Institute of Actuaries of India

Subject ST8 – General Insurance: Pricing

March 2018 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

We need to generate a value for the number of claims. Using the distribution method, the probabilities for a Poisson (2) distribution are:

P(0) = exp(-2) = 0.1353

P(1) = exp(-2)*2^1/1! = 0.2707

P(2) = exp(-2)*2^2/2! = 0.2707

P(3) = exp(-2)*2^3/3! = 0.1804 and so on.

If we generate values from U(0,1) distribution, we find the corresponding values from a Poisson(2) distribution by assigning:

Values between 0 and 0.1353 to the Poisson value Zero

Values between 0.1353 and 0.1353+0.2707 to the value 1

Values between 0.1353+0.2707 and 0.1353+0.2707+0.2707 to the value 2

And so on.

The first random number 0.27 corresponds to a Poisson value of 1. So we assume that we have 1 claim.

To find the simulated claim amounts we invert the distribution function of the Pareto distribution:

$$F(x) = 1 - \left(\frac{\lambda}{\lambda + \mathrm{x}}\right)^{\alpha} = u$$

i.e. $x = \frac{\lambda}{(1-u)^{1/\alpha}} - \lambda = \frac{1000}{(1-u)^{1/5}} - 1000$

Substituting in u = 0.82, we get a claim amount of 409.11. Applying the access, we have a net payment of 309.11.

[3 Marks]

Solution 2:

(i) Restrictions on the type of business that a general insurer can write or classes for which the insurer is authorized
(ii) Limits or Controls on the premium rates that can be charged
(iii) Restrictions on the information that can be used in underwriting and premium rating
(iv) A requirement to deposit assets to back claims reserves
(v) A requirement that the insurer maintains a minimum level of solvency, measured in some prescribed manner

(vi) Restrictions on the type of assets or the amount of particular asset that a general insurer can take into account for the purpose of demonstrating solvency

(vii) A requirement to use prescribed bases for calculating premiums or for valuing general insurer's assets and/or liabilities when demonstrating solvency

(viii) Restrictions on individuals holding key roles in the Company

[4 Marks]

Solution 3:

i) Advantages of the burning cost approach include:

- Simplicity
- Needs relatively little data.
- Quicker than other methods to perform.
- Allows for experience of individual risks or portfolios.

Disadvantages of the burning cost approach include:

- Harder to spot trends so it provides less understanding of changes impacting the individual risks.
- Adjusting past data is difficult.
- Adjusting for changes in cover, deductibles and so on may be difficult as we often lack individual claims data.
- It can be a very crude approach.

[2]

ii) Assumptions:

- Policies are written evenly over the period and the risk is constant during the policy period
- Claims occur half-way through the policy period
- Inflation rate of 6% is applicable for earlier and for all future years as well.
- Business mix is similar for all the years

Burning Cost Approach

First we need to adjust the claims to 1/1/2018 level. Further, assuming that the new exposure period runs from 1/5/2018 to 30/4/2019, we need to adjust the claims to 1/11/2018.

Start Date	Mid Period of exposure	Months to 1/11/2018 from mid-point of exposure	Claim Amounts	Inflation adjusted factor	Inflation adjusted claims
01/05/2013	01/11/2013	60	48	$1.06^5 = 1.3382$	64.23
01/05/2014	01/11/2014	48	45	$1.06^4 = 1.2625$	56.81
01/05/2015	01/11/2015	36	105	$1.06^3 = 1.191$	125.06
01/05/2016	01/11/2016	24	56	$1.06^2 = 1.1236$	62.92

01/05/2017	01/09/2017	14	24	$1.06^{14/12} = 1.0703$	25.69
Total			278		334.71

Since the premium rate to be calculated has to be per 1000 of Sum Insured, we can consider Sum Insured as the exposure measure

So, burning cost premium = (334.71 + 23)/18663 = 19.1668 per 1000 Sum Insured

[3]

iii) Before finalizing the premium, we need to determine whether there are any large losses in the historical data.
Considering the given data:

Start Date	Average Claim Amount
01/05/2013	24,194
01/05/2014	21,307
01/05/2015	43,050
01/05/2016	25,000
01/05/2017	25,974
Total	28,663

There is a clear indication of huge jump in the average claim in year 2015. It might be due to a few large claims in 2015 or data error.
Hence, the data should be double checked and possibly apply some sort of smoothing to the claims or cap any of the extremely large claims.

Other factors to consider:

- No information has been provided on the type of cover in each year within the experience. It is important to identify whether the cover in 2014 will differ from previous years
- It may be inappropriate to rely solely on the experience of one Car Rental, especially if it is not credible and therefore, some combination between book rates and experience rates should be considered.
- Need to determine whether there has been any change in the car rental market e.g. types of drivers and types of cars driven
- Need to consider any expected future external events such as changes in legislation that may impact claims costs, expenses, and commission or profit allowances.
- Determining whether the policy will be reinsured and whether reinsurance loadings should be included.
- Competitors' quotes and assumptions made by competitors. [3]

[8 Marks]

Solution 4:

i) Investigations for difference in average cost per claim

- Type of customers written by Agents as compared to the brokers
- Investigate about vehicle details like
  - Average Sum Insured,
  - Age of Vehicle,
  - Local make vs imported vehicles etc.
- Analyze Large and attritional claims separately
- Analyzing if there are any difference in the policies w.r.t. to level of excess/deductibles
- Comparison of Attachment rate of various Add-on covers sold by both distribution channels. Few Add-on covers are likely to increase the average cost per claim like Zero depreciation cover.
- ALAE and ULAE levels for both distribution channels if any
- Use of similar or different types of garages i.e. Company tied-up garages vs Others
- Use of similar or different loss assessors i.e. inhouse surveyors or external surveyors
- At what level vehicles are written off and what are the Salvage agreements?

[4]

ii) Further investigations:

- Combined ratio = Loss Ratio + Expense ratio. If loss ratios are as expected, investigate the level of actual expense ratio compared to what was expected
- This can be done by splitting expenses into fixed or variable and direct vs. indirect
- Though, Loss ratio of individual channels are as expected, the overall loss ratio may be high due to variation in expected business volume from both channels
- Check for any error in the calculation of expense ratio, loss ratio and hence combined ratio
- The company might have entered into one-off big deal having higher initial expenses which might taper off as the business grow in future
- Given the nature of the business, the combined ratio targets might be unrealistically low as compared to the rest of the industry. Investigate the combined ratio of other players in the market.

[5]

[9 Marks]

Solution 5:

i) Benefits: Loss, not exceeding the Limit of Indemnity, sustained or incurred by the insured due to defect in the Title.

Insured perils:

- any defect in or Lien on the Title
- forgery, fraud, undue influence, duress, incompetency, incapacity or impersonation
- failure of any person or Entity to have authorized a transfer or conveyance;
- a document affecting Title not properly created, executed, witnessed, sealed, stamped, acknowledged, notarized or delivered
- a document affecting Title executed under a falsified, expired or otherwise invalid power of attorney

Basis for cover: Losses occurring

Measures of exposures to which premiums are related

- SI limit
- Sales value of the property units or Land Cost+ Cost of Construction + Profits

Claim characteristics:

- Frequency is normally very low as the insurance is normally provided after a thorough Title search. Frequency may rise because of defalcation of the Title Servicing Agencies.
- Large ticket claim size limited to the SI Limit
- Faster claim reporting as most of the consumers will be well educated and aware of insurance
- Claim settlement may be slower due to court and other legal processes.
- Loss assessment may take some time and effort due to the nature of risks and time consuming legal processes.

Risk factors and rating factors:

- Jurisdiction
- Land recording or Land registration systems
- Age of the property project
- Title search documents available

[8]

ii) The reinsurance arrangements which would be suitable for this product are

- Excess of Loss Treaty with suitable limits and deductibles. Protects against large losses. The limit of the XL treaty can be set at the max SI limit offered in the product.
- Quota Share. This will be suitable when the deductible under the Excess of Loss treaty seems to be too high in respect of the average ticket size of the business. The target market of the insurance company may be all small real estate projects located in the smaller cities or sub-urban areas.
- Stop Loss. This may be purchased along with quota share treaty subject to comfortable terms and conditions. The stop loss may be used to protect further the retention under the quota share treaty providing adequate balance sheet protection.

[2]

[10 Marks]

Solution 6:

i) Curves should represent the expected indemnity cost at the various limits. Cost is made up of frequency as well as severity. So, if we assume that the underlying frequency of claims is independent of both the severity and the limit, then we can ignore frequency, and take the curve just to represent just the limited expected severity. [1]

ii) If we do not use exposure curves then we would require a separate loss distribution for each SI limit for a risk, which is not pragmatic. The exposure curve solves this problem with the assumption that certain homogeneous risks, the distribution of loss as a percentage of SI or MPL (Maximum Probable Loss) is independent of the limit/SI. [1]

iii)

a) In standard exposure curves, the y – axis is always represented by the percentage of pure expected claim costs whereas in this case, it is the proportion of total number of claims. With this curve, we can always derive the standard exposure curve but the other way around is difficult.

The benefits of having such exposure curves are

- carrying out frequency and severity approach for pricing layers beyond a certain threshold which further help understanding the underlying factors driving a premium.

- Helps in deciding certain terms and conditions of the RI contract e.g. reinstatement premiums, which requires a better understanding of number of claims expected to hit above a certain threshold limit. [3]

b) Assumptions

- Average Loss as a percentage of the SI = E(X) = 10% is constant across all the SI bands.
- Loss ratio of 60% is constant across all the SI bands.

Answer in Column G in table below

Band Id	Lower Band	Upper Band	No. of risks	Premium	Total Sum Insured	Total Losses (B*60%)	Avg. SI	Avg Loss Size(E * E(x))	No. of losses (D/F)
			A	B	C	D	E	F	G
1	0	1,00,00,000	5000	5,00,00,000	30,00,00,00,000	30000000	60,00,000	6,00,000	50
2	1,00,00,000	5,00,00,000	1000	5,00,00,000	35,00,00,00,000	30000000	3,50,00,000	35,00,000	9
3	5,00,00,000	10,00,00,000	500	40,00,00,000	32,50,00,00,000	240000000	6,50,00,000	65,00,000	37
4	25,00,00,000	50,00,00,000	250	50,00,00,000	92,50,00,00,000	300000000	37,00,00,000	3,70,00,000	8
5	50,00,00,000	1,00,00,00,000	50	45,00,00,000	27,50,00,00,000	270000000	55,00,00,000	5,50,00,000	5

[2]

c) Answer in Column K in table below

Band Id	Avg. SI	x= (100000000/E)*100	Prop of losses exceeding 10 Cr(if H>100 then 0 else [1-(1-exp(-0.1*H)]	No. of losses (D/F)	No. of large losses exceeding threshold 10 Cr. (I * G)	Total (Sum of all values in column J)
	E	H	I	G	J	K
1	60,00,000	1,666.67	0	50	-	1.34
2	3,50,00,000	285.71	0	9	-	
3	6,50,00,000	153.85	0	37	-	
4	37,00,00,000	27.03	0.07	8	0.54	
5	55,00,00,000	18.18	0.16	5	0.80	

[4]

[11 Marks]

Solution 7:

i) Splitting data into training and test groups prevents overfitting of the model because the model will always fit the training better with more parameters but it also could pick up random variation as a predictive variable.

It also allows for the testing of the predictive power of the model, if it doesn't fit the test data well it likely won't predict future outcomes well either. [1]

ii) The Gini index measures the ability of the rating plan to differentiate between the best and worst risks, i.e. lift of the insurance plan.
In insurance rating, large Gini index represents stronger risk classification power.
It is a measure of the predictive lift of the rating program. Higher Gini index equals more lift and more predictive power.

[2]

iii)

- Collect the data and input that into the model in appropriate format
- Specify time period for analysis
- Link exposure and claims data
- Identify any anomalies or errors in the data
- Identify any CAT claims or one-off large claims. These claims can be either removed or capped
- If working on net claims, allow for gross reinsurance costs in the pricing elsewhere
- Modelling can be on ground up loss or can be on claims net of excesses, salvages and recoveries other than reinsurance
- Ideally, Frequency and Severity should be modelled separately
- Poisson for frequency – need to check the level of underlying frequency level for Poisson to be appropriate
- Gamma for severity
- Check goodness of fit
- If separate model for frequency and severity are not possible, risk premium can be modelled using gamma or tweedie distn.
- Assign appropriate link function to each GLM
- Identify rating factors for modelling
- Test each factor using various tests like:
    - statistical tests like chi-squared,
    - consistency check – across time
    - reasonability checks
- Test rating factor interaction with various factors for significance
- Allow for capped and removed claims, may be by adding flat margin on base premium
- Model validation checks on final model
- Compare actual claims experience against predicted using training data
- The model should be allowed for future trends e.g. inflation
- Claims must be fully developed to ultimate/adjust for inflation
- Adjust historical claims to today's values for inflation
- Other modelling considerations like treatment of Nil claims, ULAE, ALAE etc.

[10]

[13 Marks]

Solution 8:

i) Pricing Challenges:

- The biggest challenge in pricing this cover is the availability of the data. Though, the Company might be having credible and reliable data for the similar product, but the existing product is sold at the time of sale of the phone.
- The lack of data will prevent the insurer from holding any data back to use for validation purpose, which could result in the model "over-fitting" of the data.
- There shall be high uncertainty in the incidence rate and average claim size as compared to existing product.
- There might be genuine people who want to buy this product after the day of buying the phone, but there is a large risk of Moral hazard in this product, as the people may try to buy the policy after the loss to the phone.
- This would be a low premium size product, hence not feasible to inspect every phone before giving the cover.
- If the Company decides to charge higher premium to cover the risk of Moral hazard, the cover may become very expensive and there would be higher risk of anti-selection.
- Difficulty in valuing phone after purchase, hence the level of indemnification.
- Uncertainty in the level of expenses, hence calculating true office premium
- Assumptions of volume and mix will also be particularly difficult to predict.

[5]

ii) Mitigation:

- Though it might be expensive, the product should be sold only after inspection. The cost can be reduced by selling through mobile phone sellers and taking photos of the phone underwritten.
- The cover provided may be limited such as only Theft and physical damage and excludes any kind of coverage for software malfunction.
- The level of excess can be increased.
- In case of Theft or total loss cases, the cover may be up to a fixed percentage of total sum insured say, 70% rather than paying full sum insured.
- Technologically, the insurance should be sold through Mobile App installed on the phone to be insured. This is to ensure that the cover is not bought for a stolen phone. The App should be able to read serial no. or IMEI number of the phone with policyholder's consent. [3]

iii) Personal Accident exclusion

+ While pricing the original product, these types of claims were not envisaged, hence these are latent claims and can be excluded per current pricing
+ It will work as deterrent for policyholders to get into these kinds of activities (as claim will not be payable) and thus doing social good.

- It will lead to fall in business volume if other insurers are not excluding the same
- It will lead to policyholder grievance as it shall be very difficult to prove that the accident happened while using the phone. Even if it is proved that the policyholder was using phone at the time of accident, it is difficult to prove that the accident happened due to that.
- The regulator may not allow this
- It would be administratively difficult to keep track of the policies with different conditions
- There is a risk of mis-selling
- Competitors may use this to advertise against the company and it shall bring bad repute to the company

[5]

[13 Marks]

Solution 9:

i)

Premium rate is a measure of how profitable a policy or segment of business is expected to be. Premium rate change therefore indicates the direction of a company in terms of profitability.

[1]

ii)

a) Some examples of definitions of premium rate are

- Premium income per unit of expected loss
- Premium per unit of limit
- Premium per unit of exposure
- Premium per unit of risk-adjusted exposure

[2]

b) Such risks are exposed to large losses. They have a limit and exposure details are also captured at the policy level.

Therefore Premium per unit of limit or Premium per unit of exposure will be suitable as premium rate definitions.

[2]

iii)

a) Premium rate changes can be calculated in the following ways

- direct calculation for each risk separately
- direct calculation using a standard risk
- measuring rate changes on individual renewals
- using underwriters views

[2]

b) Assumptions: The ILF table remains the same in 2016 as in 2015 and no inflation.

Premium rate per unit of expected loss is suitable here as the limits (deductibles, SI limits) and the exposures (coverages) are varying from 2015 to 2016.

Individual policy details are not available to assess the expected loss. Hence ILF table will be used to assess the expected loss.

Let x be the loss at the base limit from the ILF table.

Premium rate per unit of expected loss in **2015** = M/N

Where M= Pure Premium of Group A policies+ Pure Premium of Group B policies+ Pure Premium of Group C policies

And N= Expected Loss of Group A policies + Expected Loss of Group B policies+ Expected Loss of Group C policies

M= 20* (1500+ 1300 + 1000) = 76000

N= 20 *x *[(ILF@20- ILF@1) + (ILF@20- ILF@2) + (ILF@20- ILF@5)]

= 20*x* [3.058-1 + 3.058- 1.432 + 3.058- 2.092]

= 93x

= > M/N = 817.2043/x

Premium rate per unit of expected loss in **2016**= X/Y

Where X= Pure Premium of Group A policies+ Pure Premium of Group B policies+ Pure Premium of Group C policies

And Y= Expected Loss of Group A policies + Expected Loss of Group B policies+ Expected Loss of Group C policies

X= 20* (1450+ 1250 + 2000) = 94000

Y= 20 *x *[(ILF@20- ILF@2) + (ILF@20- ILF@5) + (ILF@30- ILF@5)]

= 20*x* [3.058-1.432 + 3.058- 2.092 + 3.288- 2.092]

= 75.76x

= > X/Y = 1240.76/x

Therefore the premium rate change = 1240.76/ 817.2043 – 1 = 0.5183. In other words, the premium rate has increased by 51.83% indicating that the profitability will rise in 2016 from the level in 2015.

[5]

iv) The premium rate in 2015 as calculated above will remain the same.

The premium rates for combined 2016 and 2017 has to be calculated.

Since the ILF table remains the same, the limit L1 at 2016 will be equivalent to L1/(1+0.05) and be applied on the ILF table.

Similarly the limit L2 at 2017 will be equivalent to L2/[(1+0.05)^2] and be applied on the ILF table.

Therefore the combined premium rate = W/Z

W= 20 * 2 *(1500+ 1300+ 1000) = 152000

Z = (Expected Loss of Group A policies + Expected Loss of Group B policies+ Expected Loss of Group C policies) @ 2016 + (Expected Loss of Group A policies + Expected Loss of Group B policies+ Expected Loss of Group C policies)@ 2017.

[2]

[14 Marks]

Solution 10:

i)

a) Within excess of loss reinsurance, reinstatements are the restoration of full cover following a claim. Normally, the number of reinstatements, and the terms upon which they are made, will be agreed at the outset. Once agreed, they are automatic and obligatory on both parties. [1]

b) There are primarily two types:

Free reinstatements- Reinstatements can be continually be made at no cost.

Paid reinstatement- Reinstatement premium must be paid before the reinstatements go ahead

The purpose of reinstatements is to limit the maximum exposure and limit the total cover offered under a reinsurance contract.

[2]

ii)

	FGU Loss(Cr)	Recovery (Cr)	Reinstatement Premium (Cr)	Cover Reinstated (Cr)
1st Loss	400	200	10 (10=50%*20)	200
2nd Loss	300	100	10 (10= 50%*100%*20)	100
3rd Loss	300	100	10 (10= 50%*100%*20)	100
4th Loss	500	200	0	0

5th Loss	100	0	0	0
Total	1600	600	30	400

[4]

iii)

Let P be the new XL premium with two paid reinstatements.

Probability of no CAT loss in a year = exp(-0.1)=0.904837

Probability of one CAT loss in a year = exp(-0.1)* 0.1 = 0.0904837

Probability of atleast two CAT losses in a year = 1- 0.904837- 0.0904837= 0.004679

Therefore, XL Premium of Rs. 40 Cr with two free reinstatements should be equivalent to

P + 0.5* P *0.0904837 + (0.5*P + P)* 0.004679 ……..ignoring the timing of the payments.

= > 40 = 1.05226035 * P

= > P= 0.95 * 40 Cr

Therefore, a discount of 5% should be given to Rs. 40 Cr for the CAT Excess of Loss Treaty with the two paid reinstatements. [5]

iv) The circumstances under which a reinsured should choose paid reinstatements instead of free reinstatements

- The chances of exhaustion of the cover limits are negligible
- Some discount can be achieved on XL premium with free reinstatements
- Immediate cashflow crunch to pay off an immediate higher XL premium
- Terms of paid reinstatements are suitable in terms of cost of the reinstatements.

[3]

[15 Marks]

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