INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

7th April 2021

Subject CM1A – Actuarial Mathematics (Paper A)

Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)

Total Marks: 100

Q. 1) Anisha takes out a loan. It must be repaid with level annual payments based on an annual coupon rate of 5%. The 6th payment consists of ₹11000 in interest and ₹4000 of principal. Calculate the amount of interest paid in the 14th payment.

(a) 1,252
(b) 5,909
(c) 7,080
(d) 7,919
(e) 9,090 **[3]**

Q. 2) Which of these are true about annuities?

I. An annuity due is one that requires payments at the end of every month.
II. A geometric perpetuity present value can be represented by K/(i-r).
III. Because an increasing annuity immediate has an annuity due part in its equation, it becomes an annuity due.

(a) I only
(b) II only
(c) III only
(d) II and III
(e) I, II, and III are all false **[2]**

Q. 3) Atharv deposits ₹100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.

(a) 4,695
(b) 5,070
(c) 5,445
(d) 5,820
(e) 6,195 **[5]**

Q. 4) You are given the following information about a company's liabilities:

- Present value: 9697
- Macaulay duration: 15.24
- Macaulay convexity: 242.47

The company decides to create an investment portfolio by making investments into two of the following three zero-coupon bonds: 5-year, 15-year and 20-year. The company would like its position to be Redington immunized against small changes in yield rate.

The annual effective yield rate for each of the bonds is 7.5%.

Determine which of the following portfolios the company should create.

(a) Invest 3077 for the 5-year bond and 6620 for the 20-year bond.

(b) Invest 6620 for the 5-year bond and 3077 for the 20-year bond.

(c) Invest 465 for the 15-year bond and 9232 for the 20-year bond.

(d) Invest 4156 for the 15-year bond and 5541 for the 20-year bond.

(e) Invest 9232 for the 15-year bond and 465 for the 20-year bond.

[5]

Q. 5) A company uses an investment return assumption of 5.5% for profit testing. The risk discount rate is same as well. During a profit testing exercise, the company decided to increase the prudence margin in reserving from 10% to 12.5%. The NPV of future profits at the inception of the contracts:

(a) Will increase by 2.5%

(b) Will decrease by 2.5%

(c) Will remain unchanged

(d) Will increase by 0.15%

[2]

Q. 6) David agrees to pay Joe a sum of USD 5,000 once Joe attains the age of 70 years and 6 months exact. At present, Joe's age is 65 years exact. Which of the options below most closely reflects the time value of the expected payout to Joe using an interest rate assumption of 5% p.a.

Further, you should assume a mortality table of PMA92C20 and assume uniform distribution of deaths.

(a) 200

(b) 1800

(c) 3600

(d) 3750

[4]

Q. 7) An insurance company sells a with-profits product with a sum assured of INR100,000, single premium of INR10,000 and policy term of 10 years to a policyholder aged 30 years exact. Compound reversionary bonus rate for the product is 3% p.a. and are attached on each policy anniversary. Upon death of the life assured during the policy term or upon maturity, the benefit payable is sum assured plus accrued bonuses at the end of the policy year.

i) When calculating reserves, the regulations only require insurance company to hold a reserve in respect of bonus accrued until the valuation date; and make no allowance for future bonuses. Assuming a reserving interest rate of 4.5% p.a. and mortality rate based on 90% of AM92 ultimate table; which of the below most closely reflects the reserve required just after the 8^{th} policy anniversary.

(a) 119,500

(b) 116,000

(c) 116,500

(d) 114,000

(4)

ii) The regulations have changed recently. Under the revised regulations, the insurance companies are required to reserve for future expected bonus declarations as well; and accordingly set an interest rate assumption, which reflects expected real world earnings. Therefore, the company has revised the reserving basis such that: future bonus rate assumption for reserving remains at 3% p.a. and reserving interest rate is 6.5%. Without performing additional calculations, explain whether this would result in an increase or decrease in reserves.

(2)

[6]

Q. 8) Describe the following:

i) Describe the cashflows for an investor who purchases an index-linked bond. (2)

ii) Describe main features of an endowment assurance contract. (2)

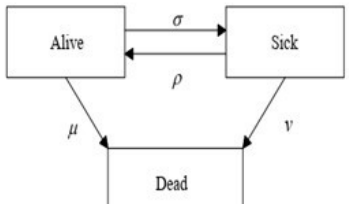
iii) Describe the effective rates of interest and discount. Calculate annual effective rate of interest that is equivalent to a simple interest rate of 4% over 5 years. (3)

iv) Describe in detail what is meant by the independent probability of decrements and dependent probability of decrements. (3)

[10]

Q. 9) A three-state transition model is shown in the following diagram: Assume that the transition probabilities are constant at all ages with $\mu = 2\%$, $v = 4\%$,$\rho = 1\%$ and sigma = 5% . Calculate the expected present value of a sickness benefit of £2,000 p.a. paid continuously to a life now aged 40 exact and sick, for this period of sickness only, discounted at 4% pa and payable to a maximum age of 60 exact.

A three-state transition model is shown in the following diagram:



(a) 18,653

(b) 19,751

(c) 25,056

(d) 20,762 **[3]**

Q. 10) Two lives, each aged exactly 40, are independent with respect to mortality and are each subject to a constant force of mortality of 0.01 per annum. Calculate the value of ${}_{5|17}q^{1}_{40:40}$?

(a) 0.12405

(b) 0.11040

(c) 0.13040

(d) 0.14030 **[4]**

Q. 11) The Government of Indiana has come up with a proposal that those buying house for the first time will be given a state relief of Rs 2,50,000 in 5 equal yearly installments starting from the end of first year of taking the loan.

Mr Gary purchases his first house and takes a loan of Rs 35,00,000 for 20 years @ 12% p.a. convertible monthly from Indiana Bank. He agrees to repay the loan in equated monthly installments.

i) Calculate the monthly installment of the loan considering with and without state relief. (6)

ii) What is the total amount of saving if state relief is availed? (2)

At the end of 3rd year, i.e. just after the paying of state relief amount, pandemic broke out and the entire country went into a lockdown. Several people lost their jobs including Gary and the EMI could not be paid to the bank. In order to meet the crisis, the Government came up with an idea of providing a moratorium period of 24 months during which no EMIs will be paid and the bank can charge a simple interest of 10% p.a. during that period on the outstanding loan. However, the state relief will continue to be paid on due dates which will help reduce outstanding loan and simple interest on it.

At the end of the moratorium period, the outstanding loan along with simple interest will then be considered for revised loan agreement.

Gary, having availed the moratorium period and also received state relief, entered into a fresh agreement with the bank for the balance period of the original tenure @12% p.a.

iii) Calculate the revised EMI that Gary has to pay at the end of moratorium period (10)

iv) Calculate the additional amount that Gary had to pay by availing the moratorium period (2)

[20]

Q. 12) A life insurance company issues a four-year unit-linked endowment assurance policy to a male life currently aged 55 exact. If the policyholder dies during the term of the policy, a death benefit of 125% of the bid value of the units is payable at the end of the policy year of death. The policyholder may only surrender the policy at the end of the first policy year or later. On surrender, the bid value of units less a surrender value penalty of INR 900 is payable. On maturity, 100% of the bid value of the units is payable. Level premiums of INR 5,500 per annum are payable annually in advance throughout the term of the policy or until earlier death with 97.5% of each premium being allocated to units. A policy fee of INR 60 is deducted from the bid value of units at the start of each policy year. The units are subject to a bid-offer spread of 5%. An annual management charge of 1% of the bid value of units is deducted at the end of each policy year from end of year fund value. These management charges are deducted from the unit fund before death, surrender and maturity benefits are paid. The company uses the following assumptions in carrying out profit tests of this policy:

Rate of growth on assets in the unit fund - 4% per annum in year 1

3.5% per annum in year 2 and onwards

Rate of interest on non-unit fund cash flows - 2.5% per annum

Mortality - AM92 Select

Surrenders - 10% of all policies in force at the end of policy year 1

5% of all policies in force at the end of the policy year 2 and onwards

Initial expense - 265

Renewal expense - 75 on the second premium date, 0 after that

Initial commission - 5% of first premium

Renewal commission - 2.5% of the second premium and onwards

Death claim expense - 100

Maturity claim expense - 60

Risk discount rate - 5% per annum

i) Calculate the non-unit fund cash flows components in the first year of the policy if the policyholder:

a) dies in the first year of the contract (3)

b) surrenders in the first year of the contract (2)

c) survives to the end of the contract (2)

ii) Calculate the non-unit fund cash flows in the second year of the policy if the policyholder:

a) dies in the third year of the contract (1.5)

b) survives to the end of the contract (1.5)

iii) Calculate the expected present value of profit of the policy? (8)

Develop all cashflows rounded to nearest two decimal places.

[18]

Q. 13) A life insurance company has launched a protection product, which in case of death of the life assured during the policy term, pays the beneficiary the Basic Sum Assured immediately upon death. Additionally, the beneficiary will also receive monthly income of 1% of the Basic Sum Assured, for a fixed period of 10 years. The monthly income shall be payable in arrears and commence one month after the death of the life assured. The policy term extends till the assured attains age of 70 years exact.

Premiums are payable annually in advance over the policy term. Basic Sum Assured is INR 5,000,000.

Other details regarding the basis are provided below:

- Age of policyholder – 40 years exact

- Mortality: AM92 Ultimate
- Initial Expense = INR2000 plus 20% of first year premium
- Renewal expense = INR500 per annum (assume no inflation), incurred from start of the second policy year and until the policy remains in-force.
- Claim expenses are 0.5% of benefit payout.
- Commission = first year commission is 15% of first year premium; and renewal commissions are 3% of premium
- Interest rate = 4% p.a.

i) Using the principle of equivalence, calculate the gross premium. (12)

ii) The insurance company is looking to launch another variant of the same product. Under this variant, death benefit will be payable until the policyholder attains the age of 99 and monthly income benefits upon death are no longer payable. All other details remain the same as above.

Calculate the gross premium, using the principle of equivalence (4)

iii) Without performing further calculations, comment on how premium in part (ii) will change in the following scenarios

a) Premiums are paid quarterly in advance (1)

b) Benefits are payable whole of life (1)

[18]

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