INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

6th September 2018

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) An insurer wishes to use a GLM to analyse the claim amounts on its Motor portfolio. It has collected the following data on claim amounts Xi, i=1,2.......30 from three different classes of products:

Class I: 10, 15, 20,8,1,2,4,1
Class II: 20,25,12,10,8,12,18,30,12,1,1,2
Class III: 12,14,18,1,1,1,5,7,12,40

The insurer wishes to analyse the data using Exp(μ) model.

i) Show that the Exponential distribution is a member of exponential family of distributions. (2)

ii) The insurer decides to use a Model (Model I) for which:

$$\text{Log } \mu = \begin{cases} -\frac{1}{2\alpha} \; for\ i = 1\ to 15 \\ \beta \; for\ i = 16\ to\ 30 \end{cases}$$

Derive the likelihood function for this model and find the likelihood estimates for α and β. (6)

iii) Calculate the scaled deviance for Model I (4)

[12]

Q. 2) A life insurer issues a micro insurance group policy covering a group consisting of 1250 small shopkeepers and 250 hawkers. The policy provides death benefit of Rs. 50000 on death of a shopkeeper and Rs. 20000 on death of a hawker. The policy covers the risk of deaths only over one year time and the probability of death over one year is 0.8% and 1.2% respectively for each of the shopkeepers and hawkers. The aggregate claim payable under the policy is assumed to conform to the individual risk model. The yearly premium for the group is agreed as Rs. 20 lacs.

i) State the individual risk model formula for modelling the aggregate claim amount for the portfolio and the assumptions underlying it. (2)

ii) Show, from first principles, that if X denotes the claim amount payable for an individual member during the coverage period then
$E(X) = bq$ and $\text{var}(X) = b^2q(1- q)$
Where, q is the probability of death of the member during the year and b is the benefit amount. (3)

iii) Calculate the mean and variance of the total claim amount arising out of the policy over the policy year. (2)

iv) Calculate the probability that the aggregate claims payable in a given year will exceed 50% of the premium, using a normal approximation with no continuity correction. (3)

v) Comment on the likely accuracy of your approximate answer to part **(iv)**. (1)

[11]

Q. 3) Number of claims (n sample observations available) in a motor insurance company follows type 2 negative binomial distribution with parameter 2 and p. The prior distribution of p follows beta distribution with parameters α and β.

i) Find maximum likelihood estimate of p. (3)

ii) Find posterior distribution of p and express the posterior estimate in the form of credibility estimate under squared error loss. (5)

iii) If total number of claims is 150 for 20 sample data, $\alpha = 5$ and $\beta = 3$ then calculate posterior estimates for p under squared-error loss, all-or-nothing loss and absolute error loss. (5)

[13]

Q. 4) **i)** Explain with the help of examples how the lack of stationarity in a time series may be caused? (3)

ii) A student is using the Box Jenkins approach to model X(t), an observed time series of a ARIMA(p,d,q) process.

a) Write down the general equation for X(t), if it is a ARIMA(p,d,q) process. (2)

b) What are the main steps involved in the Box-Jenkins methodology. (2)

c) The following table shows information regarding the dth order difference of the observed time series

Sample Auto correlation coefficient	d = 1	d = 2	d = 3
r 1	-0.32	-0.60	-0.80
r 2	-0.02	-0.47	0.54
r 3	0.12	0.15	-0.12
r 4	0.20	0.25	0.20
Sample variance	5.0	50	120

Explain with reasons which value of d you consider most appropriate if this series is to be modelled using an ARIMA(p,d,q) model. (2)

d) Classify each of the following processes as ARIMA (p,d,q) process, where e_t denotes the white noise with mean=0 and variance= σ^2

i. $X_t = 0.8e_{t-1} + e_t$

ii. $X_t = 2X_{t-1} + e_t + 0.5\, e_{t-4}$

iii. $X_t = 1.5X_{t-1} + 0.2X_{t-2} + e_{t-1} + e_{t-2}$ (6)

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Q. 5) The table below shows the cumulative claims incurred on a class of insurance policies, subdivided by accident year and development year. The total claims paid by end of year 2016 is Rs 1.9 million

Claims incurred ('000s)	Development Year				
		1	2	3	4
Accident year	2013	400	426	469	525
	2014	380	460	490	
	2015	508	540		
	2016	490			

i) State the assumption underlying Bornhuetter-Ferguson method to estimate outstanding claims reserves. (2)

ii) The earned premium in respect of the accident years 2013-2016 is shown in the following table. Use the Bornhuetter-Ferguson method to estimate the outstanding claims reserve as at December 31, 2016. (7)

Accident year	2013	2014	2015	2016
Earned premium (000's)	675	700	890	870

iii) If the rate of inflation is 5% per annum, estimate the outstanding claims reserve as on December 31, 2016 using inflation adjusted basic chain ladder method. Assume all the figures are as at mid of the year. (7)

[16]

Q. 6) **i)** Describe Linear Congruential Generators. (2)

ii) Generate two random numbers each from the Beta distribution (with parameter 3 and 2) using Box-muller and Polar Method (pseudo random numbers 0.346, 0.762). (4)

iii) Find out the minimum number of random numbers to be generated for estimating mean under part **(ii)** so that the error is less than 0.1 with probability 97% under absolute error method and relative error method. (3)

[9]

Q. 7) A General Insurance Company offers insurance for loss of valuable arts in an Art Gallery. Individual claim amounts follow gamma distribution (with parameter 20 and 0.4) and number of claims follow poisson distribution (with parameter 0.5). Along with each claim, expenses incurred which follows pareto distribution (with parameter 5 and 100) and independent of the claim amount. The Company has sold 1,000 policies under the above plan.

i) Find out mean and variance of aggregate net outgo for the company. (5)

ii) Calculate the annual premium to be charged so that the company generates profit with probability of at least 95%. (2)

[7]

Q. 8) A General Insurance Company has written a block of health insurance policies where the number of claims out of the portfolio follows Poisson distribution with parameter λ and the individual claim amounts follow exponential distribution with parameter β. The company reinsures the block of policies immediately so that the reinsurer pays 80% of total claim amount out of the block of policies exceeding the retention level of M. The premium loading factor used by the insurer and reinsurer are 25% and 60% respectively.

i) Describe the range of M such that the insurer does not incur any losses $[\beta = 0.01]$ (5)

ii) Deduce the moment generating function $M_Y(R)$; Y is the net claim payable by the insurer. (3)

iii) Hence prove that the adjustment co-efficient equation will follow the below equation:

$0.8\beta^2 e^{MR} - \beta e^{M\beta}(\beta - 0.2R) + (\beta - 0.2R)(\beta - R)[1.25e^{\beta M} - 1.28] = 0$ (3)

iv) Find out adjustment co-efficient, R using M = 50, β = 0.01

Use the following approximation:

$e^x = 1 + x + \frac{x^2}{2}$ (2)

v) If the insurance company had a proportional reinsurance with 20% retention by the insurer then find out the R (2)

vi) If the company's initial surplus is 100 then calculate maximum long-term ruin probability under part **(iv)** and **(v)** and comment. (2)

[17]

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