# Institute of Actuaries of India

## Subject CT5 – General Insurance, Life and Health Contingencies

## September 2018 Examination

# INDICATIVE SOLUTION

**Introduction**

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

**<u>Solution 1:</u>** We are required to calculate ${}_{19.5}p_{35.5} = {}_{.5}p_{35.5} * {}_{19}p_{36}$

Now ${}_{19}p_{36} = l_{55}/l_{36} = 91217/97057 = 0.939829$

(i) Assume deaths are uniformly distributed, then ${}_tp_{x.}\ \mu_{x+t}$ is constant
Then ${}_{.5}q_{35.5} = (1- .5) * q_{35}/ (1-.5* q_{35}) = 0.00058$
Thus, ${}_{19.5}p_{35.5} = {}_{.5}p_{35.5} * {}_{19}p_{36} = (1- 0.00058)* 0.939829=0.939284$

(ii) Assume that force of mortality is constant across age 35 to 36
Then, ${}_{.5}p_{35.5} = e^{-.5*\mu 35}$
$\mu_{35} = - \ln (1- q_{35}) = - \ln (1-0.00116) = 0.001161$
Therefore, ${}_{.5}p_{35.5} = e^{-.5*0.001161} = 0.99942$
Hence ${}_{19.5}p_{35.5} = {}_{.5}p_{35.5} * {}_{19}p_{36} = = 0.99942*0.939829 = 0.939284$

**[5 Marks]**

**<u>Solution 2:</u>** The Expected Present Value of Benefits payable under this contract = 1000000 * $A_{47}$
Or 1000000 * $A_{47}$ = 1000000 * 0.29635 = 296350

The variance of benefits payable under this contract is = $1000000^2*( {}^2A_{47} - (A_{47})^2)$
$= 1000000^2*( 0.10778 - (0.29635)^2) = 141268.1^2$ **[4 Marks]**

**<u>Solution 3</u>:** By definition

$a_{x:\overline{n|}} = v\, p_x + v^2\, {}_2p_x + \ldots + v^n\, {}_np_x$

Each of the items on RHS can be expressed as multiple of factor $vp_x$ i.e.

$v^2{}_2p_x = (vp_x)*(vp_{x+1})$

$v^3{}_3p_x = (vp_x)*(v^2{}_2p_{x+1})$

and so on

Thus $a_{x:\overline{n|}}$ can now be re-expressed as

$a_{x:\overline{n|}} = vp_x * (1+ v_1p_{x+1} + \ldots + v^{n-1}{}_{n-1}p_{x+1})$

$= vp_x * \ddot{a}_{x+1:\overline{n|}}$

**[3 Marks]**

**<u>Solution 4:</u>**

**i)** $A^1_{40:\overline{25|}} = A_{40} - v^{25} * {}_{25}p_{40} * A_{65}$
$= 0.23056 - [(1.04\text{^}-25) * (8821.2612/9856.2863) * 0.52786]$
$= 0.053344$

[2]

**ii)** $\ddot{a}^{(4)}_{39:\overline{21|}} = \ddot{a}^{(4)}_{39} - v^{21} * {}_{21}p_{39} * \ddot{a}^{(4)}_{60}$

$= (\ddot{a}_{39} - (3/8)) - [v^{21} * {}_{21}p_{39} * (\ddot{a}_{60} - (3/8))]$

$= (20.219 - (3/8)) -$

$[(1.04\text{^}{-21})*(9287.2164/9864.8688)*(14.134 - (3/8))]$

$= 14.15965$ [2]

**iii)** $(IA)_{30:\overline{15|}} = (IA)^1_{30:\overline{15|}} + n * A_{30:\overline{15|}}{}^{1}$

$= \{(IA)_{30} - (D_{45}/D_{30}) * [(IA)_{45} + 15*A_{45}]\} + (15 * (D_{45}/D_{30}))$

$= \{6.91559 - (1677.97/3060.13) *[8.33628 + (15*0.27605)]\}+$

$(15*(1677.97/3060.13))$

$= 8.299017$

[3]

**[7 Marks]**

**Solution 5**: $K_{xy}$ is the curate joint life expectation of x and y

$K_{xy}$ = integer part of $T_{xy}$

$P[K_{xy} = k] = P[k \leq T_{xy} < k+1]$

$= F_{T_{xy}}(K+1) - F_{T_{xy}}(K)$

$= (1 - {}_{k+1}p_{xy}) - (1 - {}_{k}p_{xy})$

$= {}_{k}p_{xy} - {}_{k+1}p_{xy}$

$= {}_{k}p_{xy} - {}_{k}p_{xy}p_{x+k:y+k}$

$= {}_{k}p_{xy}q_{x+k:y+k}$

$= {}_{k|}q_{xy}$

**[4 Marks]**

**Solution 6:**

**i)** In most insurance contracts level premiums are charged throughout the policy term to cover the cost of benefits and expenses payable under a policy.

The probability that the benefit would be payable early in the policy is low as the life is in good health. This probability of making a claim payment increases with age and thus the expected claim cost increases year by year.

Let us take the example of an endowment assurance contract where the expected claim cost increases with age as the death probability increases with age. Also upon maturity a lump sum is payable to all those who survive and is associated with a survival probability which is quite high.

In summary level annual premiums are charged to cover cost of benefits which increase with age. So the premiums charged in the earlier years are more than enough to pay for the cost of benefits. However the same cannot be said for the later years where the uniform premium would fall short of the expected amount to be paid especially the maturity claim payment.

If the excess money generated in the initial years is not kept aside to meet future outgoes the company may find it difficult to fund for the increasing claim payments as and when they arise. This could lead to insolvency as the company would not be able to meet the claim payments as and when due.

To ensure that this does not happen, the company sets up reserves for a policy rather than using the excess generated in the early years to pay to shareholders. [4]

**ii)** The conditions for equality of the reserves are:

(i) If the mortality and interest basis used is the same as that used to determine the gross premium at the date of issue of the policy
(ii) If the expenses valued are the same as that used to determine the original gross premium

(iii) If the gross premium used in the reserve calculations is the one determined using the original basis (mortality, interest, expense) using the equivalence principle.

[3]

**[7 Marks]**

**Solution 7**: To calculate the net premium reserves we would need to calculate the Net Premium. Let the net premium be P.

Thus, $P * \ddot{a}_{30:\overline{20|}} = 400000 * A_{30:\overline{20|}}$

P = 400000 * 0.315817 / 12.08705 = 10451.42

We need to calculate the net premium prospective reserve at time 7. To do so we would need the bonus that has vested in the policy till date. As the bonuses are assumed to be in line with expectations till time 7, 7 years of simple bonus at the rate of 40 per thousand SA should be added.

So the total bonus that has been added till date equals 7 * 4% * 400000 = 112000 (16000 is added for each year gone by)

So the guaranteed SA now is = 400000 + 112000 = 512000

Thus the prospective net premium reserves at time 7 is

$512000 * A_{37:\overline{13|}} - 10451.42 * \ddot{a}_{37:\overline{13|}}$

$= 512000 * 0.471638 - 10451.42 * 9.333835$

$= 143926.89$ **[6 Marks]**

**Solution 8:**

**i)** The premium equation assuming that the annual premium equals P is

$P * \ddot{a}_{58:56} = 100000 * A_{58:56}$

Or $P = 100000 * A_{58:56} / \ddot{a}_{58:56}$

Now the annuity value from the table is $\ddot{a}_{58:56} = 15.180$

Also we know, $A_{58:56} = 1 - d * \ddot{a}_{58:56} = 1 - 0.04/1.04*15.180 = 1 - 0.38462 * 15.180$

Or $A_{58:56} = .416154$

Therefore $P = 100000*.416154/15.180$

Or $P = 2741.46$ [3]

**ii)** The reserve for a policy as at 31st March 2018 can be expressed as

${}_8V = 100000 * A_{66:64} – 2741.46 * \ddot{a}_{66:64}$

From the tables we have $\ddot{a}_{66:64} = 11.845$

Similarly we know that,

$A_{66:64} = 1 - d * \ddot{a}_{66:64} = 1 – 0.04/1.04*11.845$

Or $A_{66:64} = 0.544423$

Thus, ${}_8V = 100000 * 0.544423 – 2741.46 * 11.845$

Or ${}_8V = 21969.7$

So the death strain at risk is SA - ${}_8V = 100000 - 21969.7 = 78030.30$

The probability that a claim will be made in the FY 17-18 requires the failure of the joint life status xy during the FY 17-18. In other words at least one of the lives should die within a year.

The probability that at least one of the lives dies during the FY 17-18 can be calculated as
(1 – probability that both lives survive the FY 17-18 )

Therefore, Probability of a claim in FY 17-18 = 1 – prob both lives survive in FY 17-18
= 1 – prob male survives * prob female survives
= $1 - p^m_{65} * p^f_{63}$
= 1 – (1-0.006032)*(1-0.003401)
= 0.009402

Since 5000 policies were in force as at 1st April 2017, the expected number of claims over the FY 17-18 = 5000 * 0.009402 = 47.06243

Expected death strain = Expected number of claims over FY 17-18 * DSAR
= 47.06243 * 78030.30
= 3672295.35

Actual death strain = Actual number of deaths * DSAR
= 10 * 78030.30
= 780303.03

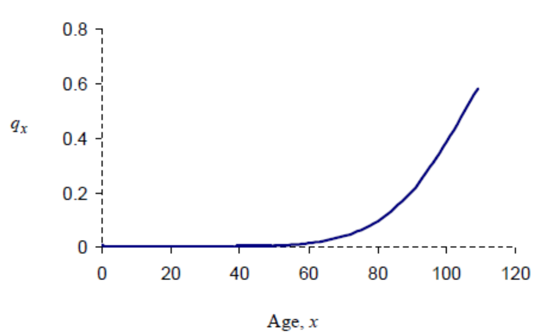
Thus, Mortality profit over the FY 17-18 = Expected death strain – Actual death strain
= 3672295.35 - 780303.03
= 2891992.32

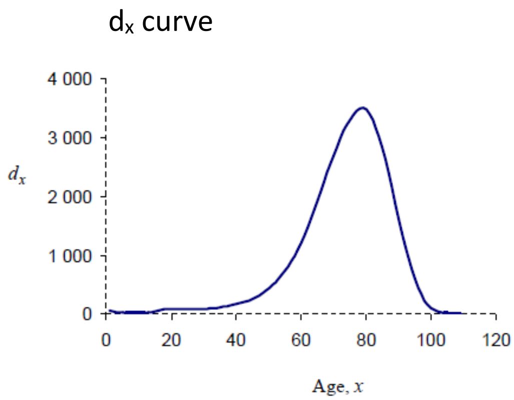
[11]

**[14 Marks]**

**Solution 9:**

**i)** $q_x$ curve






[2]

**ii)** Risk classification is used as an underwriting tool by life insurance companies to divide policyholders into different homogenous risk groups according to factors that affect mortality.

The extent to which rating factors can be used is limited by:

- the ability of prospective policyholders to provide accurate responses to questions
- the cost of collecting information
- from a marketing point of view, proposers are anxious that the process of underwriting should be straightforward and speedy.

[3]

**[5 Marks]**

**Solution 10:**

**i) Temperate zone:**

Crude death rate

Total deaths / Total population at risk = 9720/920000 = 0.010565

Standardised Mortality Rate

We apply climate-specific mortality rates to the whole world population structure:

(0.001×14400 + 0.002×15600 + 0.004×17200 + 0.026×12800)/60000 = 0.007453

Standardised Mortality Ratio (SMR)

Expected deaths are calculated using the whole world mortality:

(0.001×800 + 0.002×1600 + 0.006×3800 + 0.024×3000) × 100 = 9880

Actual deaths / Expected deaths = 9720 / 9880 = 0.983806

**Mediterranean zone:**

Crude death rate

Total deaths / Total population at risk = 8160/860000 = 0.009488

Standardised Mortality Rate

We apply climate-specific mortality rates to the whole world population structure:

(0.003×14400 + 0.002×15600 + 0.005×17200 + 0.021×12800)/60000 = 0.007153

Standardised Mortality Ratio (SMR)
Expected deaths are calculated using the whole world mortality:
(0.001×400 + 0.002×1800 + 0.006×3600 + 0.024×2800) × 100 = 9280
Actual deaths / Expected deaths = 8160 / 9280 = 0.879310 [5]

**ii) Temperate Zone:**
Population at risk = ( (80/0.001) + (320/0.002) + (1520/0.006) + (7800/0.024) )/100
= 8183.33
SERR = 8183.33 / 9200 = 0.889493

**Mediterranean Zone:**
Population at risk = ( (120/0.001) + (360/0.002) + (1800/0.006) + (5880/0.024) )/100
= 8450
SERR = 8450 / 8600 = 0.982558

Both the crude death rate and the standardised death rate of Temperate zone exceed that of Mediterranean zone. The difference in the standardised rates is smaller however, due to the slightly different population structures that serve to exaggerate the difference in the underlying mortality levels.

The SMR for each climatic zone is lower than one, indicating that the climates suffer lighter mortality than average. Again, Mediterranean zone has the lower figure of the two.

The new approach of SERR gives a slightly different picture suggesting that Temperate zone suffers lighter mortality than Mediterranean zone.

These results show that while single figure indices provide useful summary information, they can be misleading when viewed in isolation as they only paint part of the picture.

[6]
**[11 Marks]**

**Solution 11:**

**i)** Past service Pension:

PV = 55000 x $S_{40}/S_{39}$ x 20/50 x ${}_zM^{ra}_{40}/{}_sD_{40}$
= 55000 x 7.814/7.623 x 20/50 x 128026/25059
= 115213.83

Future Service Pension
PV = 55000 x $S_{40}/S_{39}$ x 1/50 x ${}_zR^{ra}_{40}/{}_sD_{40}$
= 55000 x 7.814/7.623 x 1/50 x 2884260/25059
= 129780.92

Return of current shadow fund on death
PV = (15000 x ${}_jM^{d}_{40}$) / ($1.03^{20}$ x $D_{40}$)
= (15000 x 323) / ($1.02^{20}$ x 3207)
= 1016.70

Return of future contributions on death

PV= 0.05 x 55000 x $S_{40}/S_{39}$ x ${}_{sj}R^{d}_{40}/{}_{s}D_{40}$

= 0.05 x 55000 x 7.814/7.623 x 16258/25059

= 1828.87

PV of total benefits = 247840.32

[7]

**ii)** Construct a multiple decrement table

| Age | Number of Lives | Number of Deaths | Number of withdrawals over year | Number of withdrawals year end |
|---|---|---|---|---|
| 40 | 100000.00 | 97.50 | 4997.50 | 4745.25 |
| 41 | 90159.75 | 87.91 | | |

At age 40 number of deaths = 100000 × 0.001 × (1 - 0.5 × (0.1 × 0.5)) = 97.50

No of withdrawals over the year = 100000 × (0.1 × 0.5) × (1 - 0.5 × 0.001) = 4997.50

No of withdrawals over year end = 100000 × (1 - (0.1 × 0.5)) × (1-0.001) × (0.1 × 0.5)

= 4745.25

At age 41 number of deaths = 90159.75 × 0.001 × (1 - 0.5 × (0.1 × 0.5)) = 87.91

Required Probability = 87.91 / 100000 = 0.0008791 [4]

**[11 Marks]**

**Solution 12**:

- Three health insurance contracts are: income protection insurance, critical illness insurance and long term care insurance.
- An income protection insurance contract pays an income to the policyholder while that policyholder is deemed as being "sick" (with the definition of sickness being carefully specified in the policy conditions). If the policyholder recovers, the cover under the policy usually continues, so that subsequent bouts of qualifying sickness would merit further benefit payments.
- Such policies are usually subject to a deferred period (eg 3 months) of continuous sickness that has to have elapsed before any benefits start to be paid, and during which no benefit is payable.
- Premiums for these policies would normally be regular (eg monthly) and would typically be waived during periods of qualifying sickness. This means that premiums would not be paid at the same time as benefits are payable.

**[4 Marks]**

**Solution 13:**

**i)** Calculating the dependent probabilities:

| $x$ | $(aq)_x^d$ | $(aq)_x^s$ | $(ap)_x$ | ${}_{x-42}(ap)_x$ |
|---|---|---|---|---|
| 42 | 0.000922 | 0.099908 | 0.899170 | 1.000000 |
| 43 | 0.001150 | 0.099885 | 0.898965 | 0.899170 |
| 44 | 0.001327 | 0.000000 | 0.998673 | 0.808323 |

Calculating the unit fund:

| Year | Unit Fund at Start | Allocated Premium | Bid Offer Spread | Interest | Annual Management charge | Unit Fund at End |
|---|---|---|---|---|---|---|
| 1 | 0.00 | 3000.00 | 75.00 | 263.25 | 39.85 | 3148.40 |
| 2 | 3148.40 | 8000.00 | 200.00 | 985.36 | 149.17 | 11784.59 |
| 3 | 11784.59 | 9250.00 | 231.25 | 1872.30 | 283.45 | 22392.19 |

Calculating the non unit cashflows:

| Year | Unallocated Premium | Bid Offer Spread | Expenses | Commission | Interest | Annual Management charge | Death Cost | Maturity Cost |
|---|---|---|---|---|---|---|---|---|
| 1 | 7000.00 | 75.00 | 2000.00 | 1000.00 | 285.25 | 39.85 | 15.54 | 0.00 |
| 2 | 2000.00 | 200.00 | 750.00 | 300.00 | 80.50 | 149.17 | 9.45 | 0.00 |
| 3 | 750.00 | 231.25 | 750.00 | 300.00 | -4.81 | 283.45 | 0.00 | 3354.37 |

| Year | End of year cashflow | Probability Inforce | Discount Factor | Expected PV of profit | Premium signature | Expected PV of premiums | Profit Margin |
|---|---|---|---|---|---|---|---|
| 1 | 4384.56 | 1.000000 | 0.884956 | 3083.46 | 10000.00 | 24287.61 | 12.70% |
| 2 | 1370.22 | 0.899170 | 0.783147 | | 8991.70 | | |
| 3 | -3144.48 | 0.808323 | 0.693050 | | 8083.23 | | |

[12]

**ii)** Reserves do not reduce profits; they only defer the release of profits.

So, if the interest earned on reserves is equal to the risk discount rate than the there is no effect of deferring the profit as the opportunity cost on money held as reserves is zero.
Therefore, the profit margin will be same as calculated earlier. [3]

**iii)** Since the mortality rates and surrender rate is the same in the valuation basis and pricing basis, we will be able to use the dependent probabilities derived in i).

To calculate the expected provisions at the end of each year we have:

${}_2V = 4532.25 / 1.07 = 4235.75$

${}_1V \times 1.07 - (ap)_{43} \times {}_2V = -(-760.10)$ $\rightarrow$ ${}_1V = 4269.06$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 2 $4235.75 \times (ap)_{43} = 3807.79$
Year 1 $4269.06 \times (ap)_{42} = 3838.61$

[4]

**[19 Marks]**

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