Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

September 2017 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) $s_{55:\overline{10|}}$

= 1.06^10 * $a_{55:\overline{10|}}$ / $_{10}p_{55}$

= 1.06^10 * (($\ddot{a}_{55}$ - 1) - 1.06^(-10) *(l_{65}/l_{55})* ($\ddot{a}_{65}$ - 1)) / (l_{65}/l_{55})

= 1.06^10 * ((13.057 - 1) - 1.06^(-10) *(8821.2612/9557.8179)* (10.569 - 1)) / (8821.2612/9557.8179)

= 13.82616 [2]

ii) $\ddot{a}^{(4)}_{[40]:20\overline{7}}$

= $\ddot{a}^{(4)}_{[40]}$ - v^{20} * ($l_{60}/l_{[40]}$) * $\ddot{a}^{(4)}_{60}$

= ($\ddot{a}_{[40]}$ – 3/8) – 1.06^(-20) * ($l_{60}/l_{[40]}$) * ($\ddot{a}_{60}$ – 3/8)

= (15.494 - 3/8) – 1.06^(-20) * (9287.2164/9854.3036) * (11.891 - 3/8)

= 11.735 (rounded) [2]

[4 Marks]

Solution 2:

μx is defined as the force of mortality which represents the instantaneous death rate for a life aged x.

Kx is a discrete random variable called as curtate future lifetime of a life currently aged x. It represents the complete number of future years survived and ignores the fractional part.

Tx is a continuous random variable representing the complete future lifetime of a life aged x.

[3 Marks]

Solution 3:

i) Difference between Net premium reserves (NPR) and Gross premium reserves(GPR) –

NPV has no explicit allowance for future expenses and is based on the Net premium. Implicitly it applies that the difference between the future gross premium and net premium would be used to meet the expenses in the respective future years.

GPR allows explicitly for expected future expenses and takes into consideration the gross premiums (office premium) which has loading for future expenses.

In case of with profit products, NPV does not allow for any future bonuses whereas GPV explicitly allows for some level of future bonuses since GPV is based on gross premium, which in case of with profit products has loadings for future bonuses along with future expenses. [2]

ii) Components of gross premium for with profit product:

- Guaranteed death, surrender, maturity and other benefits
- Loading for initial and renewal expenses
- Loading for initial and renewal commissions
- Loading for some level of future bonuses [2]

iii) Suitability of GPR without future bonuses:

Calculation of GPR without any allowance for future bonuses for a with profit product would lead to under reserving, since the gross premiums included in GPR calculation would generally have some loadings towards future bonuses. If no allowance is made for future bonuses than it would lead to release of these loadings being treated as surplus and hence leading to under reserving.

However if there is no allowance for future loadings in the gross premium, it might be fine to calculate GPR without any allowance for future bonuses. [2]

[6 Marks]

Solution 4:

Expected PV = 2,500,000 * $A^1_{45:20\urcorner}$ + 1,000,000 * $A_{45:20\urcorner}^{\ 1}$

Now:

$A^1_{x:n\urcorner} = \int_x^{x+n} v^t * tpx * \mu_{x+t} * dt$

Given a constant μ = 0.025 and force of interest = Ln(1.05) we have

$A^1_{45:20\urcorner} = \int_0^{20} \exp(-\ln(1.05)) * \exp(-\mu) * \mu * dt$

$A^1_{45:20\urcorner} = \int_0^{20} \exp(-\ln(1.05)) * \exp(-0.025) * 0.025 * dt$

= 0.025 * $\int_0^{20} \exp(-0.07379)\, dt$

=(0.025/0.07379)* (1-exp(-1.475803))

= 0.261351

$A_{45:20\urcorner}^{\ 1}$ = exp(-ln(1.05)*20) * exp (-0.025 *20)

= exp (-1.475803) = 0.228595

Expected PV = 2,500,000 * 0.261351 + 1,000,000 * 0.228595

= Rs. 881,972.5

Now for calculation of Variance, we have to Evaluate the above factors with force of interest = 1.05^2 – 1 = 10.25%.

This gives a force of interest of Ln(1.1025) = 0.09758

So

$^2A^1_{45:20\urcorner} = 0.025 * \int_0^{20} \exp(-0.09758 - 0.025)\, dt$

= (0.025/0.12258) * (1- exp(-2.451607))

= 0.186377

And

${}^2A_{45:20\overline{7}}^{\ 1}$ = exp (-2.451607)

= 0.086155

Now Variance is give as:

= 2,500,000^2 * ${}^2A_{45:20\overline{7}}^{1}$ + 1,000,000^2 *${}^2A_{45:20\overline{7}}^{\ 1}$ – EPV^2

= 2,500,000^2 * 0.186377+ 1,000,000^2 *0.086155 – 881,972.5^2

= (687,848.65)^2 = 473,135,759,244

[8 Marks]

Solution 5:

Let the minimum single premium be S

Insurers' loss random variable is given by:

L = PV of Future outgo – PV of Future income

= 500,000 * $a_{\overline{Kx|}}$ + 5% * S + 200*$a_{\overline{Kx|}}$ - S

Where K_x is the curtate future expectation of lifetime of life currently aged x.

For annuity product more the lifetime of policyholder, greater would be loss for the insurer.

We need to find a value of t such that

P (L > 0) = 0.1

➔P (T < t) = 0.9

Using PMA92C20 we require

l_{60+t} / l_{60} <= 0.9

l_{72}/l_{60} = 8968.099/9826.131= 0.912678

l_{73}/l_{60} = 8803.265/9826.131= 0.895903

So K_x = 73 – 60 = 13

Hence we need the minimum premium such that

L = 500,000 * $a_{\overline{13|}}$ + 5% * S + 200*$a_{\overline{13|}}$ - S (@4%) = 0

= 500,200*9.9856 - 0.95*S = 0

S = Rs. 5,257,681

[8 Marks]

Solution 6:

i) Following could be the reasons:

- Term assurance portfolio must have witnessed huge number of lapses during these years. These could be because of many reasons like, policyholders switching to other companies which might offer better rates, changes in the personal situation of individuals, leading to no further need of term assurance, in such scenarios the policyholder could lapse the policy specially given that it's a regular premium contract and each year of premium is like a cost of cover for one year. This would not be the case with immediate annuity policies, as the companies will not generally offer surrender values and it would not make sense for policyholders in such case to lapse and forfeit the future benefits.
- Small differences could also be on account of difference in death rates for two portfolios. [2]

ii) Reserve for Annuity portfolio:

= 9,900 * 250,000 * a_{60}
= 9,900 * 250,000 * (15.632 – 1)
= 9,900 * 3,658,000
= Rs. 36,214.2 Million

Calculation of reserves for term insurance:

First calculate per policy annual net premium = P

P * $\ddot{a}_{40:\overline{25|}}$ = 2,500,000 * $A^{1}_{40:\overline{25|}}$

P * $\ddot{a}_{40:\overline{25|}}$ = 2,500,000 * (A_{40} - 1.04^(-25) * (l_{65}/l_{40}) * A_{65})

P = 2,500,000*(0.23056-1.04^(-25)*(8821.2612/9856.2863) * 0.52786)/15.884

P = Rs. 8,396

Reserves for Term Insurance portfolio

= 8,500 * (2,500,000 * $A^{1}_{50:\overline{15|}}$ - P * $\ddot{a}_{50:\overline{15|}}$)

= 8,500*(2,500,000*(A_{50}-1.04^(-15)* (l_{65}/l_{50}) * A_{65}) – 8,396 * 11.253)

= 8,500*(2,500,000*(0.32907-1.04^(-15)*(8821.2612/9712.0728)*0.52786)
- 8,396 * 11.253)

= Rs. 532.5 Million

Total Reserve = 36,214.2 + 532.5 = Rs. 36,746.7 Million [6]

iii) Following could be possible sources of surplus for a life insurance company:

- Mortality surplus
- Lapse/surrender surplus
- Expense surplus
- Interest surplus [2]

iv) Calculation of mortality profit:

Annuity portfolio:

Death strain for each policy surviving till 31st Dec 2016

DSAR = 0 – (250,000 + 3,658,000) = - 3,908,000

Actual death strain (ADS) = Actual deaths * DSAR = 4 * - 3,908,000 = -15,632,000

Expected death strain (EDS) = Expected deaths * DSAR
= Policies at start of 2016 * q_{59} * DSAR
= (9900+4)* 0.002110 * (-3,908,000)
= - 81,667,196

Mortality Profit= EDS – ADS
= Rs - 66,035,196 (i.e a loss)

Term assurance portfolio:

Death strain for each policy surviving till 31st Dec 2016

DSAR = 2,500,000 – 62,650 = 2,437,350

Actual death strain (ADS) = Actual deaths * DSAR = 5 * 2,437,350 = 12,186,751

Expected death strain (EDS) = Expected deaths * DSAR
= Policies at start of 2016 * q_{49} * DSAR
= (8500+5)* 0.002241 * 2,437,350
= 46,455,172

Mortality Profit= EDS – ADS
= Rs. 34,268,421 (i.e a profit)

Total mortality profit = 34,268,421 - 66,035,196
= Rs. -31,766,774 (loss of 31,766,774) [6]

[16 Marks]

Solution 7:

EPV = 1,000,000 * ($\bar{a}_{\overline{65(m):60(f)}}$ - $\bar{a}_{65(m):60(f)}$) + 200,000 * $\bar{a}_{\overline{5}|}$ * $\bar{A}_{\overline{65(m):60(f)}}$

Now:

$\bar{a}_{\overline{65(m):60(f)}}$ = $\bar{a}_{65(m)}$ + $\bar{a}_{60(f)}$ - $\bar{a}_{65(m):60(f)}$
= (13.666 - 0.5) + (16.652 – 0.5) – (12.682 – 0.5)
= 17.136

$\bar{A}_{\overline{65(m):60(f)}}$ = (1 - δ * $\bar{a}_{\overline{65(m):60(f)}}$)
= 1 - Ln(1.04) * 17.136
= 0.327914

So
EPV = 1,000,000 * (17.136 – (12.682 – 0.5)) + 200,000 *((1-1.04^(-5))/ δ) * 0.327914

EPV = 1,000,000 *(17.136-(12.682-0.5))+200,000 *((1-1.04^(-5))/Ln(1.04))*0.327914

= 5,251,764

[6 Marks]

Solution 8:

Let ${}_t(ap)_x'$ and ${}_t(aq)_x'$ represent dependent probabilities that ignore the decrement of surrender.

Further, let decrement of death due to accident be denoted by a, death from any other cause be denoted by d and surrender bye denoted by s. We can then write:

$(aq)_{40}^{a} = {}_{1/2}(aq)_{40}^{a\prime} + {}_{1/2}(ap)_{40}'\left(1 - q_{40+1/2}^{s}\right) {}_{1/2}(aq)_{40+1/2}^{a\prime}$ (Similar for $(aq)_{40}^{d}$)

${}_{1/2}(ap)_{40}' = exp\left[-(\mu^{a} + \mu^{d})\, 1/2\right] = e^{-0.032\times 0.5} = 0.984127$

${}_{1/2}(aq)_{40}^{a\prime} = {}_{1/2}(aq)_{40+1/2}^{a\prime} = \frac{\mu^{a}}{\mu^{a}+\mu^{d}}\left(1 - {}_{1/2}(ap)_{40}'\right) = \frac{0.002}{0.032} \times (1 - 0.984127)$
$= 0.000992$

Similarly, ${}_{1/2}(aq)_{40}^{d\prime} = {}_{1/2}(aq)_{40+1/2}^{d\prime} = \frac{0.03}{0.032} \times (1 - 0.984127) = 0.014881$

$(aq)_{40}^{a} = 0.000992 + 0.984127\ (1 - 0.15)\ 0.000992 = 0.001822$

$(aq)_{40}^{d} = 0.014881 + 0.984127\ (1 - 0.15)\ 0.014881 = 0.027328$

$(aq)_{40}^{s} = {}_{1/2}(ap)_{40}'\, q_{40+1/2}^{s} = 0.984127 \times 0.15 = 0.147619$

$(ap)_{40} = 1 - (aq)_{40}^{a} - (aq)_{40}^{d} - (aq)_{40}^{s} = 0.823231$

Assuming that on average decrement a and d occur halfway through the year,

Let the Single Premium to be charged be P. Using the principle of equivalence:

$P = 20000\ (1.01)(aq)_{40}^{a} v^{1/2} + 10000(1.01)\ (aq)_{40}^{d} v^{1/2} + 0.7P(1.01)(aq)_{40}^{s} v^{1/2}$
$+ 5000(1.01)(ap)_{40} v + 0.02P$, where $v = 1.03^{-1}$

$P = 36.26 + 271.97 + 0.102836P + 4036.23 + 0.02P$

$0.877164P = 4344.46$

$P = 4952.85$

A Single Premium of Rs. 4,952.85 should be charge on the contract.

[10 Marks]

Solution 9:

i)

a) An accrued benefit (sometimes referred to as past service benefit) is a benefit that has been earned as a result of pensionable service (or credited service) prior to the valuation date e.g. a pension of n/80th of final average salary where n is the number of years of past pensionable service at the valuation date.

b) A future service benefit is a benefit that is expected to be earned as a result of pensionable service after the valuation date e.g. a pension of (65-x)/80th of final average salary on age retirement at NPA of 65 for a member aged x at the valuation date.

c) A prospective service benefit does not depend on either past or future service, although it may depend on total expected pensionable service, e.g. a pension of m/80th of final average salary at date of ill-health retirement where m is the total expected pensionable service before NPA. [3]

ii) a) $\frac{1}{3}\ (500000)\ \frac{s_{42}}{s_{41}}\ \frac{{}^{z}M_{42}^{ia}}{{}^{s}D_{42}} = \frac{1}{3}\ (500000)\ \frac{8.129}{7.980}\ \frac{56093}{22757}$

= Rs 4,18,481.86 [3]

b) $\frac{500000}{10}\ \frac{s_{42}}{s_{41}}\ \left(\frac{17\ {}^{z}M_{42}^{ra} + {}^{z}\bar{R}_{42}^{ra} - {}^{z}\bar{R}_{60}^{ra}}{{}^{s}D_{42}}\right)$

$= \frac{500000}{10}\ \frac{8.129}{7.980}\ \frac{(17)(128026) + 2628208 - 323739}{22757}$

= Rs 1,00,28,951.82 [3]

c) $1000000\ \left(\frac{M_{42}^{i} + M_{42}^{r}}{D_{42}}\right) = 1000000\ \left(\frac{M_{42}^{i} + M_{42}^{r}}{D_{42}}\right)$

$= 1000000\ \left(\frac{356+782}{2799}\right)$

= Rs 4,06,573.78 [2]

[11 Marks]

Solution 10:

i) The unit fund will be projected as under:

Policy year	Premium allocated	Cost of allocation	Plus fund b/fd	Fund before charge	Fund management charge	Policy admin. Fees	Fund c/fd
1	17500.00	15750.00	15750.00	16852.50	84.26	500.00	16268.24
2	27500.00	24750.00	41018.24	43889.52	219.45	500.00	43170.07
3	27500.00	24750.00	67920.07	72674.47	363.37	500.00	71811.10
4	27500.00	24750.00	96561.10	103320.38	516.60	500.00	102303.78

The expected profit will be calculated by projecting the non unit cashflows as under:

Policy year	Profit on allocation	Expenses	Non unit interest	Fund management charge	Policy admin. fees	Non unit death cost	Profit in each year
1	9250.00	3500.00	402.50	84.26	500.00	730.98	6005.78
2	250.00	1100.00	-59.50	219.45	500.00	327.45	-517.50
3	250.00	1100.00	-59.50	363.37	500.00	0.00	-46.13
4	250.00	1100.00	-59.50	516.60	500.00	0.00	107.10

[10]

ii) The non-unit reserves required are:

Start of year 3: 46.13/1.07 = 43.11
End of year 2: 43.11 × 0.985 + 517.50 = 559.96
Start of year 2: 559.96/1.07 = 523.33
End of year 1: 523.33 × 0.985 = 515.48

The calculations of revised profit are as under:

Policy year	Non unit reserve b/fd	Interest on reserves	Increase in reserves	Profit ignoring reserves	Profit allowing for reserves
1	0.00	0.00	515.48	6005.78	5490.30
2	523.33	36.63	-480.87	-517.50	0.00
3	43.11	3.02	-43.11	-46.13	0.00
4	0.00	0.00	0.00	107.10	107.10

[9]

[19 Marks]

Solution 11:

a) Temporary initial selection occurs when heterogeneity is present in a group that was selected on the basis of a criterion whose effects wear off over time. The differences are temporary. E.g. getting through the medical underwriting hurdle of a life company gives policyholders particularly good mortality, but that this effect wears off as time since the medical examination increases.

b) Class selection refers to a factor which is permanent in its effect with respect to mortality i.e. the source of the heterogeneity, in this case, is due to a permanent attribute of the individuals concerned. E.g. gender with categories of male and female, occupation with categories of manual and non-manual employment.

c) Within a population mortality (or morbidity) varies with calendar time. This effect is usually observed at all ages. The usual pattern is for mortality rates to become lighter (improve) over time, although there can be exceptions, due, for example, to the increasing effect of AIDS in some countries. E.g. Individuals living 20 years ago experienced higher mortality rates than individuals of the same age living today.

d) Adverse selection is characterised by the way in which the select groups are formed rather than by the characteristics of those groups. It usually involves an element of self-selection, which acts to disrupt (act against) a controlled selection process which is being imposed on the lives. E.g. If prospective policyholders know that a company does not use smoking status as a rating factor while underwriting, then lives who smoke will opt to buy a policy from this company.

e) Ascribing mortality differences to groups formed by factors which are not the true causes of these differences is termed spurious selection. E.g. Increasing the strictness of underwriting for life insurance products will lead to a lighter mortality experience. This will give the false impression that mortality is improving at a quicker rate than it really is. **[5 Marks]**

Solution 12: A pension fund includes following different classes of lives:

- active members of the scheme, i.e. those not in receipt of any benefits and for whom contributions are being paid
- deferred pensioners, i.e. those for whom no contributions are currently being paid and who are entitled to a pension at some future date
- pensioners who retired at the normal retirement age and are now receiving a pension
- pensioners who retired under the ill-health retirement rules of the scheme and are now receiving a pension benefit
- pensioners who retired under the early (before normal pension age) retirement rules of the scheme and are now receiving a pension benefit.

The following selection will be present:

- Class selection: The mortality of those who retired early (but in good health) or at normal retirement age is likely to be lower than that of ill-health retirement pensioners.
- Class selection: Different sections of a large scheme, e.g. works and staff, may exhibit different levels of mortality.
- Temporary initial selection: The mortality of ill-health retirement pensioners is likely to depend on duration since retirement for a few years following the date of retirement, and subsequently only on age attained.
- Among the active members of the scheme ill-health retirement acts as a selective decrement, resulting in lighter mortality among the remaining active members.
- If voluntary resignation is the cause of withdrawal this tends to select those with lighter mortality (and ill-health retirement) rates.
- Time selection: If redundancy is the cause withdrawal rates tend to vary markedly over time as economic conditions vary.

[4 Marks]

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