

**INSTITUTE OF ACTUARIES OF INDIA**

**EXAMINATIONS**

**22<sup>nd</sup> March 2022**

**Subject CM2A – Financial Engineering and Loss  
Reserving (Paper A)**

**Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)**

**Total Marks: 100**

- Q. 1)**
- i) A European put option is purchased for Rs. 2 with strike price Rs.100 and its underlying current share price is Rs. 101.The Intrinsic Value and Time Value are?
- I.** Intrinsic Value is 0 and Time Value is 1
  - II.** Intrinsic Value is 1 and Time Value is 1
  - III.** Intrinsic Value is 0 and Time Value is 2
  - IV.** Intrinsic Value is 2 and Time Value is 2
- (2)
- ii) In an Efficient Stock Market, the following statements are made about the stocks?
- a. The price of every security fully reflects all available information
  - b. The price of every security is their TRUE Value
  - c. There is no information asymmetry
- I.** a is Correct
  - II.** a and b are Correct
  - III.**a and c are Correct
  - IV.**All are correct
- (2)
- iii) Which of the following invalidates continuous log normal assumption in modelling investment returns
- a. Mean reverting nature of returns
  - b. Volatility is constant over time
  - c. Momentum effect of returns
  - d. Drift is constant over time
- I.** a & b
  - II.** c & d
  - III.** a & c
  - IV.** a, b & d
- (2)
- iv) Which of the following statement(s) is(are) correct.
- a. random variables are defined in respect of sample spaces
  - b. probabilities measures are associated with sample spaces
  - c. probabilities measures are associated with event spaces
  - d. event space is a set containing the subsets of sample space
- I.** a & b
  - II.** c & d
  - III.** Only c
  - IV.** a, b & d
- (2)
- v) A portfolio manager is building a portfolio with forward contracts which pays twice of share price at maturity. She is trying to vega hedge the portfolio. If the portfolio is vega hedged then
- a. The portfolio will be insensitive to changes in volatility of the underlying share
  - b. The portfolio will be insensitive to changes in interest rates
  - c. Could not estimate volatility of the underlying shares accurately
  - d. The portfolio will be sensitive to changes in volatility of the underlying share

- I. only d
- II. only b
- III. both b and d
- IV. both a and c

(2)  
[10]

Q. 2) i) According to Utility Theory, the minimum and maximum premium charged by an insurer with an initial capital “a” and potential loss “X” is given by

- I.  $E [ U(a-X) ] = U (a - P) \ \& \ E [ U(a+P-X) ] = U (a)$
- II.  $E [ U(a+P-X) ] = U (a) \ \& \ E [ U(a-X) ] = U (a - P)$
- III.  $E [ U(a+P-X) ] = U (a - P) \ \& \ E [ U(a+P-X) ] = U (a)$
- IV.  $E [ U(a-X) ] = U (a) \ \& \ E [ U(a+P-X) ] = U (a - P)$

(2)

ii) The investor is having a portfolio of the following options

- Purchased an European call option for Rs. 3 with Strike price of Rs. 48 and the price of underlying share at expiry is Rs. 49
- Purchased an European put option for Rs. 5 with Strike price of Rs. 55 and the price of underlying share at expiry is Rs. 50

The profit or loss for the investor of his portfolio at expiry is

- I. Loss of Rs. 2
- II. Profit of Rs. 2
- III. Loss of Rs. 8
- IV. Loss of Rs. 3

(2)

iii) Which of the following statements about options are true.

- a. Early exercise of an American call option on a non-dividend paying share is not optimal.
- b. American call option on a dividend paying share is always worth as much as its intrinsic value.
- c. European call option on a dividend paying share is worth less than its intrinsic value.
- d. Early exercise of an American put option is attractive if risk-free rate of interest increases.

- I. All the four
- II. a and b only
- III. a only
- IV. d only

(2)

iv) A Gambler wins 100 when a coin comes up heads and loses 150 when the coin comes up tails. If ‘p’ is the probability of the coin showing ‘heads’ then which of the following is a ‘submartingale’.

- a. Gambler’s fortune over time when  $p = 0.45$
- b. Gambler’s fortune over time when  $p = 0.5$
- c. Gambler’s fortune over time when  $p = 0.6$
- d. Gambler’s fortune over time when  $p = 0.65$

- I. a & b
- II. c & d

**III.** a, b & c

**IV.** b, c & d

(3)

**[9]**

**Q. 3) i)** Which of the following is the suggestions as per Myopic Loss aversion?

**I.** Investors are more concerned by losses than equivalent gains and tend to focus on short-term returns & volatility rather than long term earnings.

**II.** Investors are more concerned by losses than gains and tend to focus on minimizing the short term volatilities.

**III.** Investors are more concerned by short term volatilities and tend to maximize their return in the short term.

**IV.** Investors are more concerned by short term volatilities and tend to minimize their investments in equities.

(2)

**ii)** At the beginning of a month, an investor has a portfolio of Rs. 10 Crores. Considering the volatility in the share market, his portfolio may worth Rs. 9 Crores or Rs. 11 Crores at the end of month.

The Utility Function of the portfolio is assumed to be  $U(w) = \sqrt{w}$ .

**a)** Determine the investor's certainty equivalent for the above scenario.

(2)

**b)** IF there is an insurance available to protect the portfolio then how much is the maximum premium, the investor would like to pay based on the certainty equivalent of the scenario.

(1)

**iii)** In a Stock Market, the concerned Government announced a cut in corporate tax rate.

What would be the effect of this announcement to the individual stocks traded in the market. Explain the reason for the effect through efficient market hypothesis.

(2)

**iv)** The adjustment coefficient "R" in Lundberg's inequality is negative. Then which of the following is true:

**I.** The statement is incorrect as "R" is only positive.

**II.** The statement is incorrect as "R" is unique and positive.

**III.** The statement is incorrect as "R" is unique and negative.

**IV.** The statement is correct as "R" is always a root of Lunberg's inequality.

(2)

**[9]**

**Q. 4)** Assuming CAPM holds , find the risk free rate using the following information

State	Probability	Risky Asset 1	Market of risky assets
1	0.2	0%	2%
2	0.25	15%	11%
3	0.05	2%	3%
4	0.5	7%	5%

Correlation between risky asset 1 and Market of risky assets= 0.9831

**[8]**

**Q. 5) i)** Using Black-Scholes formula calculate the price of a 3-month European put option on a non-dividend paying share with both the strike price and current share price equal to Rs. 100. It is given that risk-free interest rate is 6% pa and the volatility is 30% pa. (3)

**ii)** Calculate the value of above European put option if a dividend of Rs. 2 is expected in 2 months. (3)  
[6]

**Q. 6)** Pied Crested Cuckoo is a particular species of bird which migrates to India from Africa at around May every year. It stays & breeds for 5 months and leaves India along with its fledglings at around first week of October. This year, around 5 lakh birds are expected to visit in the month of May.

The number of birds are expected to grow at the following rates per month

- 3% with probability 0.25
- 5% with probability 0.6
- 0% with probability 0.15

**i)** Assuming independence of monthly growth rate, Choose the expected monthly growth rate from the following options.

- I.** 3.50%
  - II.** 3.70%
  - III.** 3.75%
  - IV.** 3.90%
- (1)

**ii)** Assuming independence of monthly growth rate, choose the monthly variance from the following options.

- I.** 0.030%
  - II.** 0.032%
  - III.** 0.035%
  - IV.** 0.040%
- (2)

**iii)** Assuming **fixed rate** stochastic model, choose the expected number of birds in the first week of October from the following options.

- I.** 600,934
  - II.** 601,050
  - III.** 602,124
  - IV.** 602,794
- (3)

**iv)** A Wild life conservation group has decided to increase the number of birds that depart in the first week of October. This would be done by facilitating the habitats to increase survival of fledglings during the 5 month period. The group has found that, after the facilitation, exponential of the monthly growth rate (i.e.  $e^i$ , where 'i' is the growth rate) follows log normal distribution with expected value of 1.05 and standard deviation of 10.86%. Estimate the 2 parameters of the distribution. (4)

**v)** The group would get a negative remark from its supervisor if the total birds in the first week of October is less than the value obtained in (iii) above. Assuming independence of monthly growth rate, estimate the probability of a negative remark. (4)

[14]

**Q. 7)** The claims paid on a Fire Insurance Portfolio are as follows:

Rs. In Crores				
<b>Claims paid in the year of Development</b>				
<b>Year of Accident</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>2018</b>	275	115	62	22
<b>2019</b>	315	162	82	
<b>2020</b>	180	170		
<b>2021</b>	450			

The claims are paid at the end of Development Year. Inflation rate for the past years and the expected inflation rates for the future years are as follows:

Year	2018	2019	2020	2021	2022	2023	2024
Inflation	4.00%	5.00%	3.00%	6.00%	7.00%	7.00%	10.00%

- i) Whether the Inflation figures for all the above years are useful in the calculation. If not, state with reason which year inflation has no relevance in the estimation. (1)
- ii) Calculate the outstanding claim reserve at the end year 2021 using inflation adjusted Chain Ladder Method. (8)
- iii) State any assumptions made to estimate the reserve underlying in the above Method. (2)

Round all the figures except the development factors to nearest integer. Development factors round to 3 digits.

[11]

**Q. 8)** i) a) A portfolio named Alfa consists of two assets A and B with Rs. 1000 invested in each of the assets. Both of these assets have daily volatility of 1%. The correlation coefficient between their returns is 0.3. The 4-day 99% VaR for the portfolio will be (Rounded to nearest Rupees).

- I. 100  
 II. 75  
 III. 37  
 IV. 149 (3)

b) Another portfolio named Beta is constructed similar to portfolio Alfa except the correlation coefficient between the returns of both the assets is 1. Which of the following statements will be true.

- a. The 4-day 99% VaR will be less than the portfolio Alfa  
 b. The 4-day 99% VaR will be greater than the portfolio Alfa  
 c. Beta is a diversified portfolio  
 d. Alfa is a diversified portfolio

- I. b is correct  
 II. Both b and d are correct  
 III. Both a and c are correct  
 IV. None of the above (3)

ii) The investment returns of an asset have following probability distribution.

Returns per annum	Probability
-3%	0.5
3%	0.5

The Variance and downside semi-variance of returns of the asset are

- I. 9% and 4.5% respectively
- II. 9% and 3% respectively
- III. 0 for both
- IV. 3% and 1.5% respectively (2)

iii) Which of the following statement is not true for downside semi-variance of investment risk

- I. Ignores the risk above mean
- II. It gives more weight to downside risk
- III. Based on mean as benchmark which is arbitrary
- IV. semi-variance is half of variance in all cases (1)

iv) A person who is currently having cash of Rs. 48000 wants to go for a tour at end of one year from now which will cost him Rs.50000. The cash can be invested only in two assets namely X and Y.

X gives a guaranteed return of 4% pa whereas Y gives a return which follows continuous uniform distribution over the range of 2% to 6% pa.

Find the probability that the tour will not happen if

- a) 100% of cash is invested in asset X
  - b) 80% of cash is invested in Y and remaining in X. (5)
- [14]

**Q. 9)** A portfolio manager purchased a one-month European put option on the share of a software company with exercise price of Rs. 18. The share price is expected to increase to Rs. 22 or decrease to Rs. 14 at expiry and the risk-free force of interest is 2% per month. (Assume the shares can be bought or sold in fractions)

i) Explain how to create a hedged position and evaluate the value of the European put option given the current share price is Rs. 20. (3)

ii) Assume the current share price is now Rs. 21 and other values are same as above calculate the value of European put option using risk-neutral valuation. (3)

iii) Estimate the delta of the option using the values calculated in (i) and (ii) and comment on the value of the delta. (2)

[8]

**Q. 10)** Claims occurring in a health portfolio follows Poisson Distribution with 120 claims per year. The individual claims follow generalized Pareto Distribution with the following three parameters:

$$K = 3 ; \lambda = 75 ; \alpha = 4$$

The aggregate claims for each year follow normal distributions and the company has an initial capital of 1500.

- i) Calculate the loading that the Company has to charge over its premium to sustain the first year with 98.5% probability. (6)

The company later decided to reinsure its portfolio on a proportional basis with 75% retention.

- ii) Calculate the Reinsurer's security loading that the insurer can sustain in the first year with 98.5% probability with the insurer premium loading set at 15%. (5)
- [11]**

\*\*\*\*\*