INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st September 2017

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION **Please return your answer book and this question paper to the supervisor separately.**

Q. 1) **i)** Explain with the help of formula the following measures of investment risk:
a) Value at Risk
b) Tail value at Risk (3)

ii) Describe how the risk measures listed above are related to the form of an investor's utility function. (2)

iii) An investor is contemplating an investment with a return of Rs. R, where:

R = 250,000- 100,000N and N is a Normal [1, 1] random variable.

Calculate each of the following four measures of risk:

a) Variance of return (1)

b) Downside semi-variance of return (1)

c) Shortfall probability, where the shortfall level is Rs. 50,000 (2)

d) Value at Risk at the 95% confidence level (3)

iv) Derive the expression for Tail Value at Risk at the 95% confidence level, conditional on the VaR in part (d) above being exceeded (3)

[15]

Q. 2) Suppose there are only three assets available on stock exchange:

Asset	Expected Return	Standard Deviation
1	.06	.10
2	.08	.15
3	.10	.20

Correlation matrix is given by

$$\begin{bmatrix} 1 & .5 & .5 \\ .5 & 1 & .5 \\ .5 & .5 & 1 \end{bmatrix}$$

An investor in this market wants to minimise the variance of this portfolio.

i) Determine the Lagrangian function from the above information that can be used to find the minimum variance portfolio for a given expected return. Define any notation used. (3)

ii) By deriving the partial derivatives of the function in (i) state the five equations that could be solved to determine the minimum variance portfolio associated with an expected return of 9%. (4)

iii) Determine the composition of the corner portfolio where asset 1 is not present. (5)

[12]

Q. 3) **i)** Within the context of Capital Asset Pricing Model (CAPM), explain what is meant by the "market price of risk". (2)

ii) In the market where CAPM is assumed to hold, the expected annual return on the market portfolio is 12%, the variance is 4%% and the effective risk free annual rate is 4%. An Agent wants an expected annual return of 18% on a portfolio worth of Rs.12,00,000/-.

a) Calculate the standard deviation of return on the corresponding efficient portfolio. (2)

b) Calculate and explain the amount of money invested in each component of Agent's. (3)

[7]

Q. 4) **i)** State the assumptions underlying the Black Scholes Model. (2)

ii) How realistic are these assumptions? (2)

iii) Determine and calculate the lower and upper bounds for a 3 year European Call option on Share X if the Share price is Rs. 60, Strike price is Rs. 50 and risk free rate is 3% pa. (3)

An investor is considering selling an European call option on a share and wants to hedge some of its risk. The share is non- dividend paying and has properties as given in part (iii). Additional information is

- Volatility: 25% pa
- Vega: 29

iv) Calculate the price of the call option using Garman- Kohlhagen formula and show that boundary conditions calculated in part (iii) are being satisfied. (3)

Assume that the volatility has instantaneously increased to 27% pa, with everything else except the option price remaining the same.

v) Estimate the new option price, assuming Taylor's approximation. (2)

[12]

Q. 5) A commodity of price C is assumed to follow the process:

$dC = \mu C\, dt + \sigma C\, dW_t$

where μ and σ are positive constants and Wt is a standard Brownian motion. The continuously compounded risk free interest rate r is a constant.

You wish to value a special type of option. You construct a recombining binomial tree algorithm using a proportionate "up step" u and "down step" d for each small time interval Δt, and the stock price at time 0 is S_0.

i) Specify fully the first step of the binomial process, giving formulae for the up and down probabilities and step sizes u and d. (3)

The initial commodity price is 80, σ is 15% per annum and r=0. You may assume that u can be approximated by $\exp(\sigma\sqrt{\Delta t})$ for small Δt.

ii) For the tree specified in (i)

a) Draw three steps of the tree with quarter- year time steps and calculate the commodity price at each node. (3)

b) Using this tree, calculate the price of a 9- month European call option with an at-the- money strike. (2)

c) By considering each possible path in the tree, evaluate the price of a 9- month European lookback call option, where the lookback period includes time 0. Note that lookback call pays the difference between the minimum value and the final value of any asset price. (5)

[13]

Q. 6) **i)** State the Martingale representation theorem (2)

ii) Using 5- Step approach prove that the value of the derivative payment at $t<T$ is $V_t = e^{-r(T-t)} E_Q[X / Ft]$ (5)

iii) What is the main disadvantage of martingale approach as compared to PDE approach? (2)

[9]

Q. 7) A company ABC issued zero coupon bonds payable in 4 years' time with a nominal face value of INR 120 cr. The current gross value of the company is INR 180 cr.

i) Give expressions for the value of the debt in four years' time and today. (3)

The continuously compounded risk free interest rate is 4% p.a. and the continuously compounded credit spread on the bond is 4% p.a.

ii) Calculate the price of the bond today. (1)

iii) Estimate to the nearest 1% the implied volatility of the value of debt while adopting a Black Scholes model for the value of ABC plc (3)

iv) Determine the implied risk-neutral probability of default by the company. (3)

An investor is faced with a choice of 3 types:

Type	Spread in basis points
Investment	120
Junk	190
Default	

v) Estimate the risk-neutral probabilities of the Investment and Junk types of corporate bonds when one year risk free zero coupon rate is 5% (3)

vi) Construct the 3x3 ratings transition matrix assuming the investment grade has 90% chance of retaining the same rating over the year whereas the junk bond has 80% chance for the same situation (2)

[15]

Q. 8) Suppose X_t is a stochastic process given by

$X_t = e^{at}x + b\int_0^t e^{a(t-s)}dW_s$ where a, b, x ε R

i) Show that the SDE of the above process is given by

$dX_t = aX_t dt + bdW_{t,}$ $X_0 = x$

Note that under regularity conditions, for a twice differentiable scalar function $f(s,t)$, Ito's Lemma implies that if

$Z_t = \int_0^t f(s,t)dW_s$ then $dZ_t = dt\int_0^t \frac{\partial}{\partial t} f(s,t)dW_s + f(t,t)\,dW_t$ (4)

ii) Under what condition does X_t follow Vasicek model?

Does it have a mean reversion? If yes, to what value?

Under conditions of X_t following Vasicek, show that the long term mean and variance are 0 and $-\frac{b^2}{2a}$ respectively

(6)

iii) Show that $E(X_t|X_{t1}) = k\,X_{t1}$ for all t1 < t , where k is a constant.
Can X_t be a martingale? (4)

iv) Explain if X_t is a Markov process. (3)

[17]

****