

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

7th September 2018

Subject CT4 – Models

Time allowed: Three Hours (15.00 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 3. Mark allocations are shown in brackets.*
- 4. Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer booklet and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

- Q. 1)**
- i)** List the major steps involved in developing an actuarial model. (4)
 - ii)** Describe the difference between a stochastic and a deterministic model. (2)
 - iii)** Outline the factors you would consider in deciding whether to use a stochastic or deterministic model to study a problem. (3)
- [9]**

Q. 2) A supervisor in quality control department of a toy factory spends average 1.5 min per toy for checking it. On average, one toy in 10 is found to be faulty. The distribution of faulty toys is assumed to follow Poisson process.

- i)** Define a Poisson process. (2)
- ii)** Determine the probability that the supervisor will not find any faulty toy in next one hour. (1)

After lunch hour supervisor is replaced. New supervisor found one faulty toy exactly after one hour ten minutes.

- iii)** Explain in how much time the new supervisor may expect another faulty toy. (1)
 - iv)** Calculate the probability that at least 2 more faulty toys will be found in next ten minutes. (2)
- [6]**

Q. 3) In the context of survival model:

- i)** Define Type I and Type II Censoring. (2)
- ii)** Give an example where the censoring is informative. (1)
- iii)** Give an example for interval censoring. (1)

In a country of Intelligentsia, students enrolling for actuarial exams complete all the exams and qualify as an actuary on an average duration of three years. The enrollment for student membership starts exactly on 1st January of every year. A group of 100 students joined the Institute on 01/01/2014. The institute wanted to study the completion rate of unmarried male students. They started the study on 01/01/2014 and completed the study on 31/12/2017.

- iv)** Describe the types of censoring present in the data collected for the above study. (4)
- [8]**

Q. 4) An old statistician has a tree at his home. At the start of the autumn season, the tree starts shedding leaves. The old man has a habit of watching and analyzing how the tree sheds its leaves. He observed that the tree sheds half of its leaves in the first twenty days of autumn season. After that he had gone out of home for ten days. After ten days, on his return, he witnessed that only half of the leaves he had seen on the tree ten days before are still on the tree.

- i)** Define the force of mortality μ_x at any age x , Initial rate of mortality and Central rate of mortality. (3)

- ii) Find the Force of Shedding the leaves for the first 20 days and the next ten days assuming that the tree is shedding leaves at a constant rates during the above two periods respectively. Assume that no new leaves are appearing during this period. (3)
- iii) The Statistician assumes that the tree sheds leaves after 30 days in a pattern that resemble with Gompertz Law. He also calculated that the $\mu_{30} = 0.01256$ and $\mu_{40} = 0.02198$. Now calculate the number of days further required for the tree to shed 95% of its leaves. Assume that no new leaves are appearing during this period. (6)
- [12]

Q. 5) A CI policy provides lump sum benefit in respect of a single occurrence from a specified list of illnesses. The policy is terminated upon happening of claim and the future cover is suspended. However if a buyback premium is paid within one month of occurrence of the event then the policy is revived for the illness not related to first claim. After the second claim the policy cannot be revived and it will lapse.

The transition rate for the hazard of the specified event is a constant 0.1. Whilst policies are eligible for revival, the transition rate for buyback of cover through paying a revival premium is 0.05.

- i) Explain whether a time homogeneous or time inhomogeneous model would be more appropriate for modeling this situation. (2)
- ii) Explain how you could model this as a Markov process. (3)
- iii) Sketch a transition diagram for the expanded system. (2)
- [7]

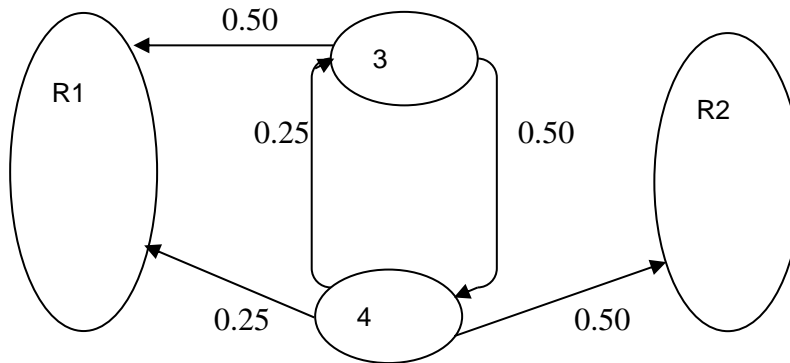
- Q. 6) i)** A Government actuary is charged with producing a national life table for general use, based on data from death registrations and censuses. Explain the role of graduation in the production of this table, and suggest a suitable procedure to use for graduating the crude rates. (5)
- ii) A life office is analyzing the graduated rates derived from its recent experience. These graduated rates must be consistent with a published standard table. What is meant by consistency here? (1)
- iii) What is the purpose of Serial Correlation test and how it works? (2)
- iv) The following table gives an extract of data from a mortality investigation conducted by a life insurance company. The raw data have been graduated by reference to a standard table of assured lives.

Age	Expected deaths	Actual Deaths
60	38.3	36
61	40.45	34
62	42.52	38
63	44.56	40
64	47.63	52
65	51.25	48
66	55.35	57
67	60.45	65

68	63.22	69
69	67.78	74
70	71.56	77

Test whether the graduated rates are reflecting the standard table of assured lives using Serial Correlation Test by assuming that death at each age follows Poisson distribution. (5)
[13]

Q. 7) Consider the following Markov Chain along with transition probabilities.



- i) Assuming the current state held by the process is 3, find the probability that the chain get absorbed in R1. (5)
- ii) Find the expected no. of steps until the chain gets absorbed in R1 or R2. (5)

[10]

Q. 8) i) State the Principle of Correspondence. (2)

- ii) One small life insurance company is selling term insurance policies in two different States of a particular country. The following data is extracted from the policy servicing department of the company:

For age 60 lbd (last birthday)	State I	State II
Number of policies in force as at 01/01/2016	15000	32000
Number of policies in force as at 01/01/2017	20000	27000
Number of deaths during 2016	264	445

Calculate the central death rate for the year 2016 for this age for the two states. (2)

- iii) A detailed analysis of the records shows that 60% of the policy holders of State I are from Rural area and 40% of the policy holders of State II are from Rural area. Moreover, 40% of the rural policy holders of State I are male and 60% of the rural policy holders of State II are male. And 60% of the urban policy holders of State I are male and 40% of the urban policy holders of State II are male. National Census data showed that for age 60, death rates for Rural population is 40% more than the death rates of Urban Population and death rates for Male are 20% more than the death rates of Female.

Estimate the central death rates for Urban Male & Rural Female in State I and Urban Female & Rural Male in State II.

State any assumptions, you made in the above estimations.

(5)

- iv) The company is charging the same premium for everyone. Comment on the pricing structure of the product based on the results of (iii).

(4)

[13]

- Q. 9)** A college student decided to sell samosa at railway stations to earn money. On the first day, he bought 50 samosas from a local vendor and entered the railway station at morning 8 am. He wanted to sell at least 30 samosas so that he can recoup the expenses. During the first day the following events happened.

At 8.20 am	Five samosas are bought by a family
At 8.40 am	Seven samosas are bought by a group of college students
At 9.00 am	The student found that five samosas are not in good condition and he throws them out
At 9.20 am	Ten samosas are bought by a group of workers.
At 9.40 am	One passenger bought three samosas but forget to pay the money.
At 10 am	Seven somas as are bought by a passenger.
At 10.20 am	One passenger bought eight samosas.
At 10.30 am	The student left the station with the remaining samosas.

- i) Estimate the time taken by the student to sell at least 30 samosas using Nelson-Aalen estimator.

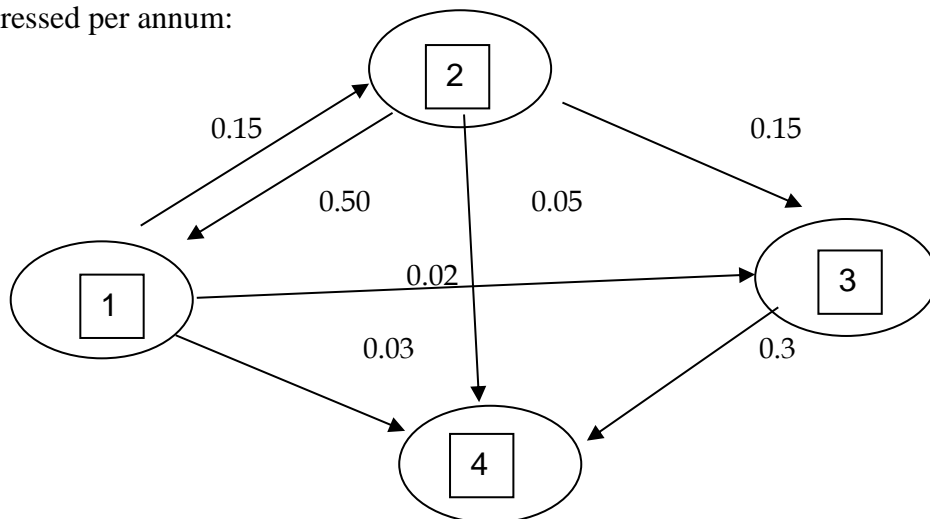
(6)

- ii) Comment whether the above estimate would be a good basis for the student to predict his future sales.

(3)

[9]

- Q. 10)** A continuous-time four state Markov process is defined in following diagram with transition rates expressed per annum:



- i) Derive from the first principles a differential equation for ${}_t p_x^{13}$ stating all the assumptions made.
[${}_t p_x^{13}$ denotes the probability that a person who is in state 1 when aged x , will be in state 3 at age $x + t$] (5)
- ii) Derive expression for the probability of a person aged x of remaining in state 1 for t years continuously. (4)
- iii) Calculate the probability of a person remaining in state 1 for 4 years continuously. (1)
- iv) Give integral form of the backward Kolmogorov equation for the probability of a person in state 2 at time s moving to state 4 at time t without going to state 3. (3)
- [13]**
