Institute of Actuaries of India

Subject CM1A – Actuarial Mathematics (Paper A)

March 2021 Examination

INDICATIVE SOLUTION

Introduction
The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1: Correct answer (e)

Payment= 11,000 +4,000=15000

Principal repaid grows by 1.05 with every payment.

Principal in 14th payment is: 4000(1.05)^8 = 5909.82

Interest Paid: 15000 – 5909.82 = 9090.18 **[3 Marks]**

Solution 2: Correct answer (b) - II only **[2 Marks]**

Solution 3: Correct answer: (e)

From basic principles, the accumulated values after 20 and 40 years are:

$100[(1+i)^{20} + (1+i)^{16} + (1+i)^{12} + \ldots + (1+i)^{4}] = 100\,(1+i)^{4}]\ [\frac{(1+i)^{20}-1}{(1+i)^{4}-1}]$ ----(A)

$100[(1+i)^{40} + (1+i)^{36} + (1+i)^{32} + \ldots + (1+i)^{4}] = 100\,(1+i)^{4}]\ [\frac{(1+i)^{40}-1}{(1+i)^{4}-1}]$ ----(B)

Ratio is 5, thus

$5= [\frac{(1+i)^{40}-1}{(1+i)^{20}-1}]$

Let $(1+i)^{20}$ = x

Therefore

$5= [\frac{x^2-1}{x-1}]$

$\Rightarrow x^2 - 1 = 5(x-1)$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow x^2 - x - 4x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0$

Only 2nd root gives positive solution. Thus

$x = (1+i)^{20} = 4$

$\Rightarrow (1+i)^{4} = 4^{\frac{1}{5}} = 1.31951$

Therefore

$X = 100\,(1+i)^{4}]\ [\frac{(1+i)^{40}-1}{(1+i)^{4}-1}]$

$= 100 * 1.31951 * [\frac{(4)^2-1}{1.31951-1}]$

$= 100 * 1.31951 * [\frac{15}{0.31951}]$

= 6195

= Answer **[5 Marks]**

Solution 4: Correct answer: (a)

Let x, y, and z represent the amounts invested in the 5-year, 15-year, and 20-year zero-coupon bonds, respectively.

Since the company has decided to invest two types of bonds only, in this problem, one of the above three variables is 0.

The present value, Macaulay duration, and Macaulay convexity of the assets are, respectively,

x+y+z

$\frac{5x+15y+20z}{x+y+z}$

$\frac{5^2x+15^2y+20^2z}{x+y+z}$

We are given that the present value, Macaulay duration, and Macaulay convexity of the liabilities are, respectively, 9697, 15.24, and 242.47.

Since present values and Macaulay durations need to match for the assets and liabilities, we have the two equations:

x+y+z = 9697

$\frac{5x+15y+20z}{x+y+z} = 15.24$ (1)

Note that 5 and 15 are both less than the desired Macaulay duration 15.24, so z cannot be zero.
So, we try either the 5-year and 20-year bonds (i.e. y = 0), or the 15-year and 20-year bonds (i.e. x = 0).
In the former case, substituting y = 0 and solving for x and z yields
x = 3077.18 & z = 6619.82 (1)

We need to check if the Macaulay convexity of the assets exceeds that of the liabilities.
The Macaulay convexity of the assets is

$$\frac{5^2 * 3077.18 + 20^2 * 6619.82}{9697} = 281.00$$

which exceeds the Macaulay convexity of the liabilities, 242.47 (2)

The company should invest 3077 for the 5-year bond and 6620 for the 20-year bond.
Note that setting x = 0 produces y = 9231.54 and z = 465.46 and the convexity is 233.40, which is less than that of the liabilities. (1)

[5 Marks]

Solution 5: Correct answer (c)
Since investment return is same as discount rate no change in NPV. **[2 Marks]**

Solution 6: **Option C**
Probability of payout = ${}_{5.5}p_{65} = {}_{5}p_{65} \times {}_{0.5}p_{70}$
${}_{5}p_{65} = l_{70}/l_{65}$ = 9238.134/9647.797 = 0.957538; and
${}_{0.5}p_{70} = 1 - {}_{0.5}q_{70} = 1 - 0.5 \times {}_{1}q_{70}$ = 1 – 0.5 x 0.013605 = 0.993198
Therefore, probability of payout = 0.957538 x 0.993198 = 0.951025
Present value of payout is = 5000 x 0.951025 x $1.05^{(-5.5)}$ = 3636 **[4 Marks]**

Solution 7:

i) **Option B**

Benefit upon completion of 8 policy anniversaries = 100000 x $(1+3\%)^8$ = 126,677
Age of policyholder = 38
q_{38} = 90 % x 0.000813 = 0.000732
Reserve = 126,677 x $(0.000732/1.045 + (1-0.000732)/1.045^2)$
=116006 [4]

ii) The reserves will increase.

- Currently the effective discount rate is 4.5%. Under the revised regulations, the benefits will increase at 3% p.a. and be discounted at 6.5%, resulting in an effective discount rate of ~3.5%.

As effective discount rate is lower, the reserves will increase. [2]

[6 Marks]

Solution 8:

i) At the outset the investor has a negative cashflow [½]
In return the investor receives a series of regular interest payments linked to an index [½] reflecting the effects of inflation [½]. A final capital repayment [½], which is also linked to an index [½] and may be subjected to a lag. [2]

ii) An endowment assurance is a contract that provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1]
The benefits are provided in return for a series of regular premiums (or a single premium). [½]

The sum assured payable on death or survival need not be the same, although generally they often are. [½] The term of the contract could be fixed or up to certain age. [½] [2]

iii) An investor may lend an amount of INR 1 at time 0 in return for a repayment of (1+i) at time 1. In this case, i is the interest paid at the end of the year and hence considered to be the effective rate of interest per unit time. [1]

In case an investor lends and amount of (1-d) at time 0 in return for a payment of 1 at time 1. In this case d is called the effective rate of discount per unit time. [½]

The accumulation factor for 4% pa simple interest rate over 5 years is:

A(5) =1+5*0.04=1.2

[½]

The accumulation factor for effective interest rate over 5 years is:

A(5) = (1+i)^5

[½]

If the two rates are equivalent, then they result in the same accumulation factors:

(1+i)^5 = 1.2

⇨ 0.03714 [½] [3]

iv) An independent probability of decrements is a purely theoretical quantity that assumes that there are no other decrements operating. For example, qxd is the probability that a life aged x will leave the active population before time x+1 through the decrement d, where d is the only decrement operating.

[1½]

A dependent probability of decrements on the other hand takes into action the action of other decrements operating on the population as well. For example, aq_x^d is the probability that a life aged x will leave the active population before time x+1 through the decrement d, where all the other decrements are also operating.

[1½] [3]

[10 Marks]

Solution 9: Correct answer – A

The probability of a life aged x , who is currently sick, staying in the sick state for at least t years is given by:

$$ {}_t p_{40}^{\overline{SS}} = \exp\left(-\int_0^t \left(\rho_{x+s} + \nu_{x+s} \right) ds \right). $$

Since the transition intensities are assumed to be constant, the expression simplifies to:

$$ {}_t p_{40}^{\overline{SS}} = e^{-t(\rho+\nu)}. $$

The expected present value of the sickness benefit is then:

$$ 2,000\int_0^{20} e^{-\delta t} \ {}_t p_{40}^{\overline{SS}} \, dt = 2,000\int_0^{20} e^{-(\delta+\rho+\nu)t} \ dt $$

$$ = \left[-\frac{2,000}{\delta+\rho+\nu} e^{-(\delta+\rho+\nu)t} \right]_0^{20} $$

$$ = \frac{2,000}{\ln 1.04 + 0.05}\left[1 - e^{-20(\ln 1.04 + 0.05)} \right] $$

$$ = £18,652.72 $$

[3 Marks]

Solution 10: Correct answer (c)

The probability that, of two lives aged 40, one particular life dies first and the death occurs between 5 and 22 years from now (i.e. between age 45 and 62).

$$
\begin{aligned}
{}_{5|17}q^{1}_{40:40} &= \tfrac{1}{2} \times {}_{5|17}q_{40:40} \\
&= \tfrac{1}{2} \times \left[{}_5P_{40:40}(1 - {}_{17}P_{45:45}) \right] \\
&= \tfrac{1}{2} \left({}_5p_{40}^2 - {}_{22}p_{40}^2 \right)
\end{aligned}
$$

$$
\begin{aligned}
\text{and } {}_tp_x &= e^{-\int_0^t \mu_{x+s} ds} \\
&= e^{-\int_0^t 0.01 ds} \\
&= e^{-0.01t}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow {}_{5|17}q^{1}_{40:40} &= \tfrac{1}{2}\left[(e^{-0.05})^2 - (e^{-0.22})^2 \right] \\
&= \tfrac{1}{2}[0.9048374 - 0.6440364] \\
&= 0.13040
\end{aligned}
$$

Alternative solution:

$$
\int_5^{22} {}_tp_{40}\, {}_tp_{40} \mu_{40+t} dt
$$

$$
= \int_5^{22} e^{-0.02t} 0.01 dt
$$

$$
= \frac{1}{2}\left[e^{-0.02t} \right]_{22}^{5}
$$

$$
= \frac{1}{2}\left(e^{-0.1} - e^{-0.44} \right) = 0.13040
$$

[4 Marks]

Solution 11:

i) Without state relief:

Interest rate $i^{(12)} = 12\%$ p.a.

Let i be the effective monthly rate of interest

$i = 12\% / 12 = 1\%$ per month

Let monthly installment be Y

$35,00,000 = Y\, a_{\overline{240}|}^{@\, i}$

where $a_{\overline{240}|}^{@\, i} = 90.819416$

Y = 38,538 per month. (2)

<u>With state relief:</u>

i= 1% per month

$i_{yly} = (1.01)^{\wedge 12} - 1 = 12.68\%$

Present value of state relief:

Per year state relief = 2,50,000 / 5 = 50,000

PV = 50,000 $a_{\overline{5}|} @ i_{yly}$

Where, $a_{\overline{5}|} @ i_{yly}$ = 3.544650

Hence, PV = 1,77,233 (2)

Let monthly installment be X

35,00,000 = PV + X $a_{\overline{240}|} @ i$

X = 36,586.53 per month. (2)

[6]

ii) Total Savings if state relief is availed

= Y *240 – X *240

= 4,68,355 **[2]**

iii) Loan outstanding at the end of 3rd year:

= X $a_{\overline{204}|} @ i$ + 50,000 $a_{\overline{2}|} @ i_{yly}$

= 36,586.53 * $a_{\overline{204}|} @ i$ + 50,000 $a_{\overline{2}|} @ i_{yly}$

= 36,586.53* 86.864707 + 50,000 * 1.675015

= 32,61,829 (4)

Simple Int = 10% p.a.

Loan at the end of 4th year = 32,61,829 * (1.10)

= 35,88,013 (1)

Loan outstanding after state relief payment at the end of 4th year = 35,88,012 – 50,000

= 35,38,012 (1)

Loan at the end of 5th year = 35,38,012 * (1.10)

= 38,91,813 (1)

Loan outstanding after state relief payment at the end of 5th year = 38,91,813.08 – 50,000

= 38,41,813.08 (1)

Balance Term = 15 years = 180 months

Loan amount outstanding = 38,41,813.08

Let Revised EMI at the end of 5th year for balance term (i.e. after 24 months from pandemic) be Z

Hence 38,41,813.08 = Z * $a_{\overline{180}|} @ i$

Where,

(2)

$a_{180\rceil}^{@\,i}$ = 83.321664

Hence Z = 46,108.21 per month

[10]

iv) Total amount paid availing the moratorium benefit:

= 46,108.21 *180 + 36,586.53 * 36

= 96,16,592.88

Total amount paid without availing the moratorium benefit:

= 36,586.53 * 240

= 87,80,767.20

Additional amount paid

= 96,16,592.88 - 87,80,767.20

= 8,35,825.68

[2]

[20 Marks]

Solution 12:

Decrement Table

Year	Lx	qx	wx	No. of deaths	No. of surrenders	No. of maturity	No. survivors
1	1	0.003358	10%	0.003358	0.0996642		0.8969778
2	0.896978	0.004903	5%	0.00439788	0.044629		0.847951
3	0.847951	0.00565	5%	0.00479092	0.042158		0.801002
4	0.801002	0.006352	5%	0.00508796		0.79591404	0

Unit Fund

Year	Premium	Allocated premium	Bid offer Spread	Fund before deduction	Pol Admin charge	Fund after deduction	Growth rate	FmC	Fund after growth and FMC
1	5,500.00	5,362.50	5,094.38	5,094.38	60.00	5,034.38	4%	52.36	5,183.40
2	5,500.00	5,362.50	5,094.38	10,277.78	60.00	10,217.78	3.50%	105.75	10,469.65
3	5,500.00	5,362.50	5,094.38	15,564.03	60.00	15,504.03	3.50%	160.47	15,886.20
4	5,500.00	5,362.50	5,094.38	20,980.58	60.00	20,920.58	3.50%	216.53	21,436.27

Non Unit Fund

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non-Unit Investment Income	Profit
1	405.62	60.00	265	275.00	4.69	-89.7	0	52.36	-1.86	61.13
2	405.62	60.00	75	137.50	13.32	-44.8	0	105.75	6.33	396.68
3	405.62	60.00	0	137.50	23.00	-44.7	0	160.47	8.20	518.49
4	405.62	60.00	0	137.50	34.68	-	59.62	216.53	8.20	458.56

i)

a)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	1395.85	0	0	52.36	-1.86	-1,419.73

(3)

b)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	0	-900	0	52.36	-1.86	876.12

(2)

c)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	0	0	0	52.36	-1.86	-23.88

(2)

ii)

a)

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
2	405.62	60	75	137.5	0	-	0	105.75	6.33	365.20

(1.5)

b)

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
2	405.62	60	75	137.5	0	0	0	105.75	6.33	365.20

(1.5)

iii)

Profit vector per policy	Profit signature	NPV@5%
		1,062.94
61.13	61.14	1054.95
396.68	355.82	751.88
518.49	439.65	349.82
458.56	367.31	-

(8)

[18 Marks]

Solution 13: Age = 40. Policy term = 30.

i)

Based on principle of equivalence,
PV Premium = PV Claims + PV Expenses + PV Commissions

(0.5)

PV Premium = P x $a^{due}_{40:30}$

$a^{due}_{40:30} = a^{due}_{40} – v^{30}$ x l_{70}/l_{40} a^{due}_{70} = 20.005 – 1.04^{-30} x 8054.0544/ 9856.286 x 10.375 =17.3911

PV Premium = 17.3911x P

(2)

PV Commissions

P x (0.15 + v x 0.03 x l_{41} /l_{40} x $a^{due}_{41:29}$)

$a^{due}_{41:29} = a^{due}_{41} – v^{29}$ x l_{70}/l_{41} a^{due}_{70} = 19.784– 1.04^{-29} x 8054.0544/ 9847.051x 10.375 =17.06299

l_{41} /l_{40} = 9847.051/ 9856.286= 0.999063

So PV Commission = 0.641741 x P

(2.5)

PV Initial Expenses = 2000 + 0.2 x P

(1)

PV Renewal Expense = 500 x v x l_{41} /l_{40} x $a^{due}_{41:29}$ = 500 x 16.39135= 8195.676 (based on calculations performed for PV Commissions already)

(1)

PV Claims = (5,000,000 + 12 x 0.01 x 5,000,000 x $a^{(12)}_{10}$) x $A^{1}_{40:30}$ (continuous)

(1)

$a^{(12)}_{10} = (1- v^{10}) / i^{(12)} = 8.258599$

(1)

$A^1_{40:30} = A_{40} - A_{70} \times v^{30} \times l_{70}/l_{40} = 0.23056 – 0.60097 \times 1.04^{-30} \times 8054.0544/ 9856.286 = 0.07915$

$A^1_{40:30}{}^{(continuous)} = 1.04^{0.5} \times 0.07915= 0.080718$

(1)

PV Claims = (5,000,000 + 12 x 0.01 x 5,000,000 x 8.258599) x 0.080718
= 803,561

(1)

PV Claim expenses = 0.5% x PV Claims = 0.005 x 803561 = 4017.8

(1)

So based on principle of equivalence we have,
17.3911 x P = 0.641741 x P + 2000 + 0.2 x P + 8195.676 + 803561 + 4017.8
16.54936 x P = 817774
P = 49414

[12]

ii) Under this variant,

PV Claims = (5,000,000) x $A^1_{40:59}{}^{(continuous)}$

(0.5)

$A^1_{40:59} = A_{40} – A_{99} \times v^{59} \times l_{99}/l_{40} = 0.23056 – 0.90139 \times (1.04)^{(-59)} \times 143.712 / 9856.286= 0.229261$

$A^1_{40:59}{}^{(continuous)} = 1.04^{0.5} \times 0.229261 = 0.233801$

(1.5)

PV Claims = 1,169,005

(0.5)

PV claim expenses = 0.005 x 1,169,005 = 5845.023

(0.5)

So, using elements from premium equation in part a)
16.54936 x P = 2000 + 8195.676 + 1,169,005 + 5845.023 = 1,185,045
P = 71,606.7

(1)

[4]

iii)

a) Premium payable will be higher. This is because company is able to earn higher investment income when full year's premium is received at start of year rather than spread over the year. (1)

b) Premium payable will be higher. Under whole of life, probability of claim payment will increase resulting in higher expected claims and claims expenses. (1)

[18 Marks]

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