INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

19th November 2019

Subject CS2B – Risk Modelling and Survival Analysis (Paper B)

Time allowed: 1 Hour 45 Minutes (14.45 – 16.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Mark allocations are shown in brackets.*

2. *Attempt all questions, beginning your answer to each question on a new page.*

3. *Attempt all sub-parts of the question in one document only, unless otherwise instructed to do so.*

4. *Ensure to copy and paste R codes and output at regular intervals onto the MS word file.*

5. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

6. *Ensure that you click the "Submit" button only when you have completed the question paper and final submission has to be made.*

AT THE END OF THE EXAMINATION

Please return this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you. You are requested to save and submit the work before leaving the examination premises.

Paste your R Code, Outputs and plots/ graphs in your answer sheet.

Q. 1) Monthly totals of airline passengers are given in time series data 'AirPassengers'.

    i) Plot the data as time series with frequency =12. (2)

    ii) Plot the acf function. (2)

    iii) Decompose the trend, seasonality and randomness for the data.

        (Hint: use decompose function.) (3)

    iv) Fit Arima model.

        (Hint: order=c(1,0,0), list(order=c(2,1,0), period=12 as parameters.) (3)

    v) Predict the data for next 24 months. (2)

    vi) Find upper and lower bound of fitted data and plot the same along with predicted data with legends. (8)

                                                                        [20]

Q. 2) **i)** Draw the graph of hazard function for the gamma distribution with parameters $\alpha = 0.75$ and $\lambda = 0.5$. (3)

    ii) Draw the graph of hazard function for the gamma distribution with parameters $\alpha = 1$ and $\lambda = 0.5$. (1)

    iii) Draw the graph of hazard function for the gamma distribution with parameters $\alpha = 1.5$ and $\lambda = 0.5$. (1)

    iv) Comment on the thickness of the tails of the distribution of (ii) with the tails of (i) and (iii). (2)

        (For the graphs, the value of x varies from 0 to 10).

                                                                    [7]

Q. 3) Mortality of a group of lives follows Gompertz's law. The value of the parameters is known as c = 1.128 & B = 7.29 x 10^{-6}.

Use the approximation $q_x = 1 - e^{-\mu(x+\frac{1}{2})}$, calculate an approximate of the curtate value for age 55 or e55. Assuming 100 is the maximum age. **[7]**

Q. 4) An life insurance company is selling Endowment, Pure Term and Unit Linked products wherein 80% of the business comes from Endowment, 5% from Term Insurance and 15% from Unit Linked products. The sum assured ranges between 30000 to 200000 with multiple of 5000. The sum assured are normally distributed with below parameters - Endowment (mean 50000, standard deviation 10000), Pure Term (mean 100000, standard deviation 25000), Unit Linked (mean 60000, standard deviation 5000). One policy is randomly picked from the system and the sum assured of that policy is 75000.

    i) Create a sequence ranging from 30000 to 200000 with an increase of 5000 and plot the density functions of the above three distributions in a single graph. (5)

    ii) Find the probability that the policy (75000 sum assured) comes from the Endowment product portfolio and the probability that the policy comes from Unit Linked Product portfolio. (7)

[12]

Q. 5) Signals are mathematically represented by a function that takes into account the magnitude and frequency.

$x_t = A \cos(2\pi\omega t + \phi) + w_t$, where ω is equal to 1/50 and w_t is the white noise with mean zero and SD of 5. A and ϕ need to be estimated.

In this case the parameters appear in a nonlinear way, so we use a trigonometric identity and write
$A \cos(2\pi\omega t + \phi) = A \cos(\phi) \cos(2\pi\omega t) - A \sin(\phi) \sin(2\pi\omega t)$,

$$= \beta_1 \cos(2\pi\omega t) + \beta_2 \sin(2\pi\omega t),$$

$$= \beta_1 * z1 + \beta_2 * z2$$

If we can estimate β_1 and β_2, then A and ϕ can be estimated.

Therefore, $x_t = \beta_1 \cos(2\pi\omega t) + \beta_2 \sin(2\pi\omega t) + w_t$

    i) Simulate and plot a series x_t, using A=2, $\phi = 0.6\pi$ and SD of white noise as 5 (for t= 1 to 1000). (3)

    ii) Generate two series z1 and z2 (for t= 1 to 1000). (3)

    iii) Fit a Linear model to between x_t, z1 and z2 and estimate parameters β1, β2 , A and ϕ. (7)

    iv) Plot x, fitted line. (3)

[16]

Q. 6) Suppose the transition probabilities are a function of the age of a person. The transition probability of a person aged x moving:

- from Healthy to Sick state is 0.007x
- from Healthy to Death state is 0.001x
- from Sick to Death state is 0.002(100-x)
- from Sick to Healthy is 0.006(100-x).

Assuming 100 is the maximum age. Calculate the probability of:

    i) Healthy person aged 30 will be in Sick state after 4 years. (9)

ii) Sick person aged 25 will be in Death state after 7 years. (3)

[12]

Q. 7) An extract from the data for 15 patients monitored in hospital after every 15 minutes post giving them a dosage of life saving drug is shown in the table below.

Person	Last time at which patient was observed (time unit in 15 Minutes)	Outcome
1	6 (i.e. 90 minutes)	Died
2	6	Discharged
3	12	Died
4	18	Discharged
5	27	Died
6	27	Discharged
7	30	Discharged
8	36	Discharged
9	39	Died
10	39	Discharged
11	54	Discharged
12	57	Died
13	60	Survived
14	60	Survived
15	60	Survived

i) Use the data to create the Kaplan-Meier estimate of the survival function. Use the "Survival" library in R to create the summary of the model (survival rates, standard deviation, lower 95% confidence interval, upper 95% confidence interval). (8)

ii) Use the results of the summary created in (i) and input the values of tj, dj and nj in the model to calculate the estimated integrated hazard and hence the Nelson - Aalen estimate of the survival model. (5)

iii) Create a table to compare the above two analysis. (3)

[16]

Q. 8) Consider a set of data generated by tossing a fair coin, letting $x_t = 1$ when a head is obtained and $x_t = -1$ when a tail is obtained. Then, construct y_t as

$y_t = 2 + 1.5x_t - 0.5x_{t-1}$

Simulate data for three cases, one with a small sample size (n = 10), another one with a moderate sample size (n = 100) and lastly with large sample size (n = 1000).

i) Plot the series for all three cases. (8)

ii) Find the mean and variance and comment on results. (2)

[10]

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