INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

15th March 2018

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) Player A and B throws an unbiased die consecutively in a zero sum game. Player A receives Rs.10 from Player B if the absolute difference of the outcomes is less than equal to 1.

i) Deduce payoff matrix. (2)

ii) Derive optimum payoff matrix. (4)

iii) Derive value for Player B under optimal randomised strategy. (1)
[7]

Q. 2) i) Define Proportional and Non Proportional Reinsurance. (2)

ii) Define 2 sub categories each of Proportional and Non Proportional Reinsurance. (2)

iii) An Agriculture Insurer in India has taken a proportional reinsurance with retained proportion of 20%. Claims are expected to occur as a Gamma (α, λ) with parameters $\alpha = 10$ and $\lambda = 3$. Find the following:-

a) Mean and Variance of the amount paid by the insurer on an individual claim. (3)

b) Mean and Variance of the amount paid by the reinsurer on an individual claim. (3)

iv) A Motor Insurer expects that the claim amounts, X, on its passenger car portfolio follows an exponential distribution with a mean of 10,000. The Insurer purchased an excess of loss reinsurance with retention of 2500, subject to maximum reinsurance claim amount of (M – 2500).

Calculate M such that expected reinsurance claim amount is 5000. (7)
[17]

Q. 3) i) Describe how ‘Bornhuetter-Ferguson’ method improves the use of loss ratio as compared to basic chain ladder method. (2)

ii) Earned premium for the year 2014 to 2017 are 120, 150, 140 and 180 crores respectively and expected loss ratio for year 2014 to 2017 are 80%, 90%, 105% and 95% respectively. The Company has a 1 year waiting period to pay the claim amount from date of intimation of the claim. Cumulative loss ratios for various development years are summarized in the table below:

Accident Year	**Development Year**			
	0	**1**	**2**	**3**
2014	40%	55%	70%	75%
2015	35%	50%	75%	
2016	60%	70%		
2017	55%			

Estimate the outstanding claim reserve using the Bornhuetter-Ferguson method. (9)
[11]

Q. 4) The number of claims per 1000 insured vehicles for a large motor insurer in three different geographic locations is given in the table below for past 10 years. The number of claims is assumed to follow Poisson distribution.

	Year									
Region	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
Metro	2	5	1	7	2	3	NA	8	4	5
Rural	5	3	NA	2	6	2	1	2	NA	8
Urban	4	6	3	5	NA	NA	12	NA	2	9

*NA – Data not available

i) Determine Poisson parameter for each region using Maximum Likely Estimator (MLE) (4)

ii) Test whether claim rates are same under the three regions. (4)

[8]

Q. 5) A general insurance company is considering using Weibull (with second parameter 2) and Exponential distribution to model the distribution of claims. It is given that 1 out of 250 claims will exceed 200 and the "Threshold Limit" (Z) is defined as P(Claim > Z) = 0.0001. Calculate the "Threshold Limit" for both the distributions and comment. **[6]**

Q. 6) Let h(x) follow exponential distribution with mean λ and f(x) follow chi-square distribution with α degrees of freedom.

i) State an algorithm to generate a sample from h(x) using inverse transforms method. (1)

ii) Construct an algorithm to generate a sample from f(x) by using acceptance-rejection method and h(x). (6)

iii) Determine optimum value of λ under which maximum values will be accepted in the above algorithm. (5)

[12]

Q. 7) **i)** State and explain Lundberg's inequality. (2)

ii) Derive Lower Bound for R (adjustment coefficient) in Lundberg's inequality assuming M to be the upper limit of an individual claim. (4)

iii) On a portfolio of motor Insurance policies, claims occur according to a Poisson process at a rate λ. All claims are for a fixed amount d and premiums are received continuously. The Insurer's initial surplus is U and the annual premium income is 1.5λd. Derive an expression of the probability for ruin occurring at first claim. (5)

iv) In question (iii) above, instead of claim amount being a fixed amount, the claim amount follows a distribution with density function $f(z) = 0.05^2 z e^{-0.05z}$; z>0. The insurance company calculates premiums using a premium loading of 30%. Calculate the adjustment coefficient if the insurer has initial surplus U. (6)

[17]

Q. 8) During a national holiday, 3 major online sales portal viz. Freekart.com, Appdeal.com and E-mazon.com are planning to organize special discount sales of Myphone, a major mobile phone brand. All 3 portals have varying incentive plans for dealers of Myphone, which ultimately will impact the payoffs of the dealers.

A dealer of Myphone has the option for one day of selling the phones on either of 3 portals that day however his payoffs will depend on the demand of Myphone on the portals. The dealer believes that the demand is equally likely to be High (H), Medium (M) and Low (L) and estimates his profits under each possible scenario to be:-

	H	M	L
Freekart.com	28	19	14
Appdeal.com	9	30	15
E-mazon.com	5	16	29

i) Determine the minimax solution to the problem. (2)

ii) Determine the decision which will maximize the maximum profit. (1)

iii) Determine the Bayes criterion solution to this problem (2)

iv) The dealer believes that there are equal chances of High or Low demand however there is more than an even chance of it being medium demand. Hence, determine the revised Bayes criterion solution. (3)

[8]

Q.9) **i)** What are the uses of Generalized Linear Models in Insurance? Give an example. (2)

ii) What are the 3 key components of Generalized Linear Model? (2)

iii) An Insurer would like to model its claims costs from Crop Insurance using a simple generalized linear model based on 3 factors viz. A (rainfall) , B (temperature) and C (Humidity):

$$A_i = \begin{Bmatrix} i = 1 \text{ for Above normal } rainfall \\ i = 0 \; for\; Below\; normal\; rainfall \end{Bmatrix}$$

$$B_j = \begin{Bmatrix} j = 1 \; for\; Above\; normal\; temperature \\ j = 0 \; for\; Below\; normal\; temperatutre \end{Bmatrix}$$

$$C_k = \begin{Bmatrix} k = 1 \; for\; Above\; normal\; humidity \\ k = 0 \; for\; Below\; normal\; humidity \end{Bmatrix}$$

The insurer is considering 3 possible models for the linear predictor:-

Model 1: A + B +C

Model 2: A + B + C + A.B

Model 3: A*B*C

a) Write each of these models in parameterized form, stating how many non zero parameter values are present in each model. (5)

b) The table below shows the calculated values of the scaled deviance for these three models and the constant model.

Model	Scaled Deviance
1	40
A + B +C	11
A + B + C + A.B	7
A*B*C	0

Calculate the number of degrees of freedom for each model and determine which model is most suitable. (5)

[14]

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