INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

8th April 2021

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)

Total Marks: 100

Q. 1) Select the most appropriate choice from amongst the given options.

i) Consider the following inequalities relating to the probability of ruin for a claims process operating in continuous time: Which of these inequalities are correct?

[Here, u, u1 and u2 represent initial surplus and t, t1 and t2 represent time.]

I. Prob(u,t1) <=Prob(u,t2) if 0 <=t1<=t2
II. Prob(u1,t) <=Prob(u2,t) if 0 <=u1<=u2
III. 0<=Prob(u,t) <= 1

a) I and II only

b) I and III only

c) II only

d) I , II and III (2)

ii) Which of the following phenomena are attributable to overconfidence?

I. The tendency for people to think that they have very smart friends.
II. The tendency for people to think that their work is better than others'.
III. The tendency to avoid anything that you might regret later.

a) I and II only

b) I and III only

c) II only

d) I , II and III (2)

iii) Which of the following components is not part of Daniel Kahneman's and Amos Tversky's Prospect Theory?

a) When an outcome is certain and becomes less probable, loss in utility is much higher than a similar deduction if previous outcome was probable.

b) A value function that is concave in gains and convex in losses, with a kink at the reference point that separates the gains- from the loss-region.

c) An overconfidence-factor that is directly applied to the value function in the loss-region.

d) A probability weighting function that overvalues small probabilities and undervalues high probabilities. (2)

iv) Behavioural explanation of the equity puzzle can be explained when Investors

a) are risk seeking and do not realize the risks they are taking

b) are averse to short term losses.

c) focus on the long-term because they are saving for retirement

d) are over-confident and never re-evaluate their portfolio allocations (2)

v) A finding that ___________ would provide evidence against the weak form of the EMT.

I. Changes in stock prices are somewhat predictable from past data.
II. Changes in stock prices are random and unpredictable.
III. Insiders of company can consistently profit from insider trading.

a) I and II only

b) I and III only

c) I only

d) I , II and III (2)

[10]

Q. 2) Label the following behaviours of insured policyholders as 'Adverse selection' or 'Moral hazard':

I. A person with co-morbid conditions availing a group term insurance through the employer
II. Healthy life not paying the renewal premium for a term insurance in the last 2 years
III. A vehicle owner parking the car below a tree during windy rainy season
IV. A biker riding the bike carelessly after buying additional accidental cover

[2]

Q. 3) Match the following risk incidents to their appropriate category. Also state the recommended risk mitigation action for each category / incident from amongst {avoid, ignore, control, insure}.

Event Category	**Risk incident**	**Action**
Low-frequency Low-severity	Fire at the head office of a life insurance company	
High-frequency Low-severity	An earthquake striking a nuclear reactor situated in an earthquake-prone area	
Low-frequency High-severity	An employee getting locked in the office restroom for a few minutes	
High-frequency High-severity	Undetected shoplifting at retail store e.g. a biscuit packet opened and eaten in the store	

[4]

Q. 4)

i) Compare value at risk and expected shortfall as a risk measure.
Also list at least four other measures of risks, similar to expected shortfall.

(4)

ii) Mr. Tourist wants to go for world tour after 5 years to celebrate his 50th Birthday. The cost of the tour is Rs. 25 lakhs. He chooses to save for the tour by buying non-dividend paying stock whose price S_t follows the Geometric Brownian Motion:

$$S_t = S_0 \exp[(\mu - \sigma^2/2)t + \sigma Z_t]$$

where Z_t is a standard Brownian motion, $\mu = 8\%$, $\sigma = 16\%$, and $S_0 = 1$.

- If Mr. Tourist invests Rs. 19.5 lakhs in this stock now, what is the shortfall probability after 5 years?

- Also compute the expected value of his investment after 5 years.

(6)

[10]

Q. 5) The annual returns, i, on a fund are independent and identically distributed, with a mean of 10% and a standard deviation of 4%. Each year, the distribution of 1+i is lognormal with parameters μ and σ^2.

i) Calculate μ and σ^2. (4)

ii) Calculate the amount that should be invested in the fund immediately to ensure an accumulated value of 200,000 at the end of 10 years with a probability of 0.99. (5)

[9]

Q. 6) An analyst uses a two state continuous time model to study the credit risk of zero coupon bonds issued by different companies.

Company A has issued 10 year zero coupon bonds. The risk neutral transition intensity function for company A is as follows:

$\lambda_A(s) = 0.05s$, $0 \le s \le 10$

i) The risk-neutral probability that the bond will not default between times 5 and 10, given that it hasn't defaulted till time 5, is:

a) 0.1362

b) 0.1534

c) 0.8466

d) 0.8825 (2)

Similarly, company B has issued 10 year coupon bonds. The risk neutral transition intensity function for company B is as follows:

$\lambda_B(s) = 0.0015s^2 + 0.006s$, $0 \le s \le 10$

ii) Compute the risk-neutral probability that the bond will default by time 10. (2)

iii) The analyst observes that the credit spread on a 10 year bond issued by company A is 2 times that of a 10 year bond issued by company B.
Given that the risk free force of interest is 5% pa and the average recovery rate in the event of default, δ, where $0<\delta<1$ is same for both the companies, calculate δ.

(7)

[11]

Q. 7) Consider the function $A(t) = e^{-0.08t+0.02B(t)}$, where B(t) is a standard Brownian motion.

i) Verify that the function above is a strictly positive supermartingale. (5)

Assuming A(t) satisfies the Stochastic differential equation of the form:

$dA(t) = A(t)\,[\mu_A(t)\,dt + \sigma_A(t)\,dB(t)]$

ii) The form of the function $\mu_A(t)$ is:

a) $\mu_A(t) = -0.08$

b) $\mu_A(t) = -0.0798$

c) $\mu_A(t) = -0.0802$

d) $\mu_A(t) = -0.0820$ (2)

iii) The form of the function $\sigma_A(t)$ is:

a) $\sigma_A(t) = 0.0200$

b) $\sigma_A(t) = 0.0040$

c) $\sigma_A(t) = 0.1414$

d) $\sigma_A(t) = 0.0202$ (2)

iv) Assuming that zero coupon bond prices can be modeled using the formula B (t,T) = $E_P[A'(T)/F_t] / A'(t)$, where P is a suitable probability measure and A'(t) satisfies $dA'(t) = A'(t) [-\mu_A(t)\, dt + \sigma_A(t)\, dB(t)]$, calculate the price (at time of issue) of a risk-free 10-year zero coupon bond issued at time 2

a) 0.4493

b) 0.4484

c) 0.5281

d) 0.4502 (2)

v) The price (per 100 nominal) of a risk free 10 year bond (issued at time 0) at time 5 that pays a coupon of 5% at the end of each year:

a) 78.11

b) 77.99

c) 77.87

d) 79.41 (2)

[13]

Q. 8) A general insurer is conducting its reserve review for a financial year.

i) Choose the method(s) in which inflation of claims is allowed for explicitly:

I. Basic chain-ladder method
II. Bornhuetter-Ferguson method
III. Inflation adjusted chain-ladder method

a) I only

b) I and II only

c) III only

d) I , II and III (2)

ii) The table below gives the cumulative incurred claims by year and premiums received for a third party claim of 2 wheeler policy (Figures in thousands). Claims paid till date total 1.70 crores. The ultimate loss ratio is expected to be in line with the 2016 accident year.

	Development year				
Accident Year	0	1	2	3	Premiums received
2016	3340	3750	4270	4400	4500
2017	3670	4080	4590		4900
2018	3690	4290			5050
2019	4150				5200

Use the Bornhuetter-Ferguson method to calculate the total reserve required to meet the unpaid claims, assuming that the claims are fully developed by the end of development year 3. (Ignore inflation.)

(9)
[11]

Q. 9) Uday's utility function is defined by U(w) = $\frac{(w^{\lambda} - 1)}{\lambda}$ w > 0

i) If relative risk aversion of the above utility function is zero, find λ. (3)

An insurer is having a utility function which is exactly that of Uday's. However, the Insurer's wealth is 2 crore whereas Uday's wealth is 1 lakh.

The probability of occurrence of any health related expense is 0.3 and if incurred the expense is 30000. Uday intends to purchase a health Insurance policy.

ii) Find the maximum premium Uday can pay. (2)

iii) Find the minimum premium the Insurer can offer. (2)

During the year, a new pandemic affected many people in the country. But luckily, Uday did not get infected. However λ in his utility function reduced to 0.9. On the other hand, Insurer's λ in its utility function has reduced to 0.7. Given that the probability of the event happening and the loss upon the event remain the same.

iv) Find the maximum premium he is prepared to buy for a new health insurance policy if Uday's wealth stood at 50000. (6)

v) Comment on the level of Minimum premium the Insurer can offer now as compared to (iii). (1)
[14]

Q. 10) The current share price of a stock is Rs. 20. There are 12-month expiry options with a Strike price of Rs. 22. The continuously compounded risk-free interest rate is of 12% per annum.

c_t : European call option price
C_t : American call option price
p_t: European put option price
P_t: American put option price

i) Determine, with brief explanations, the minimum and maximum possible values of the current option prices: C_t , p_t, P_t (6)

ii) If p_t is 10,what is the value of c_t

a) 0.49

b) 2

c) 10

d) 10.49

(2)

iii) Which of the below relationships is correct for the greeks {Rho, Lambda, Theta} for an option?

a) $r*Rho_C+(T-t)*Lambda_C+q*Theta_C=r*Rho_p+(T-t)*Lambda_p+q*Theta_p$

b) $q*Rho_C+r*Lambda_C+(T-t)*Theta_C=q*Rho_p+r*Lambda_p+(T-t)*Theta_p$

c) $r*Rho_C+q*Lambda_C+(T-t)*Theta_C=r*Rho_p+q*Lambda_p+(T-t)*Theta_p$

d) $(T-t)*Rho_C+q*Lambda_C+r*Theta_C=(T-t)*Rho_p+q*Lambda_p+r*Theta_p$ (2)

iv) Explain the term 'Dynamic delta hedging'. (1)

v) Explain complete market & its use for derivatives (2)

vi) State whether the price of a put option increases or decreases with the following:

a. Decrease in underlying share price
b. Increase in dividend rate
c. Reduction in time to expiry
d. Increase in strike price
e. Reduction in risk-free interest rate
f. Increase in volatility of share price

(3)

[16]

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