INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

19th September 2017

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Mr Sam wishes to purchase a level annuity of INR 1,20,000 per annum payable quarterly in arrear for five years. Calculate purchase price of the annuity using

    i)   12% per annum effective rate of interest   (2)

    ii)   Interest rate of 12% per annum convertible half yearly   (2)

    iii)   Interest rate of 12% per annum convertible quarterly   (2)
                                   [6]

Q. 2) Compare and contrast ordinary shares and preference shares.   **[3]**

Q. 3) The force of interest at time t (where t is measured in years) is given by:
$\delta (t) = 0.07$ for $0 \leq t < 4$
$\delta (t) = 0.06$ for $4 \leq t < 8$
$\delta (t) = 0.05$ for $8 \leq t < 20$

    i)   Derive expressions for v(t), the present value of 1 due at time t.   (5)

    ii)   Calculate the accumulated value at time 15 of an investment of INR 5,000 made at time 3.   (2)

    iii)   What constant force of interest would produce the same accumulation as in (ii) for an investment of INR 5,000 over a period of 12 years?   (2)

    iv)   Calculate the effective annual rate of interest that would have the same effect as the varying force of interest given above, over a period of 20 years.   (2)

    v)   Calculate the present value at time 0 of an annuity of INR 1,000 per annum payable annually in advance for 10 years.   (4)
                                     [15]

Q. 4) Ravi wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of INR 12,000 per annum. The payments increase at each anniversary by 3% per annum. Ravi would like to buy the annuity with a single premium 10 years from now.

    i)   Calculate the single premium required in 10 years' time to purchase the annuity assuming an interest rate of 6% per annum effective.   (5)

Ravi wishes to invest a lump sum immediately in an investment product such that, over the next 10 years, it will have accumulated to the premium calculated in (i). The annual effective returns from the investment product are independent and $(1+i_t)$ is lognormally distributed, where i_t is the return in the tth year. The expected annual effective rate of return is 6% and the standard deviation of annual returns is 15%.

    ii)   Calculate the lump sum which the individual should invest immediately in order to have a probability of 0.98 that the proceeds will be sufficient to purchase the annuity in 10 years' time.   (7)

    iii)   Comment on your answer to (ii).   (2)

[14]

Q. 5) An insurance company has liabilities consisting of eleven annual payments of INR 1,00,000 with the first payment due to be made in 10 years' time and the last payment due to be made in 20 years' time. The rate of interest is 6% per annum effective.

    i) Show that the discounted mean term of these liabilities, is 14.42 years. (3)

The insurance company holds two zero-coupon bonds, one paying INR X in 10 years' time and the other paying INR Y in 20 years' time.

    ii) Find values of X and Y such that Redington's first two conditions for immunisation from small changes in the rate of interest are satisfied. (5)

    iii) Explain, without making any further calculations, whether you would expect Redington's third condition for immunisation to be satisfied for the values of X and Y calculated in (ii). (2)

[10]

Q. 6) i) Explain what is meant by the expectations theory for the shape of the yield curve. (2)

    ii) Short-term, one-year annual effective interest rates are currently 8%; they are expected to be 7% in one year time, 6% in two year time and 5% in three year time.

        a) Calculate the gross redemption yields of 1-year, 2-year, 3-year and 4-year zero coupon bonds assuming the expectations theory explanation of the yield curve holds. (4)

        b) The price of a coupon paying bond is calculated by discounting individual payments from the bond at the zero-coupon bond yields in (a).
Calculate the gross redemption yield of a bond that is redeemed at par in exactly four years and pays a coupon of 5 per annum annually in arrear. (4)

    iii) A two-year forward contract has just been issued on a share with a price of 400. A dividend of 4 is expected in exactly one year. Using the rates given in (ii), calculate the forward price using the above spot rates of interest, assuming no arbitrage. (2)

[12]

Q. 7) A computer manufacturer is to develop a new chip to be produced from 1st January, 2018 until 31st December, 2030. Development began on 1st January 2016. The cost of development comprises of INR 9,00,000 payable on 1st January 2016 and INR 12,00,000 payable continuously during the calendar year 2017.

From 1st January, 2018 the chip will be ready for production and it is assumed that income will be received half yearly in arrear at a rate of INR 5,00,000 per annum.

    i) Calculate the discounted payback period at an effective rate of interest of 9% p.a. (6)

    ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, less than or equal to that given in part (i) if the effective interest rate were substantially greater than 9% per annum. (2)

[8]

Q. 8) A fixed interest security that currently has a remaining term of 15 years pays coupons of 8% per annum half-yearly in arrear and is redeemable at par. An investor, not subject to tax, negotiates a forward contract in which he agrees to buy INR 100,000 nominal of the security in six years' time. The gross redemption yield of the fixed interest security is currently 7% per annum and the risk-free force of interest is assumed to be 6% per annum over next 10 years.

Calculate the forward price of the contract.   **[5]**

Q. 9) At time $t = 0$ an investor purchased a three year annuity-certain contract which paid her INR 10,000 per annum annually in arrear for three years. The purchase price paid by the investor was INR 25,000.

The value of the retail price index at various times was as shown in the table below:

Time (t) years	t=0	t=1	t=2	t=3
Retail price Index	170.7	183.3	191.0	200.9

Calculate, to the nearest 0.1%, the following effective rates of return per annum achieved by the investor from her investment in the annuity:

i)   The real rate of return; and   (3)

ii) The money rate of return   (2)

                                                                                            [5]

Q. 10) On the planet of Actuaria, the forest department is considering of venturing into a project. They are considering of purchasing a small land which may be used for sheep rearing, goat breeding or for forestry. The details of the project and the estimated cash-flows associated with the project are:

Sheep rearing:
The Initial cost is $ 20,000 and the annual income from the project is $ 1,100 payable annually in arrears for twenty years. The sale price of the land after twenty years is $ 20,000.

Goat breeding:
The Initial cost is $ 20,000 and the annual income from the project is $ 900 payable annually in arrears for twenty years. The sale price of the land after twenty years is $ 25,000.

Forestry:
The planting cost is $ 20,000 and sale of trees after twenty years is $ 57,300

i)   Calculate the internal rate of return (IRR) from each of these projects (to the nearest 0.1%).   (6)

The forest department does not have enough capital for any of these ventures and is therefore considering of borrowing a capital of $ 20,000 at 5% per annum payable annually in arrear from the Bank of Actuaria. The loan is repayable in twenty years' time, there being no early repayment option. If the forest department requires any further loans they will be granted by the bank on the same interest basis and will be repayable by the same date as the original loan. If the forest department has any money after paying bank interest, they may invest it to secure at 4% per annum effective.

ii)   Which of the three projects will give the highest profit in twenty years' time?   (7)
                                                                                                        [13]

Q. 11)   Mr Raj has taken out three fixed interest loans to finance his start up. The borrowed capital will be paid by making regular annual payments over the term of the loan. The table below shows the date of each loan, the dates of first and last payments, the amount borrowed and the interest rate on which the repayment schedule was based.

Loan	Date of Loan	Date of First payment	Date of Last Payment	Amount borrowed (INR)	Interest rate
1	1st Jan 2012	31st Dec 2012	31st Dec 2021	6,00,000	10% per annum effective
2	1st July 2014	31st December 2014	31st December 2018	2,50,000	9% per annum convertible half yearly
3	1st July 2016	30th June 2017	30th June 2021	1,50,000	9.5% per annum effective

On 1st January, 2017 Mr Raj wishes to consolidate these loans by repaying the amount of capital outstanding under each loan and taking out a new loan for this amount which will be repayable by ten equal annual payments starting on 31st December, 2017. If the repayments under the new loan are calculated using an effective annual interest rate of 8% per annum, determine the amounts of the repayments required under the new loan. (Ignore expenses)   **[9]**

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