

# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

**20<sup>th</sup> March 2018**

**Subject CT8 – Financial Economics**

**Time allowed: Three Hours (15.00 – 18.00 Hours)**

**Total Marks: 100**

### *INSTRUCTIONS TO THE CANDIDATES*

- 1. Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

#### AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

**Q. 1)** Explain the following in the context of behavioral finance:

- i)** Framing (and question wording) (2)
  - ii)** Myopic loss aversion (2)
  - iii)** Anchoring and adjustment (2)
- [6]**

**Q. 2)** Discuss the following in the context of EMH:

- i)** In the strong form of efficient market, rules pertaining to company employees and management over ban in stock trading eliminates benefit from insider trading (2)
  - ii)** Can technical analysis be applied in semi strong form of efficient market? (2)
  - iii)** In semi strong form of efficiency, information level differs for different group of investors or institutions (2)
- [6]**

**Q. 3) i)** State the martingale representation theorem, including conditions for its application, defining all terms used. (3)

**ii)** The model for the price of a non-dividend paying asset at time  $t$ ,  $S_t$  is given by:

$$dS_t = \mu S_t dt + \sigma S_t dB_t,$$

By applying Ito's lemma to the function  $f(S_t) = \log S_t$ , solve the stochastic differential equation defining the above geometric Brownian motion. (4)

**[7]**

**Q. 4) i)** Define first order and second order stochastic dominance. (3)

**ii)** Find the following measures for an asset  $X$ , for which the returns are exponentially distributed with mean 4.

- a)** Variance (2)
  - b)** Downside Semi Variance (3)
  - c)** Shortfall Probability (2)
  - d)** Expected shortfall below a return  $k$  ( $=2$ ) conditional on shortfall occurring (3)
- [13]**

**Q. 5)** There are three assets in a market with the following characteristics:

| Asset | Expected Return | Volatility |
|-------|-----------------|------------|
| 1     | 4%              | 6%         |
| 2     | 6%              | 12%        |
| 3     | 8%              | 18%        |

The correlation between Asset 1 and 2 is 0.5 while the correlation between Asset 3 and other two assets is zero.

- i) State the Lagrangian function that can be minimised to find the minimum variance portfolio associated with a given expected return, defining any notation used. (3)
- ii) By taking five partial derivatives of this function, calculate the minimum variance portfolio which yields an expected return of 7%. (5)
- [8]

**Q. 6)** Consider the following table of prices of European call and put options on a non-dividend-paying stock Iota Ltd.:

| Strike (K) | Price of call option | Price of put option | Time to expiry |
|------------|----------------------|---------------------|----------------|
| 70         | 13.334               | 0.120               | 3 months       |
| 75         | 8.869                | 0.568               | 3 months       |
| a          | 6.899                | 1.055               | 3 months       |
| 80         | b                    | 1.789               | 3 months       |
| 85         | 2.594                | c                   | 3 months       |
| 90         | 2.569                | 7.909               | 6 months       |

- i) Calculate the price of a 3-month forward on Iota Ltd. (4)
- ii) Also, calculate the forward rate for delivery between 3 months from now and 6 months from now. (2)
- iii) Fill in the missing values a, b, c in the above table. (3)
- [9]

**Q. 7)** Sheldon has decided to use an n-step recombining binomial tree for pricing a derivative on a non-dividend-paying stock Epsilon Ltd. Each time period in the tree is one month. Interest rates can be assumed zero.

- i) Show that the risk-neutral up-step probability  $q$  that Sheldon should use is less than 0.5. (3)
- ii) Starting from the annual volatility of the stock, Sheldon calibrates  $q$  to be  $\frac{1}{3}$ . Find the annual volatility with which Sheldon started. (2)
- iii) Use this tree to price a derivative, which expires one year from now, with the following pay-off at expiry (T):

$$Payoff = \sqrt{\frac{S_T}{S_0}}$$

( $S_0$  is the current stock price and  $S_t$  is the price of the stock at time  $t$ ) (5)

[10]

- Q. 8)** i) Define efficient market frontier along with a diagram. (2)
- ii) Explain the difference between capital market line and security market line. (2)

- iii)** In the context of CAPM, state whether the following statements are True or False:
- a)** Risk attitude for given set of securities is identical for all investors (1)
  - b)** Only efficient portfolios lie on the security market line (1)
- iv)** What will be the market capitalization of script A in Market portfolio as per CAPM, if half of the investors hold 5% of their respective portfolio in script A and rest half of them hold only risk-free assets, given that market consists of rational and irrational investors. (2)
- [8]**

**Q. 9)** Describe the similarities and differences between the Vasicek and Hull-White models of interest rate. (4)

**Q. 10) i)** When are two measures said to be equivalent? (1)

**ii)** State the Cameron-Martin-Girsanov Theorem (and its converse). How is it used in the 5-step method? (4)

**[5]**

**Q. 11)** The current prices (per \$100 notional) of some zero-coupon bonds are presented below:

| Issuer       | 5-year maturity | 7-year maturity | 10-year maturity |
|--------------|-----------------|-----------------|------------------|
| Government   | 77.8801         | 65.7047         | 57.6950          |
| Omicron Ltd. | 77.1109         | $\kappa$        | 55.4988          |

Leonard wants to model the default of Omicron Ltd. using a continuous-time two-state model. He believes that the risk-neutral transition intensity for failure is of the form:  $\lambda(t)=\alpha t$  and the recovery rate  $\delta$ .

- i)** Define ‘recovery rate’. (1)
  - ii)** Using the prices in the table above, help Leonard calibrate values of  $\alpha$  and  $\delta$ . (6)
  - iii)** Using these calibrated values, compute the missing price  $\kappa$  in the table above. (2)
  - iv)** Sheldon suggests to Leonard that the two-state model can be generalized to the Jarrow-Lando-Turnball model. Explain how this can be done. (3)
- [12]**

**Q. 12) i)** What are ‘delta’ and ‘vega’? (2)

**ii)** Show that the ‘vega’s for a European call option and a European put option on the same stock with same strike price  $K$  and same time to expiry  $\tau$  are identical. (1)

- iii)** Given the following data, find the price of a European call and a European put option:
- Stock price = \$55
  - Strike = \$50
  - Annualised Volatility = 25%
  - Continuously compounded risk-free rate of interest = 5%
  - No dividends
  - Time to expiry = 1 year
- (3)
- iv)** What does it mean for a portfolio to be ‘delta-hedged’ and ‘vega-hedged’?
- (2)
- v)** With \$1000, construct such a portfolio using the below instruments only:
- European call option with features as in part (iii)
  - European put option with features as in part (iii)
  - A 1-year forward on the stock
- (4)  
**[12]**

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