Institute of Actuaries of India

Subject ST6 – Finance and Investment B

March 2018 Examination

INDICATIVE SOLUTION

Solution 1:

i) The bond book can be described by the following terms and conditions

Mortgage Duration	4 years	
Prepayment %	5%	per annum only allowed on payment date
Default	4%	per annum
Payment		annual
Interest rate	5%	
Interest payment payment	10%	annual simple compounding
Amount	1000	
Default recovery	30%	
Prepayment penaty	5%	
Admin Expenses % of principal outstanding	2%	Per annual
Mark up	5%	

Using the following assumption

1 Default also happens at end of the year
2 Default before the prepayment
3 Nearest 1000

We get the following cashflows

Year	**Principal**	**Capital outstanding start of the year**	**Capital outstanding end of the year**	**Interest**	**Default**	**Prepayment**	**Default recovery**	**Cashflow**
0	-1000			0	0	0	0	
1	0	1000	910	100	40	**50**	12	162
2	0	910	828	91	36	**46**	11	148
3	0	828	753	82.8	33	**42**	10	135
4	685	753	685	75.3	30	**38**	9	807

Using the discount factors calculated using 5% risk free curve the following table proves the value of the bonds and internal rate of return can be estimated using the principal amount and estimating the IRR

Year	Discount factor	Cashflows
		7.22%
Value	1,069	-1,000
1	0.952380952	162
2	0.907029478	148
3	0.863837599	135
4	0.822702475	807

Allowing for the distress sale and expenses we get the following updated cashflows.

	Principal	Capital outstanding start of the year	Capital outstanding end of the year	Interest	Default	Prepayment	Default recovery	Servicing Expense	Cashflow	Discount factor	Cashflows
											5.22%
0	-1000			0	0	0	0			866.19	-1,000
1	0	1000	910	100	40	**50**	12	20	142	0.91	142
2	0	910	828	91	36	**46**	11	18	130	0.83	130
3	0	828	753	82.8	33	**42**	10	17	118	0.75	118
4	685	753	685	75.3	30	**38**	9	15	792	0.68	792

The company incur loss of 203 cr

With return of 5.22%

[8+5+7=20 Marks]

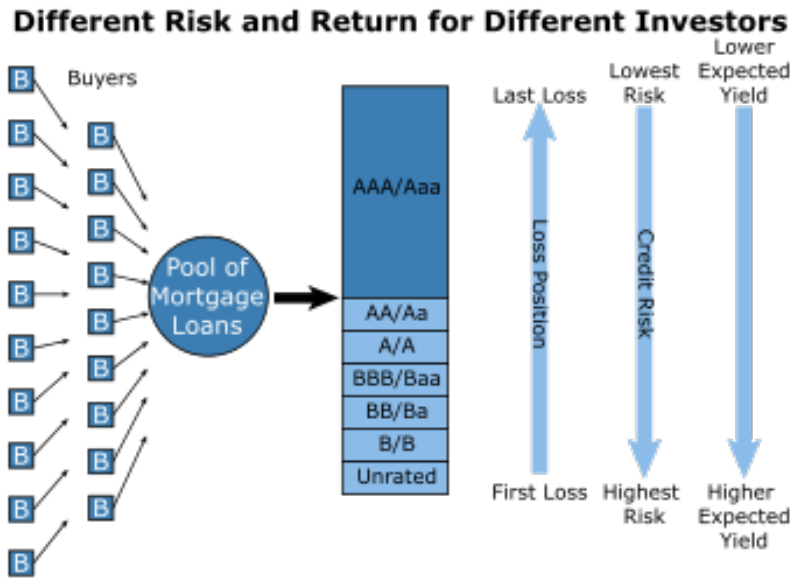
Solution 2:

The company can sell the mortgage book which would mean finding buyer in the market using various investment banks. The sale would be considered at distress hence the company is expect to receive lower valuation than the market price of these mortgages.

The securitization is to package the mortgages in SPV and issue rates notes which would serviced by the interest and prepayment from the mortgages. The SPV route would preserve the market value of the mortgages, would provide the required cashflow. Though, creating and managing SPV would required administrative expenses and company retain the equity proportion of the SPV which would be higher risk part of the model.

Structure of SPV

The structure of SPV is given in the picture below



There would be need to provide liquidity facility for the SPV for the payment of the notes. There would be equity tranche or unrated which need to be held by the company.

The legal requirement would be SPV to be separate entity compared with the company who has written the mortgages such that they do not have any recourse on the cashflows of SPV other than the equity tranche.

There would be requirement to have to have someone administrating the mortgages to provide the required cashflows for paying out the loan. The rating of the notes would depend on the risk associated with the notes.

[4+6=10 Marks]

Solution 3:

Entering into an OTC derivatives transaction almost always involves having a credit exposure to one's counterparty. Credit exposure on a derivatives transaction varies over time as the variables that drive the value of the underlying contract change.

In assessing credit risk, one must look at two issues:

1. The replacement cost of the derivative if the counterparty were to default immediately and

2. The potential size of the replacement cost if the counterparty were to default at some future time during the life of the derivative contract.

The first of these issues is simply the current mark-to-market value of the derivative contract which can be either positive or negative. In relation to the second issue, one needs to know what the replacement cost of the derivative transaction might be in the future if the variables that determine the value of the underlying contract were to move adversely.

[5 Marks]

Solution 4:

Forward Rate swap

The fixed swap rate that is associated with a forward settlement. If the yield curve is upward sloping, this rate is higher than a spot delivery swap rate. If the curve is downward sloping, the forward swap rate is lower than a spot delivery swap rate. Theoretically, this rate can be determined by two relevant spot swap rates and two relevant zero rates. The following formula illustrates this:

$$S = \left[\frac{(1+S_{A+B})^{A+B}}{(1+S_A)^A}\right]^{(1/B)} - 1$$

Calcuation for forward swap rate		
R7	7	6.75%
R4	4	6%
Using the above formula		
R 4-7	7.76%	

What is a swaption

A swaption is an option granting its owner the right but not the obligation to enter into an underlying swap. Although options can be traded on a variety of swaps, the term "swaption" typically refers to options on interest rate swaps

	Annual compounding						
Time	1.0	2.0	3.0	4.0	5.0	6.0	7.0
curve	6%	6%	6%	6%	6.25%	6.50%	6.75%

Using the forward rate from before and exposure of 100 cr we get the following swaption value with the calculation breakdown

			LIBOR yield	Discount
Strike Swap Rate (s_K)	0.0700	**Time**	**Curve**	**Factor**
Forward Swap Rate Continuous Compounding	0.0600	4.0	6.00%	0.000000
Forward Swap Rate *m*-Annual Compounding (s_0)	0.0776	5.0	6.25%	0.731616
Volatility of Forward Swap Rate (σ)	0.20	6.0	6.50%	0.677057
Notional Principal (*L*)	100	7.0	6.75%	0.623442
Number of Payments per year (*m*)	1			
Sum of Discount Factors (*A*)	2.0321			
Option Maturity (*T*)	4			
Payer Swaption (*P*) or Receiver Swaption (*R*)	P			
Swaption Price	3.22			
d1	0.46			
D2	0.06			
Swaption Price	3.224495			

[5+4+6=15 Marks]

Solution 5:

i)

a) Cross hedging is the practice of hedging exposure to the price of one asset (in this case rocket fuel) using futures with a different underlying asset (in this case ATF).

[1.5]

b) The hedge ratio is the ratio of the position taken in futures to the size of the exposure.

[1.5]

c) The minimum variance hedge ratio is the hedge ratio that minimizes the variance of the hedger's net position.

[1.5]

d) Tailing the hedge is a small adjustment to make to the hedge to allow for how futures (with daily settlements and delta ≠ 1 rather than forward deals with delta = 1) are being used as hedging instruments.

[1.5]

ii) Minimum variance hedge ratio h, is given by

$$h = \frac{\rho \sigma_S}{\sigma_F}$$

where, *h = 0.75 *(4.2/2.5) = 1.26.*

Therefore, the number of futures contracts to buy, before tailing the hedge, is
1.26 * (100000 / 1000) = 126.

Thus, the hedge can be tailed by
126 * (101.20/106.30) = 119.95 or approximately 120 ATF futures contracts.

[5]

iii) Let *r* be the risk free rate and *u* be the rate of storage costs. Then, the *t – year* futures price is given by

$$F_t \leq Se^{(r+u)t}.$$

If this is not satisfied, investors could make arbitrage profits by borrowing cash, buying oil and shorting oil futures.

However, in certain circumstances where physical storage capacity (or indeed borrowing capability) is limited, prices can remain distorted despite this arbitrage occurring.

One cannot form a no arbitrage argument from the opposite side of the arbitrage and prove

$$F_t = Se^{(r+u)t}$$

This is because,

1. There are not enough investors holding crude oil as an investment (and ready to take arbitrage profits by selling the oil and buying futures)
2. There are benefits in holding the oil physically, meaning that people holding the oil physically benefit from the "convenience yield"

Thus, futures prices can only give us a lower bound on crude oil storage costs of

$$u \geq \frac{1}{t} ln\left(\frac{F_t}{S}\right) - r.$$

[4]

[15 Marks]

Solution 6:

i) Value – at – Risk (VaR) is the expected loss on a portfolio from an adverse movement with a specified probability over a particular period of time.

A 99% one-day VaR means that there is just a 1% probability of experiencing an adverse change in the portfolio over one day in excess of the calculated amount.

[2]

ii) VaR expects that the underlying distribution is normal.

So,

- 95% tail corresponds to an *x*-axis value of 1.645, while
- 99% tail corresponds to an *x*-axis value of 2.326

Thus, taking a 95% confidence level, the VaR decreases by a factor of 1.645 / 2.326 = 0.7072.

VaR is also proportional to the square root of the time interval. Thus, taking a 5 – day period instead of a 1 – day period, the VaR increases by a factor of √5 = 2.2361.

Thus, the total effect is an increase in VaR by a factor of (2.2361 * 0.7072) = 1.5814.

[2]

iii) The three methods of calculating VaR are:

a) Parametric (Variance-Covariance):

This method assesses the VaR directly from the assumption of normality of price changes in all the constituent risk factors.

A linear combination of multiple normal distributions is also normal, so the loss distribution of the entire portfolio can be derived from a combination of underlying correlated values.

Correlations are estimated from past benchmark data and compiled into a large matrix.

The Parametric method has the advantage that VaR can be calculated quickly and simply provided there are not too many factors (i.e. matrix is not too large).

It uses only a normal distribution and hence it cannot cope with non-linear effects (“fat tails”).

It would typically suit a small linear portfolio of relatively few normally distributed instruments.

b) Historical Simulation:

This method assesses the loss distribution of the portfolio based on a set of actual historical scenarios from the recent past.

It re-runs the portfolio valuation for each day and creates a distribution of scenario outcomes.

VaR is then simply the 99% percentile of these (or whatever confidence interval).

The Historical Simulation method is good for non-linear instruments with complex interactions; but the revaluation can be complicated and new instruments have no past history.

It would suit a reasonably large portfolio of mixed instruments.

c) Monte Carlo Simulation:
This method involves modelling future price returns of the portfolio directly and then running many hypothetical trials to obtain a distribution of portfolio losses.

As with Historical Simulation, VaR is the relevant (e.g. 99%) percentile of the distribution.

This method is the most complex to apply, as future price movements and correlations have to be modelled. However more complicated but better fitting distributions other than the normal can be used, which enables better modelling of "fat tails".

This method is well suited to portfolios of complex options using the simulation-based models that are already set up for pricing the options.

[6]

iv)

a) The main problem with VaR is that

- It is backward looking.
- Past history does not predict future very accurately and, in this case, low volatility has not helped anticipate the emerging currency risk.
- VaR is supposed to model the tail of the distribution; however, most of the values are not in the tail, hence allowing for fat tails is difficult because by their nature they occur less frequently. Further, there is no information in the VaR statistic to indicate the expected severity of losses outside the confidence interval.
- VaR is not always well tailored to a portfolio if the risk factors used are generic, so that the price movements that drive P&L are badly matched.
- It is not clear how VaR and credit default can be linked, which could affect the specific situation in the question.

[2.5]

b) Stress tests could be the most useful additional risk measures as

- They give a forward-looking component and do not rely on past volatilities and correlations.
- The manager should test the portfolio on both historical as well as hypothetical scenarios.

- Other possibilities are to use implied rather than historic volatilities in the calculation. Also, one could use more tailored risk factors to improve fit of VaR to portfolio.
- A better estimate of tail risk could be attempted, such as using Extreme Value Theory.

[2.5]

[15 Marks]

Solution 7:

i)

a) Credit – Default Swap is defined as :

- The holder of a credit default swap (CDS) retains the right to sell the reference entity (bond) for its face value (to the seller of the CDS) when a credit event occurs (i.e. the reference entity defaults).
- The CDS therefore provides insurance for the holder against the risk of a default by a particular company.
- The holder makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs. These payments are typically made in arrears but sometimes payments can be made in advance.
- The settlement in the event of a default involves either physical delivery of the bonds or a cash payment.

[2]

b) A basket default swap is a product with which the investor gains either long or short exposure to a relatively small basket of credits.

Baskets typically consist of up to a dozen credit names.

The holder of a third-to-default basket swap is protected against the third default of the basket, and the seller is exposed to it. No protection is provided for the first or second defaults

The holder has the right to sell the reference entity (bond) for face value when the third credit event occurs. At this point, the swap is terminated.

The holder makes periodic payments to the seller until the end of the life of the third–to-default basket swap or until a credit event occurs.

Third-to-default baskets can be traded in funded or unfunded form.

[2]

ii) If the CDS bond basis is positive and the bond is trading at par, one can then theoretically have arbitrage. The strategy would be as follows:

- Sell CDS protection and short the corporate (risky) bond.

- Use the proceeds from the sale of a CDS as well as the bond to invest in a risk – free instrument.

Now, in the event of a default of the bond, the loss on the CDS should be exactly offset by the profit on the short corporate bond.

Thus, this portfolio would always return greater than the risk-free rate, if the CDS – bond basis is positive.

[4]

iii) In practice, such arbitrage may not exist due to the following:

- The CDS may provide greater protection in that it could pay out on credit events that are technical defaults which would not fully impact a cash bondholder.
- There may be other minor contractual differences in the CDS contract such as provisions allowing delivery of a range of bonds or how the accrued coupons are dealt with.
- The CDS spread may include a premium (or a discount) for counterparty risk. This is because, if the protection buyer defaults, the CDS will terminate and the protection seller will no longer receive the premia and the protection seller may default if the credit event occurs.
- The CDS spread may incorporate a premium to reflect the greater liquidity of the CDS compared to the bond, and this liquidity premium can only be harvested if the position is held to term.
- It may be either impractical or costly to short the underlying bond.
- The transaction costs may also remove the arbitrage.

[4]

iv) The effectiveness of these transactions in meeting the manager's objective can be examined based on the "Impact on Cost" and the Impact on Credit Risk".

- *Impact on cost:* The basket swap will be much cheaper since there is no protection given to the first or second defaults, whereas the CDS would be expected to cover defaults on half the portfolio.

  The CDS transaction would be expected to reduce the credit spread on the portfolio by around half, whereas the basket swap will reduce it by much less.

- *Impact on credit* risk: Both transactions will partially reduce credit risk in the portfolio, as required.

  In particular, the CDS will substantially reduce the credit risk by covering defaults on five of the reference entities; on the other hand, the basket swap covers only the third default event.

The basket swap will provide some protection from credit risk across the whole portfolio, whereas the CDS will not cover any credit risk for five of the holdings.

Both provide protection against increased default correlation.

The basket swap is particularly designed to protect against highly correlated default events.

As both the CDS and basket swap would be unwound on a credit event, the manager would need to purchase new derivatives to ensure the protection is maintained. This will be even more important for the basket swap, as immediately following a relevant credit event there would be no protection in place against the remainder of the portfolio.

[6]

v) In this scenario, the recommendation would be that given the large cost of the CDS and consequent reduction in exposure to credit yield, it is likely that the second transaction would be most suitable (given the requirement for "limited cost") and so the basket swap is seems the best option.

[2]

[20 Marks]

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