INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd September 2017

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (10.15* – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. ** You have 15 minutes at the start of the examination in which you are required to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*
3. *You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*
4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
5. *Attempt all questions, beginning your answer to each question on a separate sheet.*
6. *Mark allocations are shown in brackets.*
7. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION
Please return your answer booklet and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) **i)** Outline the key features of the following options as they might apply to over-the-counter contracts:

- Asian Options
- Barrier Options
- Basket Options
- Bermudan Options
- Binary (Digital) Options
- Compound Options
- Forward Start Options
- Lookback Options
- Shout Options (9)

ii) Discuss the key risks involved in transacting over the-counter options. (5)

[14]

Q. 2) The one-factor Heath-Jarrow-Morton (HJM) model of the yield curve assumes a generalized risk-neutral process for zero-coupon bond prices $P = P(t, T)$ as given below.

$$dP = r(t)Pdt + \sigma(t, T, P)dz$$

Where

- t is calendar time (from now),
- T is maturity time (from now),
- r is the short rate at time t,
- σ is the volatility of bond price P, and
- dz represents standard Brownian motion.

i) Describe how the HJM model as described above can also be expressed in terms of the forward rates $f(t, T)$ and show the explicit link between the volatility of a zero coupon bond and the drift of the forward rate. (Detailed derivation is not required). (8)

ii) Discuss the benefits of using a two-factor HJM model, i.e. with two stochastic components, dz_1 and dz_2. Recommend a possible model structure and approach to calibration. (6)

[14]

Q. 3) The table given below gives details about two corporate bonds each of which has a notional principal of Rs. 100 crores.

	Annual Coupon	Term to Maturity in Years	Clean Price (Rs. crores)
Creta Limited	7%	1	99
Statum Limited	5%	2	98

The current risk-free rate of interest is flat at 5% per annum, compounded annually for all maturities.

i) Calculate the equivalent risk free bond prices for each corporate bond. (2)

ii) Construct a risk neutral portfolio to calculate the single premium risk neutral price of a 1 – year Credit Default Swap (CDS) on the Creta Limited bond, assuming default can only take place on final maturity. (3)

iii) Calculate the single premium risk neutral price of a 2 year CDS on the Statum Limited bond, assuming default can only take place on final maturity. (2)

iv) Describe the reasons for the difference in the pricing of the CDS on the two bonds. (3)

[10]

Q. 4) The current zero coupon yield curve is given as follows. All rates are quoted with annual compounding.

Term (in Years)	Rate (% Per Annum)
1	7.25
2	7.35
3	7.65
4	7.95
5	8.2
6	8.35
7	8.5

i) Calculate the continuously compounded 3.75 year zero coupon rate and use that to estimate the bond price. (2)

ii) Calculate the fixed leg coupon of a 4-year par value annual interest rate swap. (2)

iii) Calculate the fixed leg coupon of a forward-starting 4-year par value annual to annual interest rate swap commencing in 3- years' time. (3)

[7]

Q. 5) **i)** Describe briefly what must be delivered upon exercise of exchange traded put and call options on futures contracts. (2)

ii) Demonstrate how the Black-Scholes formulae for European options on non-dividend paying stocks can be extended to apply to European options on stocks paying continuous dividend at a known dividend yield. (3)

[5]

Q. 6) A large multi – national bank with a diverse range of derivative positions has recently written a large and unusual over-the-counter (OTC) one-year put option on a basket of two equities. The bank now wants to hedge its market risks in this deal.

i) Describe in detail how the bank could price the above mentioned option using an appropriate formula. (9)

ii) For hedging purposes -

a) List Greeks that the bank will be most keen to hedge.
b) Mention the possible hedging instruments that can be used by the bank.
c) Describe the signs (positive or negative) of the Greeks for the OTC put and for each of the suggested hedging instruments. (12)

iii) Describe how the bank would hedge the risks associated with this deal. (5)

[26]

Q. 7) The Treasury division of a bank uses the Black model to value vanilla interest rate options (caps, floors and swaptions). The bank is looking to extend its range of products into more exotic interest-rate swaps and options denominated in Euro, and is considering building a term structure yield curve model for pricing and hedging these instruments.

i) Given the above information, answer the following questions:

a) Outline the reasons for choosing a full yield curve model in this situation.
b) Set out the main criteria such a model must satisfy.
c) Explain the particular relevance of a "no arbitrage" condition in this context. (9)

Assume that a one-factor yield curve model projects the short-term rate r according to the following stochastic process:

$$dr = a(t)[b(t) - r]dt + \sigma(t)dz$$

where $\sigma(t)$ is the time-dependent short-rate volatility and z is a standard Brownian motion. The bank has decided to use your implementation that sets parameters $a(t)$ and $\sigma(t)$ as constants a and σ respectively for all t.

ii) Describe how you would construct a trinomial rate tree for this model, calibrated to the term structure of zero coupon bond prices and caplet volatilities. You do not need to derive the branching equations. (10)

iii) Discuss the extent to which your implementation will fulfil the desirable criteria you identified in (i) (b). (5)

[24]

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