INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

16th March 2018

Subject CT4 – Models

Time allowed: Three Hours (15.00 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
3. ***Mark allocations are shown in brackets.***
4. ***Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer booklet and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) For each of the following processes:

- Counting process;
- General random walk;
- Poisson process;
- Markov chain;
- Markov jump Process.

i) State whether the state space is discrete, continuous or can be either. (2.5)

ii) State whether the time set is discrete, continuous, or can be either. (2.5)

Consider each of the following statistics associated with a bank account:

a) Number of times the account has been overdrawn since it was opened
b) Status (overdrawn, in credit) of the account on the last day of each month
c) Number of direct debits paid since the account was opened
d) Status (overdrawn, in credit) of the account at any time since the account was opened.

Each statistic is to be modelled by a stochastic process. In each case:

iii) State any one model from the above which may be suitable for each of the above scenarios. (2)

[7]

Q. 2) **i)** Define how the following forms of censoring arise in a survival investigation:

a) Right censoring (1)
b) Type I censoring (1)
c) Random censoring (1)

An experience analysis is conducted on the mortality of the members of group insurance policy taken by a manufacturing company for the benefit of its permanent employees.

ii) Explain which type of the censoring (Right censoring, Type I, Random) is present in the following situations.

a) For the members who change employment. (1)
b) For the members who retired. (1)
c) The policy is not renewed. (1)

[6]

Q. 3) A pet mouse is kept in artificial mouse hole which is made up of three large balls, each connected to other through pipes. The overall arrangement is triangular in shape with balls at nodes and pipes as sides. The mouse keeps moving at a very fast speed between the balls and randomly changing the direction while in the ball. The mouse cannot change direction in the pipes. Let each node be considered as state of continuous-time process with three states observed from time 0 up until the time of the 20^{th} transition. The results may be summarised as follows:

			Number of transition from State I to:		
State, i	No. of visits to state i	Time spent in state i	State 1	State 2	State 3
1	16	96	0	6	10
2	8	320	2	0	6
3	16	480	14	2	0

i) Describe the stages of model fitting and model verification in the modelling process. (2)

ii) Suppose that a Markov jump process model is to be fitted to the data set above. List all the parameters of the model and discuss the assumptions made when such a model is fitted to a data set. (4)

iii) Estimate the parameters of the model in (ii) above and write down the estimated generator matrix. (4)

[10]

Q. 4) A life insurance company is worried about the lapsation in its term assurance portfolio. The company is trying to determine the profile of the customers who may have low lapsation rate. From the policy data of recent years, the company has fitted a Cox proportional hazard model to those policyholder who lapsed their policy within first 5 years after purchase.

The following figures have been derived from the data

Covariates	Category	Parameter
Gender	Males	0
	Females	0.065
Smoker	Non-smoker	-0.035
	Smoker	0
Area of Residence	Rural	0.012
	Urban	0

i) Give the hazard function for this Cox proportional hazard model defining all the terms and conditions (3)

ii) State the features of the persons to whom the baseline hazard applies. (1)

Marketing Actuary of the company suggested that marital status could be added as an additional factor to improve the model. There is a 20% higher probability that a female, Non-smoker from rural area lapses her policy on 5^{th} policy anniversary if she is married rather than being single. Also there is 50% probability that policy of a male, Non-smoker from the City remains in-force for at least 5 years.

iii) Give the revised hazard function defining the new terms used and determine the probability that a married female, Non-smoker from rural area will not lapse the policy for at least 5 years. (5)

[9]

Q. 5) A portfolio of pensioners, aged between 90 and 92 years, are observed during the period from 1st January 2014 to 31st December 2016. The pensioners are followed from their 90th birthday until they died or celebrate their 92nd birthday. The pensioners are required to submit their survival certificate each month. Pensioners who do not submit their survival certificate are removed from the observation. The duration of survival measured in nearest months is recorded for whom death is reported during the period and those who do not submit their survival certificate. The recorded data measured in nearest months separately for males and females is given below:

Males	2*	3	3	4	4	5+	6*	8	10	12	14*	17+	20	20	21	22*	22+	23*
Females	3	4	5*	6	6+	7	7	8	8	9*	12	12	15	18*	18*	19	20+	23*

* Those who did not submit their survival certificate
\+ Those who celebrated their 92nd birthday.

i) Calculate the product -limit (Kaplan-Meier) estimate of the survival function, S(t), for males and female pensioners separately, where t is the duration under observation. (5)

ii) Calculate the probability of death of both males and female pensioners up to 4 months with a 95% confidence interval assuming normal distribution of deaths. (5)

iii) Comment whether males or females have a significantly higher probability of survival to duration of 4 months? (2)

[12]

Q. 6) An insurance company is carrying out mortality investigation of its term assurance portfolio. It records in-force policies using age label "age y last birthday as at 1st April". Information about the number of in-force policies is available for year 2015, 2016 and 2017. The number of deaths in financial year 2015 to 2017 as reported by claim department grouped by age x nearest birthday on the date of death. No unreported claims are assumed.

The following data have been supplied for the investigation:

	Age			
	55	56	57	58
No. of deaths	1150	1380	1420	1780
No. of lives at 01.04.2017	18500	20000	15000	21200
No. of lives at 01.04.2016	20500	21100	20700	20500
No. of lives at 01.04.2015	20100	20000	19700	18500

i) Estimate force the mortality for lives with age label 56 and 57, state any assumptions made. (6)

ii) Estimate initial mortality rates for lives in (i) using derived force of mortality, clearly indicating the age to which it applies to. (2)

[8]

Q. 7) A worldwide sporting agency enters into contracts with various players across the continents and organises various tournaments at various levels. The contract fee for the player depends on the current level of the player and rates that are fixed by the agency. The levels for the contracting purposes are as follows:

- Level D - Base Rate
- Level C - 20% higher than Base rate
- Level B - 50% higher than Base rate
- Level A - 75% higher than Base rate

The tournaments are then organised by grouping these players into teams according to predefined rules.

The performance of each player is tracked over the year using various sporting and fitness parameters, and a single metrics is derived using these performance parameters.

Note 1: This single parameter is not same as/or an indicator of ranking, it just signifies the overall performance of the player.

Note 2: Assume that there are no exits and entry from the current pool from date of this exercise

The following are the rules adopted by the agency for renewal of yearly contract and everybody abides by these rules.

- Player whose performance parameter increased as compared to last year move to one higher level or remain at A
- Player whose performance parameter remains unchanged as compared to last year remain at same level
- Player whose performance parameter decreased as compared to last year but has an increase in performance parameter in the previous year as compared to the year before last move one level down or remain at D.
- Player whose performance parameter decreased as compared to last year and also had a decrease in performance parameter in the previous year as compared to the year before last move two level down or remain at D

E.g if current year is 2018, last year is 2017 and year before last is 2016.

i) Explain why a process with the state space of {D, C, B, A} does not display the Markov property. (2)

ii) Define any additional state(s) required to model the system with the Markov property. (2)

The agency believes that each year player has a 50% likelihood of having an increase in their performance and a 30% likelihood of having a decrease, irrespective of their current level.

iii) Construct a transition graph of this Markov process clearly labelling all the states. (2)

iv) Write down the transition matrix for the Markov process. (2)

v) Calculate the stationary distribution. (3)

vi) Hence calculate the long run average price of each contract assuming the base price is 1 million USD. (2)

vii) Calculate the average time for player currently in D level to move to A level. (4)

[17]

Q. 8) **i)** What are the reasons for doing graduation? Give one practical example. (2)

A life office has undertaken the investigation of mortality experience of its high selling pension product. Death data from a cohort of 10000 pensioners aged exact 60 years are followed until age 70 is given below. There is no decrement other than death.

Age	No. of deaths	qx (as per pricing assumption)
60	14	0.00128
61	16	0.00144
62	13	0.00162
63	15	0.00183
64	17	0.00206
65	14	0.00233
66	30	0.00263
67	25	0.00297
68	27	0.00335
69	42	0.00378
70	35	0.00425

ii) Apply χ^2 test to find out if the actual mortality experience is consistent with pricing assumption and perform an overall goodness of fit test on the data (5)

iii) Carry out one other test to determine any overall bias. (3)

With regard to term assurance portfolio, the insurance company so far assumed that no policyholder has more than one policy. However, investigation of recent data suggest otherwise. The company therefore wants to know the impact of multiple policies on the claim experience.

iv) Show that the variance of mortality estimates in presence of duplicate policies would increase by

$$\frac{\sum_i i^2 \pi_i}{\sum_i i \pi_i}$$

where π_i is the proportion of policyholders holding i=1,2,3.. policies. (5)

[15]

Q. 9) A large bank in a developed country is envisaging implementing blockchain technology for its accounting system and in particular the payment system as current system requires too much of efforts in reconciliation.This will also help in reduction of transaction cost for international payments. Blockchain network is a collection of high end servers (nodes) that make cryptographic calculations. Each node tries to outperform each other by performing the calculations as fast as possible and node owner gets a small fee for providing their computing power to the network depending on certain success factors.

Under this system, the transactions originating anywhere are added to a queue which is managed by a queuing server. The transaction at the front of the queue is shared with all network nodes attached to the blockchain network. These nodes perform certain cryptographic calculations and determine the validity of the transaction. Once a node

validates a transaction it sends a message across network which is known as "consensus". The queuing server does not share the transaction at the front of queue with network for processing until the previous transaction is validated and added to the blockchain, i.e. queuing server has received the required number of consensuses for previous transactions.

As per current requirements, Four consensus are required to add a transaction to the blockchain. The first four network nodes from whom the consensus was received will receive the fee for consensus. There is no delay between arrival of the last consensus for previous transaction and issue of new transaction by the queuing server.

The time taken by various nodes to solve cryptographic problem follows a random process and consensus messages received by queuing server follows a Poisson process with a rate of β per minute.

i) Explain how the number of consensus received by queuing server for the current transaction can be modelled as Markov jump process. (2)

Write down, for this process:

ii) The Generator Matrix (1)

iii) Kolmogorov's forward equations in component form (3)

iv) Calculate the expected time a blockchain node that has sent the consensus will have to wait until the previous transaction is added to the blockchain and new transaction from queue is received for computing. (4)

The bank felt that average number of consensus should depend on size of the risk i.e. transaction amount. It performed certain risk analysis of the transactions and arrived at a conclusion that transactions with amount 100K and below will be accepted at Three consensus whereas those above 100k will require Six consensus for addition of that transaction to the blockchain. All transactions have equal probability of being 'more than' or 'less than and equal to' 100k and they arrive randomly at the queuing server.

v) Write down the transition matrix of the Markov jump chain describing the number of consensus received by queuing server for the current transaction after this rule change. (2)

vi) Calculate the expected waiting time for a blockchain node that has sent the consensus until the current transaction is added to the blockchain and new transaction from queue is received for computing after this rule change and compare this with your answer to part (iii). (4)

[16]

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