INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

12th September 2017

Subject CT3 – Probability & Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) The table below shows the maximum monthly temperatures (in$^\circ$ Celsius) recorded in a year for two cities, *A* and *B*.

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
City A	21	26	26	29	31	33	35	32	30	26	21	20
City B	18	25	25	32	37	68	42	32	29	25	20	16

i) Draw boxplot diagrams for the maximum monthly temperatures of these cities. (4)

ii) Use the boxplot diagrams to compare and contrast the two data sets. (3)
[7]

Q. 2) Consider the following probability density function:

$$f(x) = \frac{k}{(4+x)^2} \; ; x > 0$$

i) Determine the value of *k*. (1)

ii) Simulate three observations from the above distribution. Use the random numbers 0.914, 0.683 and 0.257 selected from *U* (0, 1) for the simulation. (4)
[5]

Q. 3) A random variable X has Poisson distribution with mean 1. Using the infinite series expansion for e:

i) Show that the probability it takes positive even values is $(\frac{1+e^{-2}}{2})$ - e^{-1}. (4)

ii) Hence, find the probability that it takes positive odd values. (2)
[6]

Q. 4) The random variable *X* has a lognormal distribution with the same mean and variance as that of the χ_9^2 distribution.

Calculate $P(X > 9)$. **[5]**

Q. 5) In a bakery, the time taken to prepare an exotic cake is normally distributed with mean 2 hours and standard deviation 15 minutes.

Calculate the probability that the time taken for two randomly selected exotic cakes differs by no more than 25 minutes. **[4]**

Q. 6) A random sample $X_1, X_2, \dots X_9$ is drawn from *N* (8, 3^2) distribution.

Calculate *P* [$\bar{X} > 6.75$ and $S < 3.75$] where $\bar{X}$ and S are the mean and standard deviation of the sample respectively. **[4]**

Q. 7) The random variable *X* has a Normal distribution with mean μ and variance 400. In a test of H_0: μ = 100 against H_1: μ = 200, it is decided to reject H_0 if the sample mean exceeds 110.

Determine the smallest sample size required to ensure that the probability of Type-I error is less than 0.01. **[4]**

Q. 8) A random variable X has a gamma distribution with parameters α (> 1) and λ.

i) Derive an expression for the mode of X in terms of α and λ. (4)

Given that α = 40 and λ = 0.1, calculate the value of $P(X > 500)$ using:

ii) The gamma- chi square relationship (2)

iii) The Central Limit Theorem (3)

[9]

Q. 9) A random variable X has a Pareto distribution with parameters α = 3 and λ = 4 and Y is a random variable such that :

$$E\ (Y \mid X\ =\ x)\ =\ 2x\ +\ 5 \text{ and } Var\ (Y \mid X\ =\ x)\ =\ x^2 +\ 3$$

Calculate the unconditional standard deviation of Y. **[5]**

Q. 10) Let $X_1, X_2, \ldots X_n$ be a random sample from a population with *pdf*

$$f(x/\theta) =\ \theta x^{\theta-1};\ 0 < x < 1, \theta > 0.$$

i) Find the moment estimator for θ. (2)

ii) Find the maximum likelihood estimator for θ. (4)

iii) Obtain the Cramer Rao Lower Bound (CRLB) for the variance of an unbiased estimator for θ. (2)

iv) Calculate from the following data:

0.32, 0.19, 0.25, 0.34,0.42

a) Moment estimate for θ (1)

b) Maximum likelihood estimate for θ and (2)

c) Maximum likelihood estimate of the population mean. (1)

[12]

Q. 11) The table below shows that the time interval (*in sec.*) between successive red cars in a free flowing traffic.

Time (in sec.)	0-20	20-40	40-60	60-90	90-120
Frequency	50	30	20	10	5

Perform a suitable statistical test whether these times can be modeled as an exponential distribution. **[9]**

Q. 12) A dietician has prepared a programme for Mr. *A* covering routine exercise and level of food intake so that he can reduce his weight. After following her instructions, he has started taking monthly readings of his weight for 10 months. His target weight is 55 *kg*.

The following table gives the recorded monthly weight.

Month (x) :	1	2	3	4	5	6	7	8	9	10
Weight in kg (y):	80	78	79	74	72	73	70	68	70	65

$\sum x = 55$ $\sum y = 729$ $\sum x^2 = 385$ $\sum y^2 = 53{,}363$ $\sum xy = 3881$

Consider the linear regression model $y = \alpha + \beta x + \epsilon$.

i) Calculate the least squares fitted regression line (3)

ii) Draw a scatter plot of y against x and comment on the validity of the model suggested. (3)

iii) Calculate an estimate of the variance of the error term. (1)

iv) Carry out a suitable test for $H_0 : \beta = 0$ against $H_1 : \beta < 0$. (4)

v) Construct a 95% confidence interval for the predicted recorded weight on month 20. (4)

vi) Construct a 95% confidence interval for his mean weight on month 20 and compare the results obtained here to the confidence interval obtained in part (v). (4)

[19]

Q. 13) The following data are collected for comparing the average yield per *hectare* of three varieties of wheat :

Variety 1: 20, 25, 31, 33, 26

Variety 2: 26, 34, 33, 37, 30

Variety 3: 19, 25, 23, 20, 23

Consider the model:

$$y_{ij} = \mu + \tau_i + e_{ij}\ , i = 1,2,3;\ j = 1,2,3,4,5\ ;\ e_{ij} \sim N(0, \sigma^2).$$

where y_{ij} is the yield per *hectare* from the variety i and replicate j.

i) Calculate the least square estimate of μ and τ_i , $i = 1,2,3$. (4)

ii) Perform an analysis of variance stating the assumption, hypothesis and your conclusion. (7)

[11]

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