

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

16th March 2022

Subject SP6 – Financial Derivatives

Time allowed: 3 Hours 30 Minutes (14.30 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
- 3. Mark allocations are shown in brackets.*

- Q. 1)** Let S_t denote the price of non – dividend paying asset at time t , satisfying the stochastic equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where the drift, μ , and the volatility, σ , are both assumed constant and W_t is the Wiener process. Let C_t be the price of a derivative based on this asset satisfying

$$dC_t = \mu_t^C C_t dt + \sigma_t^C C_t dW_t.$$

Based on the same, answer the following questions:

- i)** If at time t a trader is long ϕ unit of the asset, and short one unit of the derivative, what is the value of trader position? Show that the position is risk free only if

$$\phi_t = \frac{\sigma_t^C C_t}{\sigma S_t}. \quad (3)$$

- ii)** Assuming that the short – term interest rate is a constant r , show that the principle of no arbitrage implies that

$$\frac{\mu_t^C - r}{\sigma_t^C} = \frac{\mu - r}{\sigma}.$$

Comment on the financial significance of this relation. (3)

- iii)** Now, assume that the derivative price can be expressed in the form $C_t = C(S,t)$ where the function $C(S,t)$ has a continuous second derivative. Use Ito lemma to show that

$$\begin{aligned} C_t \mu_t^C &= \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} \\ \text{and } C_t \sigma_t^C &= \sigma S \frac{\partial C}{\partial S}. \end{aligned} \quad (4)$$

- iv)** Show that the no arbitrage condition implied that $C(S,t)$ satisfies

$$\frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = r \left(C - S \frac{\partial C}{\partial S} \right).$$

What is the implication of the absence of μ in this relation? (4)

[14]

- Q. 2)** Answer the following with respect to options on futures:

- i) Describe briefly what is delivered upon the exercise of exchange traded put and call options on futures contracts. (2)
- ii) How can you modify the Black Scholes formula to calculate the value of options on futures? (1)
- iii) Calculate the price of a 7-month European put option on a futures contract when the futures price is Rs. 57,000, the risk-free rate of interest is 12% per annum, the strike price is Rs. 63,000 and the volatility of the futures price is 20% per annum. (3)
- [6]

Q. 3) Let the following hold for an asset: Share Price, $S = \$50$, Strike Price, $X = \$50$, Time to Maturity, $T = 65$ days. Also, let volatility be 25% and the risk – free interest rate be 6%.

- i) Calculate the value of the European Call option using Black Scholes formula. Also, calculate the Delta and Gamma for the option. (4)
- ii) Describe two major disadvantages of delta hedging. (3)
- iii) The writer has sold 1000 calls and wants to have a delta neutral portfolio at the current time. To ensure delta neutrality, how many more shares does the writer need to buy and how much money would be needed by the writer for the same? (4)
- iv) Supposing the next day the share price increases to \$52 with the other terms remaining constant. How will the hedge portfolio need to be changed to ensure delta neutrality for the 1000 calls he has written? (4)
- v) On the day the option is written, the writer wants to be both Delta and Gamma neutral for the 1000 calls he has written. To do this, he finds that another option i.e. option 2 on the same is available in the market for 1.1466 and the delta and gamma for option 2 are 0.2965 and 0.0529 respectively.
- a) How many new options (option 2) and shares should the writer buy? How much money would this involve? (3)
- b) Comment on why the cash requirement is lower in this case as compared to the delta neutral case. (2)

[20]

Q. 4) Assume that performance of shares of LUH and G&P, are positively correlated with the performance of the Nifty 50. Also assume that they depend on no other stochastic variables. LUH has an expected return of 20% per annum and a volatility of 25% per annum, while G&P has a volatility of 35% per annum. Assume that the instantaneous risk – free rate of interest is 6% per annum.

- i) What is the market price of risk for Nifty 50? (3)

ii) What is the expected return from G&P? (2)

[5]

Q. 5) CFDH Bank has an exposure to \$100 million of unsecured debt issued by Dee Telefilms. CFDH Bank enters in to a CDS transaction with Vertex Bank to hedge its debt exposure to Dee Telefilms. Vertex Bank, in exchange for a premium, would fully compensate CFDH Bank if Dee Telefilms defaults. Assume that probabilities of default for CFDH Bank, Vertex Bank and Dee Telefilms are 0.3%, 0.5% and 3.6% respectively. Further, defaults of the two banks and the company are independent of each other. Given this information,

i) What is the probability that CFDH Bank will suffer a credit loss in its exposure to Dee Telefilms? (3)

ii) Assuming CFDH Bank suffers a credit loss, how much is the expected credit loss that it would be exposed to? (2)

[5]

Q. 6) A company is planning to sell an investment product which pays 6% annually on 1cr each year with first payment at end of year 11 and last payment at end of year 15 with no principal pay out. The company has decided to hedge the interest rate risk using derivatives.

i) Describe which derivative should be used to hedge the risk and what would be cashflow characteristic of that derivative. (3)

ii) Estimate the cost of hedge based on below information.

ZCB bond price at t= 0 for payment at end of year 11	0.55
ZCB bond price at t= 0 for payment at end of year 15	0.42
Sum of discount factors at t = 0 Year 11 to Year 15	2.43
Sum of discount factors at t=10 Year 11 to Year 15	4.28
Volatility of Forward Rate	25.0%
Risk Free Rate	5.3%

(5)

[8]

Q. 7) A new regulation requires estimation of expected credit default for the bond exposure for every bond and keep the capital requirement as sum of overall default allowance. A company holds 2 bonds, annual coupon 2-year maturity with following coupon, default and recovery rates.

Bond A		Bond B	
Maturity	2	Maturity	2
Coupon	5%	Coupon	6%
Interest	5%	GRY	6%
Default each year	5%	Default each year	10%
Recovery	40%	Recovery	25%

i) Estimate the capital requirement of the company. (5)

(5)

As an Investment Actuary of the company you argue to regulator that the above approach for estimating the capital requirement is conservative and we should consider correlation between bonds default rates also to estimate the capital requirements.

- ii) Explain the rationale for above argument and discuss possible issues with the above statement. (4)
 - iii) Assuming a correlation of 25% estimate the capital requirement for above 2 bonds exposure. (3)
- [12]**

Q. 8) Answer the following questions based on the information given:

- i) In an unexpectedly high inflation scenario who would benefit - bond holder or bond issuer? Why? (2)
- ii) Describe various approaches to hedge inflation risk and discuss possible downside. (3)
- iii) Trader purchased a bond at par which adjust bond coupon and principle as per ratio of payment year CPI and base year CPI. Using the information below calculate total return on investment at the end of 3 years. State the assumption in calculation.

Nominal	100	
Unadjusted coupon	3%	
CPI at date of issue	120.00	
Actual CPI		Real interest
1	130	3%
2	135	2%
3	150	1%

Nominal interest rate table			
t\term	1	2	3
0	4%	5%	6%
1	6%	6%	7%
2	7%	7%	8%
3	8%	8%	8%

Real interest rate table			
t\term	1	2	3
0	1%	2%	3%
1	2%	2%	3%
2	3%	3%	4%
3	4%	4%	4%

(5)
[10]

Q. 9) Answer the following questions pertaining to bonds.

- i) Value a 5-year bond with annual coupon of 6% annual in 3 interest rate scenarios of fixed interest rate, scenario A=5%, scenario B=6% and scenario C=7% (annually compounded). Identify in which interest rate scenario bond is at premium, at par and at discount and calculate the % premium and discount. (4)
- ii) Calculate running yields or current yield & Gross redemption yield on all the bonds. Explain the reason of variation. (4)
- iii) Estimate the value of the bond after 1 year in scenario C (7% interest rate). Explain the changes in value of the bond based on Gross Redemption yield and coupon payment. (4)

[12]

Q. 10) XYZ is a South Asian country with a deep and mature market in equities and its derivatives. Most of the listed companies in the country are majority owned by promoters and their family members. The country followed the policy of dividends declared by the companies as tax-free in the hands on the shareholders. The stock exchanges in the country did not make any explicit adjustment for dividends as far as the derivatives were concerned. The finance minister of the country has now announced that after the next three months dividends will be taxed in the hands of the recipient just like any other income.

- i) Describe in detail the impact of this announcement on the futures and options market of the country. (4)
- ii) The leading stock exchange in the country has reached out to you to suggest the proposed changes that needs to be made in the derivative segment. What recommendations would you make? (4)

[8]
