INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

13th September 2017

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) In a horse racing game (numbered between 1 to 10), bet can be placed on the below three strategies.

a. Prize of 10 times the number
b. Prize of 15 times if the number is prime else 5 times the number
c. Prize of 18 times if the number is square number else 2 times the number

On a rainy day, probability of winning a even numbered horse is twice than the odd numbered horse and on other day, probability of winning a odd numbered horse is 0.75.

i) Deduce the payoff matrix. (2)

ii) If probability of rain is 0.25 then find out the optimal strategy under bayes criterion. (5)
[7]

Q. 2) Claims from a portfolio have a lognormal distribution with mean 900 and variance 111300. An excess of loss reinsurance is in place with a retention limit of 1000. Inflation is constant at a rate of 10% per year.

i) Find the expected net claims in year 1 and year 2 paid by the reinsurer. (8)

ii) Find out the year in which the probability that a claim arising will involve the reinsurer is 80%, assuming inflation to be 10% p.a. (4)
[12]

Q. 3) An Insurance company has launched new pet insurance plan. As the risk is relatively new for the company, they have entered into an reinsurance agreement where reinsurer will pay the initial claim amount upto the limit of K and 60% of the amount in excess of K for claims greater than K. The premium loading factor used by the insurer and reinsurer are 34% and 40% respectively. Individual claim amounts follow exponential distribution with parameter 0.02 and number of claims follow poisson distribution with parameter 20.

i) Find out K for which the insurer makes profit. (7)

ii) If K = 40, then find out the adjustment coefficient for the insurer (5 significant digit). (6)

iii) If individual claim amount in next year has decreased by 30%, then calculate the net decrease in claim amount for the insurer under part (ii) and comment. (5)

iv) If initial surplus is 150, then determine $\psi_1(150,1)$ under (ii) and (iii) (assume variance is proportionately distributed between insurer and reinsurer based on the expected claim amount). (4)
[22]

Q. 4) In a country there are 4 reinsurers of motor claims portfolios. An Actuary needs to analyse the amounts paid by each of the reinsurers using the models of Empirical Bayes Credibility Theory. The actuary obtains following information about number of policies reinsured and the claim amounts paid by the reinsurers for each of the next three years.

	Year 1	Year 2	Year 3
Reinsurer A(Claim Amount)	20m	22m	18m
Reinsurer A (No. Of reinsured policies)	1000	800	1100
Reinsurer B (Claim Amount)	50m	78m	72m
Reinsurer B (No. Of reinsured policies)	2000	2100	2150
Reinsurer C (Claim Amount)	10m	12m	7m
Reinsurer C (No. Of reinsured policies)	400	500	400

i) Analyse the data using EBCT model 2 and calculate the expected payout amount for reinsurer A and B in the coming year assuming the volumes as 1200 and 2200 respectively. You are given following values working in 1000's. (8)

J	$\sum_{j=1}^{3} P\,ij = \bar{P\iota}$	$\sum Pij(Xij - \bar{X}i)^\wedge 2$	$\sum Pij(Xij - \bar{X})^\wedge 2$
1	2900	59203.97	114844.4
2	6250	158400.1	454959.7
3	1300	13602.78	23789.67
Total	10450	231206.8	593593.8

ii) Explain how the value of the credibility factor (Z) depends on the data, on Pj's and on var[m(θ)]. (4)

[12]

Q. 5) In a marine insurance, claims follow gamma distribution and depends on various factors as mentioned below:

a. Value of the ship - VS(x)
b. Average age of the ship workers – AA (y)
c. Type of sea – ST (Si, i = 1 to 12)
d. Number of floors – FL (Fi, i = 1 to 10)
e. Age of ship – AS (Ai, i = 1 to 15)

Various models considered are summarized in the below table.

Model	**Linear Predictor**	**Number of parameters**	**Scaled Deviance**
VS	a + bx	2	500
Log(AA) + AA^2			620
VS + ST			475
AA + exp(AS)			540
VS * FL + ST			420
AA + AS + AA*AS			450
VS * ST * FL			350

i) Derive the missing items in the above table. (5)

ii) Determine the optimum model based on the above data. (5)

iii) Comment before deciding the final model to be used. (2)

[12]

Q. 6) **i)** Describe purpose of practical time series analysis. (2)

ii) Define univariate time series, invertibility and markov property. (3)

iii) In a Crop Insurance company, the halfyearly productivity for last 10 years are available and follows the below time series model.
$S_t = \alpha + \beta t + h_t + X_t$, where X_t is zero mean stationary process.
Suggest two methods each to remove the seasonal variation (h_t) and any linear trend. (2)

iv) After removing the trends, the revised model is given by below equation.
$Y_t - 0.6Y_{t-1} = 0.5 + 0.2e_t + 0.7\,e_{t-1} + 0.3\,e_{t-2}$, where e_t is standard zero mean white noise process.

a) Express the above model in moving average form (3)

b) Derive ACF of the above model. (4)

[14]

Q. 7) Size of claims in a general insurance company follows weibull (c, γ) distribution and the parameter c is unknown and follows exponential distribution with parameter λ. Find the Posterior distribution for a sample of size n. **[3]**

Q. 8) X_t follows a time series model defined as below:

$X_t = 4+(2/5)X_{t-1}+e_t - (1/4)\,e_{t-1} +(1/5)\,e_{t-2}$, where e_t follows $N(0,\sigma^2)$

i) Show that the process is weakly stationary (2)

ii) Calculate the auto correlation function for lags 0, 1 and 2. (6)

iii) Explain why the process does not satisfy Markov property and state how the series can be expressed as a process which satisfies the Markov property. (4)

[12]

Q.9) **i)** Describe the similarities and differences between Normal/Normal model and EBCT Model 1. (4)

ii) State the assumptions under EBCT Model 2. (2)

[6]

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