INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st November 2019

Subject CS2A – Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 15 Minutes (14.45 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) **i)** State the advantages and disadvantages of parametric and non-parametric survival analysis methods. (2)

ii) State the name of most commonly used models / distributions in parametric, non-parametric and semi-parametric approaches of survival model analysis. (2)

[4]

Q. 2) A research firm in India is analyzing the success rate of movies in India. It has collected the data of ten randomly selected movies which were screened during 2018 and tabled the result as follows:

Sr No	**Actors**	**Director**	**Budget**	**Result**
1	Popular	Famous	Big	Hit
2	Popular	Amateur	Small	Hit
3	New	Amateur	Small	Hit
4	Popular	Famous	Big	Flop
5	New	Famous	Small	Flop
6	New	Famous	Big	Hit
7	Popular	Amateur	Small	Hit
8	Popular	Famous	Big	Flop
9	New	Famous	Small	Hit
10	Popular	Amateur	Small	Flop

i) Explain what is Gini Index. (2)

ii) Draw Binary Tree with Director and Result and calculate the Gini Index for Director. (4)

iii) If Gini Index for Budget and Actor are 0.4664 and 0.45 respectively then suggest which attribute should be used as a first root in the Decision Trees. (2)

[8]

Q. 3) The life time of a new mobile phone variety follows an exponential distribution with $\lambda = 0.125$. An actuarial student has purchased one such phone in online exactly two years from the date of manufacturing.

i) Calculate the Mean Residual Life of the phone the student has purchased two years ago by deriving the formula. (3)

ii) Using result of (i), comment on the suitability of Exponential Distribution for modelling Mean Residual Life. (2)

[5]

Q. 4) Consider the following time series

$Y_t = \mu + (\alpha+\beta)\, Y_{t-1} - \alpha\beta Y_{t-2} + e_t;$

Where $1 > \mu > (\alpha+\beta) > -\alpha\beta > 0$ and $1 > |\alpha|, |\beta| > 0$ and e_t is a white noise process with variance σ^2.

i) Identify the model as an ARIMA(p,d,q) process. (1)

ii) Determine whether Y_t is a stationary process. (2)

iii) Calculate $E(Y_t)$. (2)

iv) Calculate the auto-correlations ρ1, ρ2 and ρ3. (4)
[9]

Q. 5) A life insurance company wants to perform experience analysis of its long-term sickness policies using a three-state Markov model in continuous time. The results of these experiments are used to update the premiums that the company should be charging to the policyholders. The states are healthy (H), ill (I) and dead (D). The forces of transition in the model are $P_{HI} = \sigma$, $P_{IH} = \rho$, $P_{HD} = \mu$, ID = v and they are assumed to be constant over time.

The details of a group of policyholders observed over a 1-year period are given below:

12 deaths from State H and total time spent in State H is 2608 years;
20 deaths from State I and total time spent in State I is 176 years;
92 transitions from State H to I;
60 transitions from State I to H;

i) Derive the maximum likelihood estimate of σ. (5)

ii) Estimate the standard deviation of $\hat{\sigma}$, the maximum likelihood estimator of σ. (2)
[7]

Q. 6) **i)** The following is an extract from a term insurance policyholder's data of an insurance company.

Life	**A**	**B**
Date of Birth	31-05-1989	15-04-1989
Date of Entry	01-08-2009	01-02-2014
Date of Exit	25-12-2019	31-03-2019
Reason for Exit	Death	Withdrawal

Calculate the contributions made by each life (in days) to the central exposed to risk during the calendar year 2019. The contributions should be grouped by "Age last birthday on previous policy anniversary" and curtate duration.

Note: Date of entry should be included but not Date of exit. (4)

ii) The Pricing Actuary of a Life Insurance Company wanted to project the future mortality of the future annuitants. The base mortality table used for the projection was LIC94-96 and the base year was 1996 calendar year. The mortality rate of the person aged 62 for the base year has been estimated to be 0.005916. Following are the assumptions made for projecting mortality rate for life aged 62 in 15 years time:

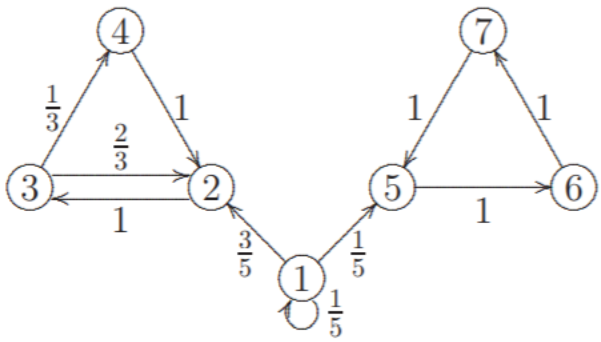
a) Minimum possible mortality rate for lives aged 62 would be 0.0010.

b) 55% of the maximum possible reduction in mortality at this age will have occurred by 8 years time.

Specify the formula for appropriate reduction factor and calculate the projected mortality rate for lives aged 62 in 15 years time. (5)

[9]

Q. 7)



i) Calculate the expected time taken to reach state 4, starting from state 2. (3)

ii) Let $X_0 = (\frac{3}{4}, 0, \frac{1}{4}, 0,0,0,0)$. Calculate the probability of the trajectory 1, 2, 3, 2, 3, 4. (2)

[5]

Q. 8) **i)** Define the term Concordance related to random variables and give two examples of measures of concordance. (2)

ii) Calculate the Spearman's rho for the following data set:

(€70, £75), (€73, £73), (€78, £71), (€74, £74), (€72, £78), (€76, £77), (€68, £86), (€60, £85), (€65, £87), (€69, £83) (4)

[6]

Q. 9) In a democratic republic of Actuaria, getting a driving license is a highly scrutinised process. It is renewed every year (t =T) after undertaking pre-specified driving and eligibility tests. There is little tolerance towards Drunk and Drive in Actuaria and if such cases are caught, the driving license is suspended along with heavy penalty. Following the incurrence of first claim, the driving license can be reinstated to its original state only through court orders. If driver is caught drunk and drive again the driving license is permanently cancelled.

The transition rate for hazard of being caught in drunk and drive case is a constant 0.1. The courts are very strict and only allows reinstatement of suspended driving licenses in few cases only, the transition rate for moving from suspended state to non-suspended (In-force) state is 0.05.

i) Explain whether a time homogeneous or time inhomogeneous model would be more appropriate for modelling this situation. (2)

ii) Explain why a model with state space {In-force, Suspended, Lapsed} does not possess the Markov property. And Suggest, giving reasons, additional state(s) system would possess to follow Markov property. (3)

iii) Draw the transition diagram with additional states. (2)

iv) Derive the probability that a driving license remained in covered state continuously from time 0 to time t (t> 0 but t < T). (2)

v) Derive the probability that a newly issued driving license at t=0 is in the Suspended state at time t (t > 0 but t < T). (4)

[13]

Q. 10) Claims on a motor insurance policy follows a Pareto distribution with $\alpha = 5$ and $\lambda = 10000$. The insurer effects an individual excess of loss reinsurance treaty with a retention limit of 4000.

i) Calculate the probability that a claim involves reinsurer. (2)

ii) Calculate the insurer's expected payment per claim. (4)

Next year, the claim amounts on these policies are expected to increase by 20%.

iii) Calculate the probability that a claim now involves the reinsurer assuming that no change in reinsurance treaty. (3)

[9]

Q. 11) Five white balls and five black balls are distributed in two urns in such a way that each urn contains five balls. At each step we draw one ball from each urn and exchange them. Let X_n be the number of white balls in the left urn at time n.

i) Compute the transition probability for Xn. (3)

ii) Find the stationary distribution and show that it corresponds to picking five balls at random to be in the left urn. (5)

iii) Identify the statistical distribution followed this chain. (1)

[9]

Q. 12) Consider the ARCH(1) process:

$$X_t = \mu + e^t\sqrt{\alpha + \beta(X_{t-1} - \mu)^2}$$

Where e^t are independent normal random variables with variance 1 and mean 0.

Show that, for s = 1, 2, … , t-1, X_t and X_{t-s} are:

i) Uncorrelated (5)

ii) Not independent (3)

[8]

Q. 13) A life insurance company has conducted the mortality investigation of its large portfolio of term insurance contracts. A graduation of the mortality experience in the age group 25 to 30 had been carried out and the following is the extract from the results:

Age	**Initial Exposed to Risk**	**Actual No. of deaths**	**Graduated Mortality Rate**
x	E_x	θ_x	q_x
25	1800	30	0.020
26	3400	70	0.020
27	2600	73	0.030
28	5400	210	0.040
29	900	37	0.040
30	4200	190	0.045

The observd value of the serial correlation coefficient (with lag one) between the ten pairs of values of the standardized deviation is 0.241.

Perform any four appropriate tests to determine whether or not graduation is satisfactory. **[8]**

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