INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

11th September 2018

Subject CT8 – Financial Economics

Time allowed: Three Hours (15.00 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) **i)** What is meant by 'Excessive volatility' under Efficient Market Hypothesis (EMH)? (1.5)

ii) Explain the Shiller's approach of testing excessive volatility. (2.5)

iii) Why was the approach criticized later? (3)
[7]

Q. 2) **i)** Define the term 'Utility' and 'Utility function'. (1)

ii) Explain how an investor's risk-return preferences influence the shape of a utility function. (4)

iii) Taking examples of commonly used utility functions (quadratic, low, power), explain the traits exhibited by each function. (3)
[8]

Q. 3) In an economy, the risk free interest rate is 4% p.a.

An asset 'Gold' provides a yield of 6% p.a. and an asset 'Silver' provides a yield of 8% p.a. An investor X decided to invest 100% in Gold, an investor Y decided to invest 100% in Silver and an investor Z decided to invest 50% in each of the asset classes.

You may assume that the default under both the assets is independent and recovery rate in case of default is zero.

i) Calculate the risk neutral probability for each asset class. (1)

ii) Calculate VaR and TailVaR at 95% level for each investor (4)

iii) Comment on the results and the appropriateness of VaR and TailVaR as measures of investment risk. (4)
[9]

Q. 4) The market price of a certain share is modelled as Geometric Brownian motion. The price S_t at time $t \geq 0$ satisfies:

$\log_e (S_t/S_0) = \mu\, t + \sigma\, B_t$

where B_t is a standard Brownian motion and μ and σ are constants

i) Show that dS_t can be written as: $dS_t/S_t = x\, dB_t + y\, dt$
where x and y are constants to be specified. (4)

ii) Derive $E[S_t]$ and $Var[S_t]$ (4)

iii) Derive $cov[S_{t1}, S_{t2}]$ where $0 < t_1 < t_2$ (6)
[14]

Q. 5) **i)** List the input parameters required to value any option. (2)

ii) State how the increase in the value of each of the parameters (independently) will impact the value of an American style call and put options respectively. (4)

iii) Imagine you are to spot an arbitrage opportunity in the world cup final football match being played between Francisco and Croatiano. The odds from two bookmakers (Baddy and Paddy respectively) involved in betting the world cup final football match are as under:

Baddy sets the odds:

Francisco wins: 1.5 Croatiano wins: 3.0

Paddy sets the odds:

Francisco wins: 1.7 Croatiano wins: 2.4

Assume that you have Rs. 1000 to bet, how you will devise a strategy to benefit from the arbitrage opportunity, if at all it exists? (4)

[10]

Q. 6) Under Mean Variance Portfolio theory, derive how variance can be reduced using diversification as the principal tool, under the given two scenarios:

i) Independent assets (4)

ii) Non-independent assets (where assets can be assumed to have positive co-variance) (3)

[7]

Q. 7) A telecom company's share price registered a steep fall on account of the payment defaults to its creditors leading to severe impact on the brand image and morale of the employees. There was mass attrition among the employees thereby impacting the new business and services to the existing customers. The top management decided to bail out the Company from such exodus and came up with an incentive scheme to the employees which could boost both sales and share price. The incentive scheme entitles the employees to a cash payment of Rs. 5000 should the company's revenue from new connections increase by more than 20% at the end of the next 6 months. In addition, the employees will be entitled to 1,000 free shares each, should the share price increase by more than 10% at the end of the next 6 months.

You are given the following data:
Current share price: Rs. 10
Risk-free rate 5% pa (continuously-compounded)
Share price volatility 25% pa
No dividends to be paid over a period of 6 months
Relevant assumption can be made for the revenue in order to calculate the value of the incentive

i) By considering the terms of the Black-Scholes call option pricing formula, calculate the value of the incentive scheme to one employee. (6)

ii) Explain the main disadvantages of this incentive scheme as an incentive for employees to accept. (2)

iii) Some shareholders are concerned that this scheme might cause an undesirable distortion to the employees' behaviour. Suggest some modifications to the scheme that will ensure that the employees' aims coincide with the long-term objectives of the shareholders. (2)

[10]

Q. 8) Consider two securities, *F* and *G*, that are both positively dependent on the same underlying source of uncertainty.

Security *F* has price *f* and Security *G* has price *g*. The process followed by them is defined by:

$df = \mu_f f dt + \sigma_f f dw$
$dg = \mu_g g dt + \sigma_g g dw$

where *w* is the same Standard Brownian motion under both the securities.

Define λ, the market price of risk as:

$$\lambda = \frac{\mu_f - r}{\sigma_f} = \frac{\mu_g - r}{\sigma_g}$$

where *r* is the risk-free rate, and let $\phi = \frac{f}{g}$

i) Define a martingale. (1)

ii) Using Ito's formula on $\ln \phi$, show that if $\lambda = \sigma_g$, then ϕ is a Martingale. (8)

iii) Suggest what might be meant by a security f being forward risk neutral with respect to a security *g*. (2)

A squared payoff call on a security with strike price *K* is an option where the payoff at exercise is the excess, if any, of the square of the stock price over *K*.

iv) By creating a suitable risk-free portfolio, or otherwise, derive the differential equation for the price of an American style squared payoff call based on security *F*. (6)

v) State the boundary conditions required to solve the equation. (2)

It has been found that the volatility of *G* is damped version of the volatility of *F*, as defined by the formula

$\sigma_g = \sigma_f e^{-\beta t}$ for some constant β

vi) Explain how you would adapt the differential equation derived in **(iv)** to price an American style squared payoff call based on *G*. (3)

[22]

Q. 9) i) State the assumptions under a Binomial Model. (2)

ii) The price S_t of a stock is assumed to follow the nodes of a recombinant binomial tree with two parameters, μ and σ, such that the process moves from value *s* at some particular node along the up/down branch to a new value:

$$\begin{cases} s\exp(\mu\delta t + \sigma\sqrt{\delta t}) & \text{if up} \\ s\exp(\mu\delta t - \sigma\sqrt{\delta t}) & \text{if down} \end{cases}$$

where δt is the small interval of time it takes for the stock to jump from one node to the other, and up and down moves are equally likely.

Let *Xn* be the total number of up jumps out of the first *n* jumps.

a) If there have been *n* jumps by time *t*, write down the relationship between *n* and *t*. (1)

b) Show that the value of S_t at time *t* is given by:

$$S_t = S_0\exp\left(\mu t+\sigma\sqrt{t}\left(\frac{2X_n-n}{\sqrt{n}}\right)\right)$$ (3)

c) By considering the probability distribution of *Xn*, explain what happens to the distribution of S_t as $\delta t \rightarrow 0$. (4)

d) Derive values for the martingale measure up and down probabilities, approximate to first order in δt. (3)

[13]

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