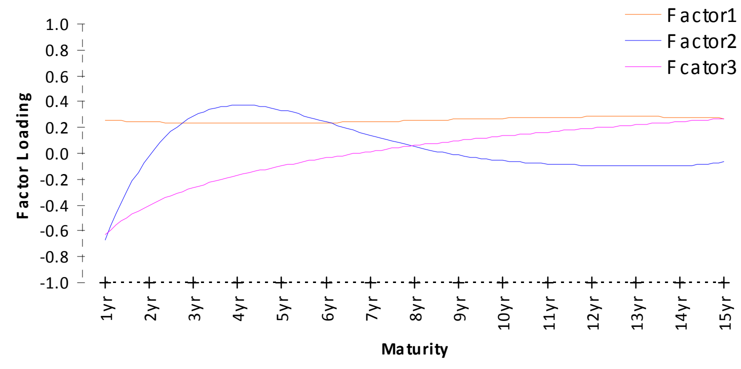
	PC 1	**PC 2**	**PC 3**
1yr	0.275	-0.714	-0.545
2yr	0.226	0.088	-0.356
3yr	0.223	0.236	-0.308
4yr	0.228	0.321	-0.233
5yr	0.234	0.313	-0.196
6yr	0.243	0.270	-0.118
7yr	0.248	0.195	-0.049

such eigen values, individual variation explanation and cumulative explanation percentage for various time periods selected. It has been noted that except for 2001 time period case and 2001-2007 time period case, first factor alone is able to explain almost 98% variation due to a shock in all rest of the maturity year rates. However, except 2001-2007 time period case, in all other time period cases, three factors together are explaining more than 99% variation.

8yr	0.267	0.061	0.085
9yr	0.264	0.030	0.162
10yr	0.271	-0.268	0.123
11yr	0.271	0.035	0.215
12yr	0.273	-0.046	0.138
13yr	0.283	-0.156	0.248
14yr	0.279	-0.079	0.300
15yr	0.276	-0.070	0.328

In Table 2, coefficient of first three most significant factors, when time period chosen was 2001-2005 have been shown. Column heading in the table are the three most significant factors, whereas row heading are various maturity year. Values inside Table 2 are coefficient of factors, which explain variation in the respective row heading's yield. As for example, in case when there is a shock on rates then for 10th row (10yr), PC1 (loading factor1) makes 0.271% variations in 10year bond rate due to the shock; PC2 (loading factor2) explains -0.268% variations in 10year bond rate due to the shock; & PC3 (loading factor3) explains 0.123% variations in 10year bond rate due to the shock. *Fig.6* is an example for one particular set of time period (2001-2005). Annexure A1.2 illustrates the plots of first three loading factors for various time periods. It has been observed that first factor (PC1) is almost constant for all the maturity years. However, 2nd factor and 3rd factor are varying with various shapes. *Fig.6* shows the three factor loadings corresponding to the three largest principal components in Table 2. The loadings we recognize as the shift, steepness and convexity factors identified by Litterman & Scheinkman. From looking at *Fig.6* it can be observed that the first factor forms almost a horizontal line over the whole time period, excluding approximately the first two years. This corresponds to a change of slope for the first two years and a parallel shift for the rest of the maturity horizon. The horizontal line is dominant for the rest of the term structure and hence the factor is recognized as the level factor.

Fig. 6: **Three Most Significant Principal Component Factors (Weekly: Jan2001-Dec2005)**



The second factor can be interpreted as the curvature factor since positive changes in it cause a decrease in yield for bonds with short and long maturities but cause an increase in yield for medium length maturities.

The third factor is the slope, which corresponds to a change of the slope for the whole term structure accounts for 0.673% of the total variation. It can be seen from the plot that the slope is decreasing as a function of maturity which fits the description of a normal yield curve. This is in accordance to the fact that the yield curve the period investigated was for most parts a normal yield cure with marginally diminishing yields.

3.3 Effect of Factors on Rates- A unit change of the ith factor causes a change a_{jt} for each maturity t-year rate. Since the factors are independent of each other we may therefore express the total change of the random variable, r_t, by

$$\Delta r_t = \sum_{j=1}^{k} a_{jt} \Delta f_j$$

Where f_j is jth factor, k is the number of factors; a_{jt} is the coefficient, identified by the eigenvector analysis, used to approximate the variance of the portfolio.

As an example lets see what effect a unit change (_f_1 = 1) of the level factor (j = 1) has on the ten year rate (t = 10). From Table2, we have $a_{1,10}$ = 0.271. so a unit change in factor 1 causes 0.271

change in the ten year rate, which means that if the ten year rate is 5% a unit change in the level factor causes it to become 5.271%.

In the same manner a unit changes of three most significance factors (Δf_1 = 1) for j = (1, 2, 3), again for ten years means:

$$\Delta r_{10} = \sum_{j=1}^{3} a_{j10} \Delta f_j = 0.271 - 0.268 + 0.123 = 0.126$$

Meaning that a 5% ten year rates would become 5.126% if a unit change occurred for all the factors.

4. Choosing the Factors for VAR model

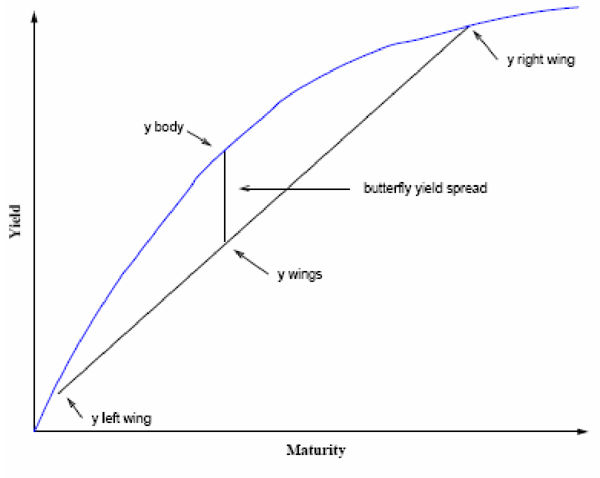
The main result from the factor analysis (Principal Component Analysis) was that three factors were to be used to construct the model. But how are the factors recognized in the VAR model? There are two methods for selecting the factors. The 1st method is a naive approach and the 2nd is butterfly method suggested by Christiansen & Lund (2007).

The former method is based on taking three positions of the yield curve, a short, medium and long term maturity. The short term rate can be chosen as a proxy for the level factor, the curvature can be chosen as the difference between two yields, a medium maturity yield minus the sort maturity yield. And finally the slope is chosen as two times the medium rate minus the long and short rate. If we note short, medium and long maturity as y_s, y_m and y_l, respectively then the factors can be denoted in the following way

level = y_s
curvature = $y_l - y_s$
slope = $2 \cdot y_m - (y_s - y_l)$

where we choose the short rate to be the 1 year rate, the medium to be the 4 year and the long to be the 15 year rate (can vary with respect to available data).
The 2nd method, butterfly method, is a bit different from naïve method. The main difference is that in butterfly approach slope of the yield curve can be chosen differently, namely by using the mechanism of the so called butterfly spread.

A butterfly spread is a portfolio which consists of a long position in an intermediate maturity bond (the body of the butterfly) and two short positions of bonds whose maturities straddle the first bond (the wings of the butterfly). Figure below shows a digram of how a butterfly spread looks for a concave (normal) yield curve and the spread s, is given as
$s = y_m - (w_1 * y_s + (1 - w_2) * y_l)$



where the weights w_1 and w_2 are chosen such that $w_1 y_s = w_2 y_l$. An example of how the weights are chosen if the maturities are 1, 4 and 15 years would be $w_1 = (4 - 1)/(15 - 1) = 3/14$ and weight 2 would become $w_2 = (15-4)/(15-1) = 11/14$. The spread shown in the figure is positive and the more concave the yield curve becomes the more positive the spread gets and vice versa. This applies for both normal and inverted yield curves. Equivalently, a negative butterfly spread indicates a convex yield curve.

By the latter method the level is chosen in the same way as before, by taking the short rate as a proxy, but the curvature is determined differently compared to the former method. The curvature in the latter method is chosen to be the difference between the long and short rate in stead of the difference between the medium and short rate before. That is done in order to keep the correlation between the curvature and the approximation of the slope at a reasonable level, according to Christiansen & Lund (2007). Using the same notation as for the former method
level = y_s
curvature = $y_l - y_s$
slope = $y_m - (w_1 * y_s + (w_s) * y_l)$.

5. VAR Model

5.1 Defining- A p^{th} order vector auto regression VAR(p) process can be expressed as

$$Y_t = C + A_1 Y_{t-1} + A_2 Y_{t-2} + A_3 Y_{t-3} + \ldots + A_p Y_{t-p} + \varepsilon_t$$

where Y_t is an (n × 1) vector of time series of random variables, C is an (n × 1) vector of constants, A_j is an (n × n) matrix of autoregressive coefficients for j = (1, 2, . . . , p) and ε_t is a vector generalization of Gaussian white noise. Since we intend to formulate a three factor VAR process, we give an example of such a process.

$$y_{1,t} = c_1 + a_{11} y_{1,t-1} + a_{12} y_{1,t-2} + a_{13} y_{1,t-3} + \varepsilon_{1,t}$$
$$y_{2,t} = c_2 + a_{21} y_{2,t-1} + a_{22} y_{2,t-2} + a_{23} y_{2,t-3} + \varepsilon_{2,t}$$
$$y_{3,t} = c_3 + a_{31} y_{3,t-1} + a_{32} y_{13t-2} + a_{33} y_{3,t-3} + \varepsilon_{3,t}$$

In this paper, 2nd method has been applied to select proxy of three factors as described in section 4.
Proxy for Factor 1 (level):
y_s = 1-yr maturity rate

Proxy for Factor 2 (curvature):
y_m = 15-yr maturity rate *minus* 1-yr maturity rate

Proxy for Factor 3 (slope):
y_l = 4-yr maturity rate *minus* [$\frac{3}{14}$ *(1-yr maturity rate) + $\frac{11}{14}$ * (15-yr maturity rate)]

5.2 Stationary Check

For all the three proxies, it has been tested whether they are stationary. It has been found that none of the three proxies are stationary. However, all the three proxies are stationary at their respective 1st difference. A 1st difference for a series, say y_t is defined as $\Delta y_t = y_t - y_{t-1}$. EViews package ahs been used to test the stationarity of the series. Augmented Dicky Fuller (ADF) test has been applied to individual series to test stationarity. Stationary has been decided on the basis of Schwartz Criteria. In Annexure A2, all the stationarity test results have been shown. In Annexure A2.1, it can be seen that neither Level, nor Curvature nor Slope is stationary. For stationary series ADF Test Statistic should be less than critical value. Only proxy for Factor 1 is stationary at 10% level of significance. Annexure A2.2 displays ADF test for 1st order difference series. All the 1st difference series are stationary at all level of significance. Estimation of parameters of VAR model will be done on the basis of these 1st order difference series.

5.3 Lag Selection and Criteria

For VAR modeling, how many lags are appropriate needs to be identified. EViews package provides facility to identify the lag selection for VAR modeling. Selection of lag has been performed using Schwartz Criteria.

5.4 Estimation of parameters

Using three series identified above viz. $\Delta(Level)_t$, $\Delta(Curvature)_t$, $\Delta(Slope)_t$ a VAR model has been set up in EViews and the parameter values have been estimated. The results can be seen in Annexure A3. The final equation of the estimation comes out to be as below

VAR MODEL: Substituted Coefficient

D_CURV = - 0.16766*D_CURV(-1) + 0.10757*D_LEVEL(-1) - 0.131497*D_SLOPE(-1) +0.00167
D_LEVEL = 0.33085*D_CURV(-1) + 0.0974*D_LEVEL(-1) + 0.32341*D_SLOPE(-1) - 0.004674
D_SLOPE =- 0.09689*D_CURV(-1) - 0.07455*D_LEVEL(-1) - 0.16468*D_SLOPE(-1) + 0.000699

In the above table shown, D_LEVEL represents $\Delta(Level)_t$, D_CURV represents $\Delta(Curvature)_t$ & D_SLOPE represents $\Delta(Slope)_t$.

From these three equations one can identify the equations for actual series very easily. Since above VAR is of order 1, for actual series of factors VAR is of order 2. The actual series, LEVEL (which was representing Factor 1 of our PCA), CURVATURE (which was representing Factor 2 of our PCA) and SLOPE (which was representing Factor 3 of our PCA) can be found from which one can identify the change in rates using equation given section 3.3.

6. Conclusion

Using PCA, one can identify the factors which are responsible for changes in yield curve. Modeling VAR is one of the ways to project the future values so that yield to maturity rates can be understood better. Analyzing the VAR process reveled that a process with lag 2 was suitable for modeling the rates, based on the results of information criteria. Investigating the stability of the VAR (2) process reviled that it was stable for the time frame of interest, but using all the data was not necessarily better. Finally, one can apply Vector Error Correction Model (VECM) to take care the shortfall of VAR model. However, along with this other modeling process can also be applied, like ARCH, GARCH or Regime-Switching models.
In these ways, investors can prepare their tools, which support the investment process related to strategic asset allocation decisions

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Annexure A1

A1.1 Explained Variance in terms of Eigen Values and Cumulative Probabilities

2001

	PC 1	PC 2	PC 3
Eigen value	10.755	0.362	0.066
% of Var.	95.798	3.226	0.584
Cum. %	95.798	99.024	99.608

2001-2002

	PC 1	PC 2	PC 3
Eigen value	23.734	0.398	0.048
% of Var.	97.863	1.641	0.197
Cum. %	97.863	99.504	99.701

2001-2003

	PC 1	PC 2	PC 3
Eigen value	37.834	0.310	0.054
% of Var.	98.848	0.811	0.141
Cum. %	98.848	99.659	99.800

2001-2004

	PC 1	PC 2	PC 3
Eigen value	35.947	0.243	0.176
% of Var.	98.573	0.665	0.482
Cum. %	98.573	99.239	99.721

2001-2005

	PC 1	PC 2	PC 3
Eigen value	28.807	0.264	0.198
% of Var.	98.076	0.898	0.673
Cum. %	98.076	98.974	99.647

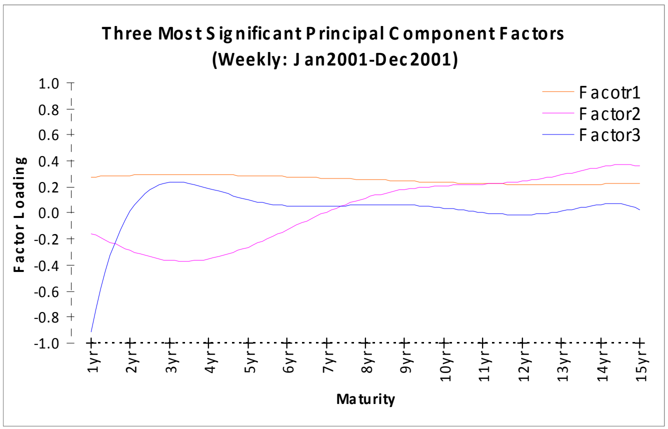
2001-2006

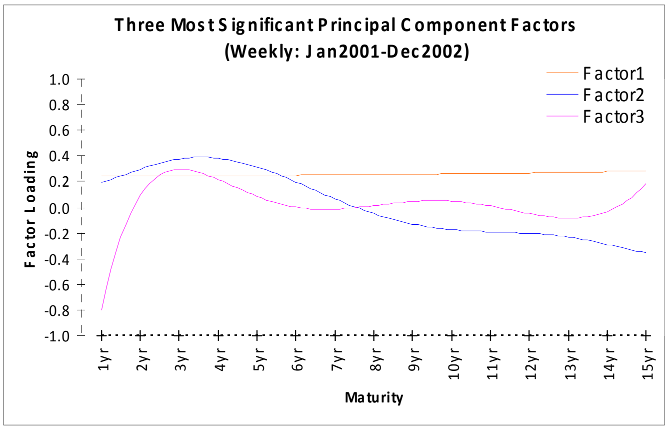
	PC 1	PC 2	PC 3
Eigen value	24.712	0.255	0.206
% of Var.	97.413	1.007	0.811
Cum. %	97.413	98.420	99.231

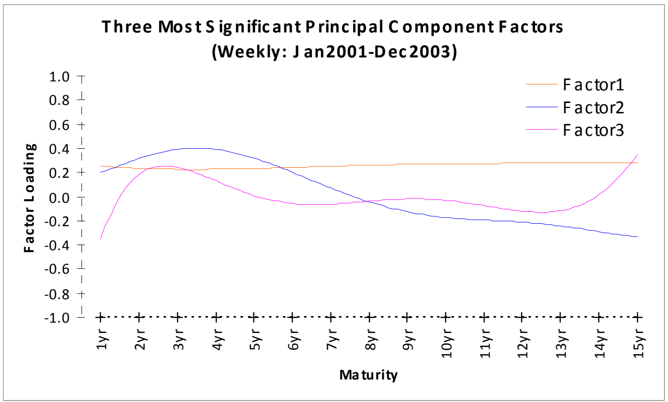
2001-2007

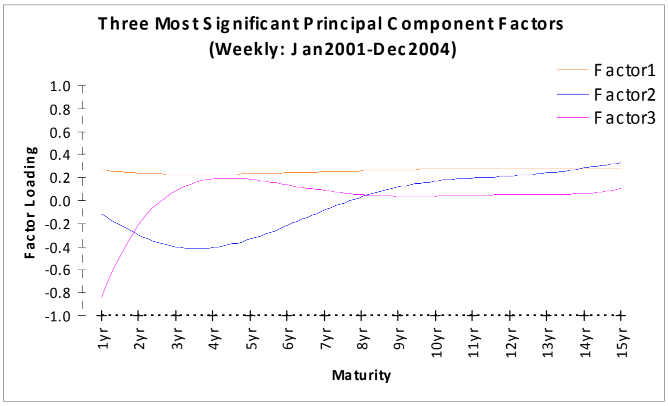
	PC 1	PC 2	PC 3
Eigen value	22.088	0.331	0.196
% of Var.	96.478	1.444	0.858
Cum. %	96.478	97.921	98.779

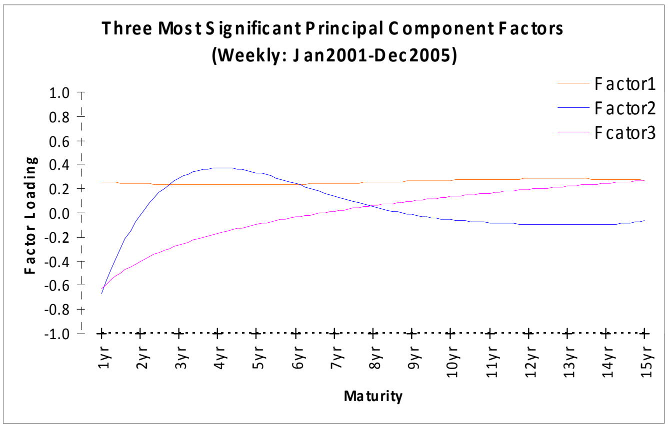
A1.2 Charts of Factor Loadings

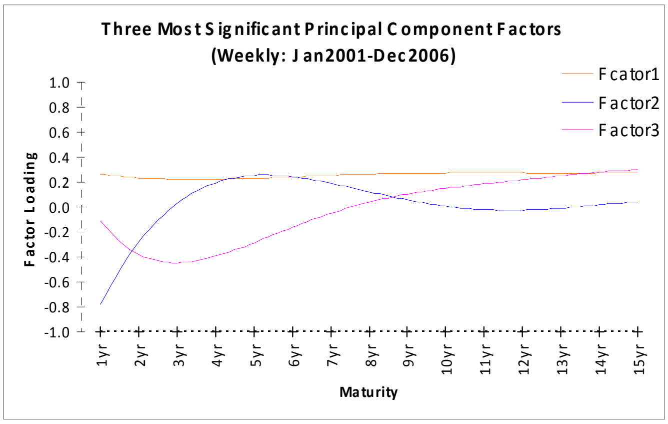


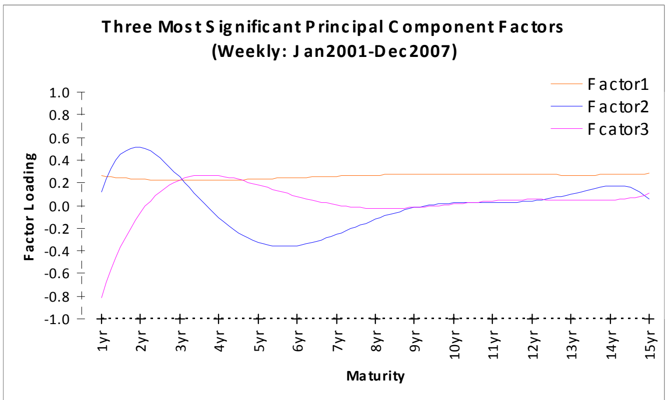












Annexure A2.
A2.1

ADF Test for Level

ADF Test Statistic	-2.581681	1% Critical Value*	-3.4506
		5% Critical Value	-2.8698
		10% Critical Value	-2.5712

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LEVEL)
Method: Least Squares
Sample(adjusted): 2/05/2001 12/10/2007
Included observations: 358 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LEVEL(-1)	-0.020808	0.008060	-2.581681	0.0102
D(LEVEL(-1))	-0.211934	0.052684	-4.022744	0.0001
D(LEVEL(-2))	-0.074128	0.053825	-1.377192	0.1693
D(LEVEL(-3))	-0.038336	0.053841	-0.712019	0.4769
D(LEVEL(-4))	0.040937	0.052700	0.776787	0.4378
C	0.128036	0.054108	2.366308	0.0185

R-squared	0.065358	Mean dependent var	-0.006927
Adjusted R-squared	0.052082	S.D. dependent var	0.205068
S.E. of regression	0.199656	Akaike info criterion	-0.367823
Sum squared resid	14.03161	Schwarz criterion	-0.302787
Log likelihood	71.84038	F-statistic	4.922943
Durbin-Watson stat	1.993118	Prob(F-statistic)	0.000228

ADF Test for Curvature

ADF Test Statistic	-2.920391	1% Critical Value*	-3.4515
		5% Critical Value	-2.8702
		10% Critical Value	-2.5714

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CURVATURE)
Method: Least Squares
Sample(adjusted): 2/05/2001 12/10/2007
Included observations: 340
Excluded observations: 18 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CURVATURE(-1)	-0.067345	0.023060	-2.920391	0.0037
D(CURVATURE(-1))	-0.228114	0.056247	-4.055587	0.0001
D(CURVATURE(-2))	-0.026011	0.057706	-0.450749	0.6525
D(CURVATURE(-3))	-0.038929	0.057717	-0.674481	0.5005
D(CURVATURE(-4))	0.067013	0.055527	1.206842	0.2283
C	0.065308	0.024741	2.639645	0.0087

R-squared	0.099348	Mean dependent var	-

			0.000500
Adjusted R-squared	0.085865	S.D. dependent var	0.199060
S.E. of regression	0.190322	Akaike info criterion	-0.462710
Sum squared resid	12.09829	Schwarz criterion	-0.395140
Log likelihood	84.66070	F-statistic	7.368478
Durbin-Watson stat	1.983061	Prob(F-statistic)	0.000001

ADF Test for Slope

ADF Test Statistic	-1.835333	1% Critical Value*	-3.4506
		5% Critical Value	-2.8698
		10% Critical Value	-2.5712

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SLOPE)
Method: Least Squares
Sample(adjusted): 2/05/2001 12/10/2007
Included observations: 358 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SLOPE(-1)	-0.023005	0.012535	-1.835333	0.0673
D(SLOPE(-1))	-0.109507	0.053465	-2.048201	0.0413
D(SLOPE(-2))	0.077878	0.053713	1.449882	0.1480
D(SLOPE(-3))	-0.025472	0.053808	-0.473396	0.6362
D(SLOPE(-4))	-0.035834	0.053496	-0.669849	0.5034
C	-0.012021	0.009604	-1.251693	0.2115

R-squared	0.034313	Mean dependent var	0.002431
Adjusted R-squared	0.020596	S.D. dependent var	0.100234
S.E. of regression	0.099197	Akaike info criterion	-1.766807
Sum squared resid	3.463671	Schwarz criterion	-1.701770
Log likelihood	322.2585	F-statistic	2.501477
Durbin-Watson stat	1.985449	Prob(F-statistic)	0.030385

A2.2

ADF Test of 1st Order Difference of Level

ADF Test Statistic	-8.788308	1% Critical Value*	-3.4506
		5% Critical Value	-2.8698
		10% Critical Value	-2.5712

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(D_LEVEL)
Method: Least Squares
Sample(adjusted): 2/12/2001 12/10/2007
Included observations: 357 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D_LEVEL(-1)	-1.271701	0.144704	-8.788308	0.0000

D(D_LEVEL(-1))	0.060571	0.128642	0.470848	0.6380
D(D_LEVEL(-2))	-0.009002	0.107946	-0.083395	0.9336
D(D_LEVEL(-3))	-0.042426	0.083639	-0.507255	0.6123
D(D_LEVEL(-4))	0.003902	0.053261	0.073270	0.9416
C	-0.009322	0.010730	-0.868757	0.3856

R-squared	0.602647	Mean dependent var	-0.000448
Adjusted R-squared	0.596987	S.D. dependent var	0.317721
S.E. of regression	0.201700	Akaike info criterion	-0.347411
Sum squared resid	14.27964	Schwarz criterion	-0.282239
Log likelihood	68.01282	F-statistic	106.4692
Durbin-Watson stat	1.989236	Prob(F-statistic)	0.000000

ADF Test of 1st Order Difference of Curvature

ADF Test Statistic	-9.234520	1% Critical Value*	-3.4517
		5% Critical Value	-2.8703
		10% Critical Value	-2.5714

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(D_CURV)
Method: Least Squares
Sample(adjusted): 2/12/2001 12/10/2007
Included observations: 336
Excluded observations: 21 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D_CURV(-1)	-1.439551	0.155888	-9.234520	0.0000
D(D_CURV(-1))	0.175199	0.138866	1.261644	0.2080
D(D_CURV(-2))	0.114681	0.115982	0.988780	0.3235
D(D_CURV(-3))	0.041954	0.089965	0.466333	0.6413
D(D_CURV(-4))	0.067202	0.056201	1.195749	0.2327
C	0.000873	0.010527	0.082955	0.9339

R-squared	0.630998	Mean dependent var	0.000506
Adjusted R-squared	0.625407	S.D. dependent var	0.315244
S.E. of regression	0.192942	Akaike info criterion	-0.435161
Sum squared resid	12.28475	Schwarz criterion	-0.366999
Log likelihood	79.10710	F-statistic	112.8608
Durbin-Watson stat	1.987825	Prob(F-statistic)	0.000000

ADF Test of 1st Order Difference of Slope

ADF Test Statistic	-8.839290	1% Critical Value*	-3.4506
		5% Critical Value	-2.8698
		10% Critical Value	-2.5712

*MacKinnon critical values for rejection of hypothesis of a unit root.
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(D_SLOPE)

Method: Least Squares				
Sample(adjusted): 2/12/2001 12/10/2007				
Included observations: 357 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
D_SLOPE(-1)	-1.134383	0.128334	-8.839290	0.0000
D(D_SLOPE(-1))	0.011501	0.113447	0.101375	0.9193
D(D_SLOPE(-2))	0.076545	0.097503	0.785052	0.4330
D(D_SLOPE(-3))	0.037083	0.079901	0.464104	0.6429
D(D_SLOPE(-4))	-0.009721	0.053175	-0.182803	0.8551
C	0.003360	0.005251	0.639889	0.5227

R-squared	0.573283	Mean dependent var	0.000623
Adjusted R-squared	0.567204	S.D. dependent var	0.150604
S.E. of regression	0.099078	Akaike info criterion	-1.769149
Sum squared resid	3.445593	Schwarz criterion	-1.703977
Log likelihood	321.7931	F-statistic	94.31187
Durbin-Watson stat	1.996479	Prob(F-statistic)	0.000000

Annexure A3

Parameter estimation under VAR

Sample(adjusted): 1/15/2001 12/10/2007
Included observations: 352
Excluded observations: 9 after adjusting endpoints
Standard errors & t-statistics in parentheses

	D_CURV	D_LEVEL	D_SLOPE
D_CURV(-1)	-0.167660	0.330859	-0.096887
	(0.08885)	(0.09078)	(0.04623)
	(-1.88708)	(3.64466)	(-2.09577)
D_LEVEL(-1)	0.107574	0.097427	-0.074549
	(0.09254)	(0.09455)	(0.04815)
	(1.16250)	(1.03043)	(-1.54827)
D_SLOPE(-1)	-0.131497	0.323407	-0.164680
	(0.11242)	(0.11486)	(0.05850)
	(-1.16970)	(2.81556)	(-2.81528)
C	0.001673	-0.004674	0.000699
	(0.01012)	(0.01034)	(0.00527)
	(0.16532)	(-0.45198)	(0.13269)
R-squared	0.078003	0.086548	0.029830
Adj. R-squared	0.070055	0.078673	0.021467
Sum sq. resids	12.49403	13.04353	3.382699
S.E. equation	0.189479	0.193601	0.098592
F-statistic	9.813853	10.99075	3.566703
Log likelihood	88.08857	80.51336	318.0461
Akaike AIC	-0.477776	-0.434735	-1.784353
Schwarz SC	-0.433871	-0.390830	-1.740448
Mean	0.000369	-0.004716	0.001026

dependent			
S.D. dependent	0.196487	0.201698	0.099668
Determinant Residual Covariance		4.10E-06	
Log Likelihood		684.6893	
Akaike Information Criteria		-3.822098	
Schwarz Criteria		-3.690384	

About the Author:

Subhash Chandra
Subhash Chandra is Senior Manager, Actuarial in Kotak Mahindra Old Mutual Life Insurance, Mumbai. The views presented in this paper are those of the authors' and are not necessarily shared by the organization.