INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

6th April 2021

Subject CS2A – Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)

Total Marks: 100

Q. 1) Find out the correct option for the below questions (for the multiple-choice questions, there is no need to mention the steps in the solution paper).

i) Please refer the below statements related to policy excess:

1. It may also be called a deductible.
2. Insured agrees to fund losses incurred up to defined level.
3. Claim amount paid by insurer reduces by the excess amount.

Choose the correct alternative.

a) Only 2 is true
b) 1 and 2 are true
c) 2 and 3 are true
d) All are true
e) None of the above (2)

ii) An insurer believes that claims from a particular type of policy follow a Pareto distribution with parameters α=2 and λ= 900 . The insurer wishes to introduce a policy excess so that 20% of losses result in no claim to the insurer. Find out the correct option for the excess amount.

a) 116.13
b) 106.23
c) 10.63
d) 30.23
e) None of the above (4)

iii) A Poisson process with rate λ is a continuous-time integer-valued process N_t, $t \geq 0$ with the following properties. Find out the correct option.

a) $N_0=0$, N_t has independent increments, N_t has Poisson distributed stationary increments: $P[N_t - N_s = n] = [\lambda (t - s)]^n e^{-\lambda (t-s)} / n!$, $s<t$, $n=0,1,\ldots.$
b) $N_0=1$, N_t has independent increments, N_t has Poisson distributed stationary increments: $P[N_t - N_s = n] = [\lambda (t - s)]^n e^{-\lambda (t-s)} / n!$, $s<t$, $n=0,1,\ldots.$
c) $N_0=0$, N_t has dependent increments, N_t has Poisson distributed stationary increments: $P[N_t - N_s = n] = [\lambda (t - s)]^n e^{-\lambda (t-s)} / n!$, $s<t$, $n=0,1,\ldots.$
d) $N_0=1$, N_t has dependent increments, N_t has Poisson distributed stationary increments: $P[N_t - N_s = n] = [\lambda (t - s)]^n e^{-\lambda (t-s)} / n!$, $s<t$, $n=0,1,\ldots.$
e) None of the above (2)

iv) For a force of mortality μ_x that is known to follow Gompertz' Law, calculate the parameters B and c if $\mu_{50} = 0.017609$ and $\mu_{55} = 0.028359$.

a) 0.9091, 5.36054
b) 0.9091, 2.06679
c) 1.1000, 0.00015
d) 1.1000, 2.06679
e) None of the above (2)

v) For a discrete time stochastic process X_n, find out the correct option which defines the terms Markov property and Martingale:

a) For a discrete process the Markov property requires that: $P[X=x|X_{t1} = x_1, X_{t2} = x_2, \ldots, X_{tm} = x_m] = P[X_t = x| X_{tm} = x_m]$ for all times $t_1 < t_2 < \ldots < t_m < t$ and all states $x_1 < x_2 < \ldots X_m < t$
A discrete time martingale X_n satisfies two conditions:
$E[| X_n |] <$ ¥ for all n and $E[|X_n | X_0, X_1, \ldots X_m] = X_n$ for all $m < n$

b) For a discrete process the Markov property requires that: $P[X=x|X_{t1} = x_1, X_{t2} = x_2, \ldots, X_{tm} = x_m] = P[X_t = x| X_{tm} = x_m]$ for all times $t_1 < t_2 < \ldots < t_m < t$ and all states $x_1 < x_2 < \ldots X_m < t$
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c) For a discrete process the Markov property requires that: $P[X=x|X_{t1} = x_1, X_{t2} = x_2, \ldots, X_{tm} = x_m] = P[X_t = x| X_{tm} = x_t]$ for all times $t_1 < t_2 < \ldots < t_m < t$ and all states $x_1 < x_2 < \ldots X_m < t$
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d) For a discrete process the Markov property requires that: $P[X=x] = P[X_{tm} = x_m]$ for all times $t_1 < t_2 < \ldots < t_m < t$ and all states $x_1 < x_2 < \ldots X_m < t$
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e) None of the above (2)

vi) Read the following statements:

1. It is possible to have a Markov chain that has more than one stationary distribution.
2. It is possible to have a Markov chain that has no stationary distribution.
3. A Markov chain with a finite state space has at least one stationary probability distribution.
4. An irreducible Markov chain with a finite state space has a unique stationary probability distribution.

Find out the correct option.

a) All true
b) All false
c) 1 = true, 2 = false, 3 = true, 4 = false
d) 1 = false, 2 = true, 3 = false, 4 = true
e) None of the above (2)

vii) Losses from a group of travel insurance policies are assumed to follow a Pareto distribution with parameters $\alpha = 4.5$ and $\lambda = 3{,}000$. Next year losses are expected to increase by 3%, and the insurer has decided to introduce a policy excess of 100 per claim.

The probability that a loss next year is borne entirely by the policyholder is

a) 0.15323

b) 0.13353
c) 0.12353
d) 0.11353
e) None of the above (4)

viii) Clients arrive at a tax consultant's office according to a Poisson process with the rate $\lambda = 1/10$ per minute. The consultant will not meet a client until at least three clients are in the waiting room. In other words, at any given point of time, there should be at least two clients waiting in the waiting room when the consultant proceeds to meet a client. After the consultant has met 15 clients on a given day, he is happy and meets all the arriving clients irrespective of the number of clients waiting.

The expected waiting time until the consultant meets the first client.

a) 45 minutes
b) 30 minutes
c) 15 minutes
d) 60 minutes
e) None of the above (2)

ix) Refer the question (viii). What is the probability that consultant does not meet any client in the first two hours?

a) 0.229%
b) 0.052%
c) 0.004%
d) 0.021%
e) None of the above (4)

x) Refer the question (viii). Today, the consultant wants to attend a family function after three hours. However, he wants to meet at least 8 clients today before he proceeds to attend the function. What is the probability that he meets at least 8 clients in three hours?

a) 98.5%
b) 1.5%
c) 0.7%
d) 99.3%
e) None of the above (5)

[29]

Q. 2) An operational researcher is analyzing switching of market share between two products. She knows that in period 1 the market shares for the two products were 55% and 45% but that in period 2 the corresponding market shares were 67% and 33% and in period 3, 70% and 30%. The researcher believes that an accurate representation of the market share in any period can be obtained using Markov processes. Assuming her belief is correct.

i) Estimate the transition matrix of how the buyers switch buying preference. (6)

ii) Calculate the market shares in period 4 using the estimated transition matrix. (2)

[8]

Q. 3) **i)** Explain what is graduation and why is it necessary to graduate crude mortality data before use? (2)

ii) What are the limitations of the graduation? (1)

iii) What are the desirable features of the graduation? (3)
[6]

Q. 4) **i)** In respect of recent pandemic of Covid-19, the diagnostic tests are conducted having different levels of false positive and false negative. Discuss the impact of false positive and false negative from the point of view of actual patient. (2)

In a sample of size 200, 15% of individuals have a particular feature.

ii) Draw up a confusion matrix for a test that can identify this feature perfectly. (2)

iii) Calculate four measures for the effectiveness of the test, based on the numbers in your Matrix. (2)

An investor purchases three 5-year bonds from different companies within the same industry sector. The probability that an individual bond default within the first year is 10%.

iv) Using a Gumbel copula with parameter $\alpha = 2$, calculate the probability that all three bonds default within the first year. (4)

v) Discuss the suitability of the Gumbel copula in this situation. (4)
[14]

Q. 5) On 1st January 2001 an insurer in country of Indiana sells 100 policies, each with a five year term, to householders wishing to insure against damage caused by fireworks. The insurer charges annual premiums of Rs.600 payable continuously over the life of the policy. The insurer knows that the only likely date a claim will be made is on the given date i.e on Diwali in the Month of November each year, when it is traditional to have an enormous fireworks display. The annual probability of a claim on each policy is 40%. Claim amounts follow a Pareto distribution (10, 9000).

Estimate the mean and the standard deviation of the annual aggregate claims. **[4]**

Q. 6) The time series X_t is assumed to be stationary and to follow an ARMA (2,1) process defined by:

$$X_t = 1 + (8/15)\,X_{t-1} - (1/15)\,X_{t-2} + Z_t - (1/7)Z_{t-1}$$

where Z_t are independent $N(0,1)$ random variables.

i) Determine the roots of the characteristic polynomial and explain how their values relate to the stationarity of the process. (2)

ii) Find the autocorrelation function for lags 0, 1 and 2 and derive the autocorrelation at lag k in the form $\rho_k = (A/c^k) + (B/d^k)$. (13)
[15]

Q. 7) **i)** List the three parameters of the Generalized Extreme Value distribution. (2)

ii) What these parameters determine? (2)

iii) What are the analogous to parameters for these in a standard distribution? (2)

[6]

Q. 8) A mobile phone manufacturer is considering a proposal to provide full warranty on high-end phones whereby it will undertake to replace new handsets for up to one year, against any manufacturing defects. Each phone costs Rs 25,000 to replace and the management has asked you to estimate the total cost of this warranty.

A customer services executive collected the contact information of buyers of 100 phones that were sold on 1 January and contacted them at the end of each month for one year.

You have the following extract from her call journals:

- 31/01 − No defects reported, 7 customers could not be reached due to incorrect contact details
- 28/02 − 1 manufacturing defect reported, 2 customers requested not to be disturbed in future
- 31/03 − No defects reported
- 30/04 − No defects reported, 2 more customers requested not to be disturbed in future
- 31/05 − 2 manufacturing defects reported, 3 customers changed their contact details and could not be reached
- 30/06 − On leave (no calls made)
- 31/07 − No defects reported for the previous two months
- 31/08 − 2 defects reported, one of which is non manufacturing defect and hence is not covered under warranty
- 30/09 − No defects reported
- 31/10 − No defects reported
- 30/11 − Two more manufacturing defects reported
- 31/12 − No defects reported

i) Use this data to compute the Nelson-Aalen estimate of the cumulative hazard function, $\Lambda(t)$, where t is the time since purchase of a new phone and thus deduce an estimate of the survival probability beyond one year. (7)

ii) The manufacturing company sells 10,000 phones on average each year. Based on your answer in part (i), calculate the expected cost of providing the warranty over one year and construct a 95% confidence interval for the same. (7)

iii) The Finance Director believes that the estimate of the expected cost that you have calculated is too high. He notes that only 6 out of 100 phones in the investigation reported manufacturing defects. Therefore, the expected cost of providing a warranty against manufacturing defects should be 6% * 10,000 phones * Rs 25,000 = Rs 1.5 crores. Explain why your estimate should be higher than this. (4)

[18]

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