

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

7th September 2018

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

- Q. 1) i)** An investment firm issues an interest rate swap to an investor. Describe how the cashflows are exchanged in an “interest rate swap”. (2)
- ii)** Describe the characteristics of ordinary shares. (3)
- iii)** A particular share pays annual dividend of 4% and is purchased immediately after a dividend payment is made. The next dividend is expected to grow at 3% per annum compound.
Inflation is expected to be 2% per annum. Assume that the equity is held indefinitely. Calculate the real return obtained on this equity. (Assume that the purchase price of the share is Rs. 1). (4)
- iv)** A bank offers the following choices for certificates of deposit:

Term (in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.75%

The certificates mature at the end of the term. The bank does not permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates.

An investor initially deposits Rs.10,000/- in the bank and needs atleast Rs. 13,000 at the end of 6 years. Which combination of terms he needs to choose to earn maximum return at the end of 6-year period and what is the accumulated amount he will get at the end of 6 years in such case. (3)

[12]

- Q. 2) i)** From first principle show that $(Ia)_{n|} = (\ddot{a}_{n|} - n v^n) / i$ (3)
- ii)** The ABC insurance company designed a single premium contract which will pay Rs. 1,00,000 in ten years' time. The annual effective rates of interest over next ten years are expected to be 7%, 8% or 10% with probabilities 0.3, 0.5 and 0.2 respectively.
- Calculate:
Single premium for the contract based on same expected rate of interest for the future years. (2)
- iii)** Expected profit at the end of the contract assuming interest rate remains constant over the ten year period. (2)

[7]

- Q. 3)** The bank of India issued a loan of Rs. 58,50,000 to an investor for starting his own firm. The loan was scheduled to be repaid over a period of 15 years using quarterly installments payable in arrears.

According to the repayment term at the end of each five year term, the quarterly installment will be increased by Rs. 40,000. The installment amounts were calculated using 12% per annum effective rate of interest.

- i) Calculate the initial amount of the quarterly repayment. (4)
- ii) On the basis of the lender's original loan schedule, calculate the amount of the principal repaid in third and the thirteenth year. (9)
- iii) Immediately after the 33rd quarterly installment, the investor requested for loan restructuring. He wanted to pay a fixed quarterly installment over the remaining loan term. Calculate the revised quarterly installment using 12% p.a. effective rate of interest. (5)
- [18]

Q. 4) Seven years ago an investment bank offered an accumulating fund where the fund investment income was reinvested into the same fund again. Following table gives the unit prices from 1st April 2011 to 1st April 2017:

Year	Price of unit on 1 st April (Rs.)
2011	10.00
2012	11.56
2013	12.89
2014	14.16
2015	15.84
2016	17.54
2017	19.36

- i) On the basis of above prices and ignoring taxation and expenses, find
- a) The time weighted rate of return for the fund over the period 1st April 2011 to 1st April 2017. (1)
- b) The yield obtained by the investor who purchased 500 units on 1st April in each year from 2011 to 2016 inclusive and sold his holdings on 1st April 2017. (5)
- c) The yield obtained by Mr. Sam who invested Rs 25,000 in the fund on 1st April each year from 2011 to 2016 inclusive and who sold his holdings to the fund managers on 1st April 2017. (5)
- ii) Suppose in order to allow for expenses, the fund managers sell units at a price which is 2% above the published unit price and buy back units at a price which is 2% below the published price. On this basis, find revised answers for (i) (b) and (i) (c). (7)
- [18]

Q. 5) The Government of Actuarialia has issued a security, redeemable at par at the end of n years. The security promises to pay annual coupons at a rate of 20% p.a. (per annum) at the end of each year. An Indian investor not subject to income tax found that he can earn an effective yield of 11% p.a., if he can purchase Rs. 100 nominal value of the stock at a price of Rs. 153, assuming no risk of default.

However, due to political uncertainty on the Actuarialia planet, the Indian investor subsequently decided to allow for the probability of default. He assumed a probability of $(0.95652)^t$ for receiving the payment at the end of t^{th} year. The probabilities in each year are assumed to be independent of the others. Calculate the present value of the inflow stream produced by holding of Rs. 100 stock, assuming an interest of 10% p.a. effective.

[6]

- Q. 6)** A security has been issued with half-yearly interest at the rate of 6% p.a. and redeemable in “n” year’s time at “C%”.

The purchase price that would give a Tax-free investor a yield of 4% p.a. is 132.55%.

The purchase price of the above security that would give an investor who is subject to only income tax of 32%, a net yield of 4% p.a. is 107.85%.

Find “n” (n is an integer, you may round your answer to nearest integer) and “C”. [5]

- Q. 7) i)** For a rate of interest of 7% per annum, convertible monthly, calculate:
- a)** The equivalent rate of interest per annum convertible half yearly, and (2)
- b)** The equivalent rate of discount per annum convertible monthly (2)
- ii)** The force of interest p.a. changes from δ_0 at time 0 to δ_m at time $t = m$ years and will thereafter remain constant at δ_m .

Calculate the value of accumulation when $m = 16$, $n = 39$, $\delta_0 = \log_e (1.04)$ and $\delta_m = \log_e (1.03)$. (6)
[10]

- Q. 8) i)** Outline the similarities and differences between deterministic and stochastic interest rate models. (3)
- ii)** The rate of interest earned in the year from time $t-1$ year to t year is denoted by i_t where $(1 + i_t)$ is log-normally distributed. The expected value of the rate of interest is 5% p.a., and the standard deviation is 11%.
- a)** Calculate the mean and Standard deviation of the lognormal distribution of $(1 + i_t)$. (3)
- b)** Calculate the probability that the rate of interest in the year from time $t-1$ year to t year lies between 4% and 7%. (3)
[9]

- Q. 9)** Describe the following terms

- i)** Instantaneous Forward rate (1.5)
- ii)** Liquidity Preference Theory (1.5)
[3]

- Q. 10)** A small technology company has taken up a new project. At the beginning of each of the first three years 18 lacs will be invested in the project. From the beginning of the first year until end of the 25th year net revenue will be received continuously. The payment of net revenue will begin at 2.5 lacs p.a. The rate of payment is assumed to grow continuously at a rate of 6% p.a. effective.

- i)** Calculate a net present value of the project at an effective rate of interest of 7% p.a. (6)

- ii) Calculate the discounted payback period of the project at an effective rate of interest of 7% p.a.

(2)
[8]

- Q. 11)** A new super market has earned a simple rate of interest of 8% p.a. over the last calendar year based on the following cash flows:

Items	In Rs.
Assets, beginning of year	2,50,00,000
Sales Revenue	X
Salaries paid	22,00,000
Other expenses paid	7,50,000

- i) Net investment income earned from above cash-flows over the year is Rs. 20,00,000. Assuming that all cash flows occur at the middle of the year, calculate the value of X.

(2)

- ii) Also calculate the effective yield of above cashflows.

(2)
[4]
