INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

17th March 2022

Subject CM1A – Actuarial Mathematics
(Paper A)

Time allowed: 3 Hours 30 Minutes (9.30 - 13.00 Hours)

Total Marks: 100

Q. 1)   Allen wishes to plan for a pocket allowance of Rs.5,000 per month for his daughter during her four years degree course starting from 1st January 2023. He is thinking of purchasing a deferred annuity where deferment period is one year and pays Rs.5,000 at the beginning of each month during the period from 1st January 2023 to 31st December 2026.

Calculate the amount Allen requires to purchase the annuity as at 1st January 2022. Assume effective rate of interest 9% pa.

a)   Rs. 203,735
b)   Rs. 186,912
c)   Rs. 184,240
d)   Rs. 209,216   **[4]**

Q.2)   **i)**   Define the terms – "discounted payback period" and "payback period"   (3)

ii)   List the disadvantages of using discounted payback period and payback period as metrics for evaluating investment opportunities.   (2)

iii)   List the disadvantages of internal rate of return as a metric for evaluating investment opportunities.   (2)
                                                                                                                       [7]

Q.3)   A life insurance policy promises to pay a benefit of Rs.50,000 at the end of policy-year of death to a life now exactly aged 50 years, provided that death occurs after attaining the age of 60 years. Calculate the expected present value of death benefit assuming that the effective annual interest rate is 6% p.a. and mortality rates are based on 100% of AM92 Ultimate table.

a)   Rs. 8,728
b)   Rs. 8,711
c)   Rs. 12,185
d)   Rs. 14,742   **[3]**

Q.4)   A whole life annuity of Rs.500 p.a. is payable annually in arrears to a life aged 70 years.
Calculate the standard deviation of the benefits from this annuity, assuming mortality rate of AM92 Select and an annual rate of interest of 6% p.a.

a)   2,154
b)   1,463
c)   1,719
d)   1,762   **[4]**

Q.5)   Select the option which equals the function: $\ddot{a}_{x:\overline{n}|} - a_{x:\overline{n}|}$

a)   1
b) $1 - v^{(n-1)}(n-1)p_x$
c) $1 - v^{(n)}np_x$
d) $1 + v^{(n)}np_x$ **[3]**

Q.6)   A three-state transition model is as below:

Healthy to ill = σ_x
ill to Healthy = þ_x
Healthy to Dead = μ_x
ill to Dead = γ_x

Assume that the transition probabilities are constant at all ages with $\sigma_x = 3\%$, $\gamma_x = 6\%$, $\text{þ}_x = 1\%$ and $\mu_x = 4\%$.

A healthy life aged 40 years exact takes out a 20-year sickness contract that provides a premium refund of Rs.15,000 if the insured remains Healthy for the full duration of the contract.
From the options provided below, select the expected present value of the premium refund at the beginning of the contract with a force of interest of 5% p.a.

a)   2,479
b)   1,265
c)   1,361
d)   912   **[4]**

Q.7)   Describe the key characteristics of a health insurance contract. Describe income protection plan, critical illness plan and long term care insurance.   **[7]**

Q.8   i)   List the considerations for assessing the suitability of a model for a particular exercise.   (5)

ii)   A life insurance company has launched a new with-profits endowment assurance product. The product has annual premium which can be Rs.50,000 or Rs.1,00,000 only. The premium term and policy term are 5 years and 10 years respectively. Sum assured is 12 times the annual premium. Benefit payable on death and maturity is sum assured plus accrued regular bonuses. The Company has developed a model to calculate the reserves for this product using a gross premium valuation approach and the table below contains all the output that this model has produced in respect of a sample model point. Analyse the output provided below and list all the issues that you can spot. (you do not need to show any calculations as part of your solution but only describe the issues)

Policy year	1	2	3	4	5	6	7	8
Number of policies	1.00	0.98	0.94	1.01	0.93	0.83	0.73	-
Death benefit per policy	720,000	720,000	720,000	720,000	720,000	720,000	720,000	720,000
Expected premium income	60,000	58,800	56,448	60,399	55,567	50,011	44,009	-
Expected commission outgo	12,000	5,880	2,822	3,020	2,778	2,501	2,200	2,200
Expected death outgo	-	14,400	27,660	50,000	58,369	61,755	60,026	-
Expected maturity outgo	-	-	-	-	-	-	528,113	-
Net cash flow	-	**38,520**	**25,966**	**7,379**	**(5,580)**	**(14,245)**	**(546,329)**	**(2,200)**
Reserves	-	-	414,822	461,529	491,985	511,004	522,310	2,096

(7)

[12]

Q.9) A fixed interest bond is to be issued bearing coupons payable quarterly in arrears at a rate of 5% per annum. Capital is to be redeemed at Rs.103 per Rs.100 nominal on a single coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. An investor who is liable to income tax at 20% and capital gain tax at 25% wishes to purchase the asset on the date of issue. A net effective yield of at least 4% per annum is required from the transaction. Calculate the price that the investor should pay to purchase Rs.100,000 nominal of this asset.   **[7]**

Q.10) **i)** Calculate $\bar{A}_{25:\overline{25}|}$ and $\bar{a}_{25:\overline{25}|}$ independent of each other, assuming AM92 Ultimate mortality and 6% p.a. interest.   (5)

ii) Verify that the usual premium conversion relationship holds approximately between the two functions noted in part (i) above.   (2)

                                                                                                                           [7]

Q.11) A loan is repayable by annual instalments in arrears for 20 years. The initial instalment is Rs 50,000 and it decreases by Rs.2,000 each year subsequently. The effective rate of interest over the term of loan is 4% per annum.

i) Calculate the original loan amount.   (3)

ii) Calculate the capital repayment in the 12th instalment.   (3)

After the 12th instalment has been paid, the borrower decides to repay the loan with level annual instalments which equal the 12th instalment in amount. There will not be any further reduction in instalment amount leading to a reduced term of the loan. Final instalment will also be reduced to the level to clear the debt.

iii) Calculate the remaining term of the loan.   (3)

iv) Calculate the final reduced payment made to clear the debt.   (3)

v) Calculate the total interest paid by the borrower during the term of this loan.   (2)

                                                                                                                          [14]

Q.12) An insurer sells a decreasing term insurance policy with a policy term of 15 years and an initial sum assured of Rs.300,000 to lives aged 50 years exact. The sum assured decreases by Rs.15,000 at the start of each year starting from the second policy year. The death benefit is payable immediately on death. Premiums are payable annually in advance throughout the term of policy. The office calculates premium using AM92 Ultimate mortality and interest rate of 4% p.a.; initial expense of Rs.500, renewal expense of Rs.50 at the start of each year except in the first policy year, and a claim expense of Rs.800.

i) Using P for the annual premium, write down the future loss random variable for the policy at the start of the term, and also just before the payment of tenth premium, assuming the policy is still in force at that time.   (4)

ii) Calculate the office premium.   (6)

iii) Calculate the gross premium prospective reserve for the policy just before the payment of tenth premium. Also clearly mention the assumption made for reserving basis.   (4)

                                                                                                                          [14]

Q.13) **i)** Explain the difference between a profit vector and a profit signature. (2)

A life insurance company issues a 4-year unit-linked endowment policy to a life aged 50 years exact which has a level premium of INR 15,000 payable annually in advance throughout the term of the policy or until earlier death.

The company uses following assumptions in carrying out profit tests of this contract.

Risk Discount Rate   : 7.50% per annum
Interest on Non-unit fund   : 5.00% per annum
Mortality   : AM92 Select

In addition, it is assumed that at the end of each of the first 3 years, 10% of all the policies still in-force surrender

For this contract the Profit Vector is as follows :

Year	1	2	3	4
Profits	7,211.25	-1,262.18	-1,063.56	-6,082.67

ii) Calculate the profit margin for the policy on the assumption that the company doesn't zeroise future expected negative cash flows. (6)

iii) Suppose the company sets up reserves so that the product is self-supporting. Select the revised cashflows from below options under this scenario:

a) (1,197, 0, 0, 0)
b) (1,553, 0, 0, 0)
c) (1,873, 0, 0, 0)
d) (1,337, 0, 0, 0) (4)

iv) Calculate the profit margin allowing for the cost of setting up these reserves. (2)

[14]

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