INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

16th March 2018

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) **i)** State the main features of :

      a) Certificates of deposits   (2)
      b) Government Index linked bond   (2)

    ii) List the characteristics of property investment.   (4)

    iii) The Life Insurance company bought a 90 day treasury bill issued by the Government of India at a simple rate of discount of 8% per annum. Calculate the effective rate of return per annum received by the company if it holds the bill to its maturity date.   (2)

                                                           [10]

Q. 2) **i)** Calculate at a rate of interest of 4% per annum effective:

      a) $_{5|}a_{32}\rceil$ (2)
      b) $_{4|}(Ia)_{15}\rceil$ (2)

    ii) Given that $\ddot{a}_{n}\rceil = 7.029584$ and $\ddot{a}_{2n}\rceil = 10.934563$, find the rate of interest and n.   (3)

                                                           [7]

Q. 3) Mr Sam purchased an index linked annuity plan from an ABC insurance company. In return of purchase price of Rs 5,00,000 Mr Sam will receive fifteen annual payments starting one year from now. The first payment will be of Rs 50,000 and future payments will be increased in line with price inflation. The insurance company also incurred an annual claim expense of Rs 500 per annum at the time of making the annuity payments. The claim expense will also increase in line with same price inflation which will be applicable to annuity payments.

If the insurance company assumes a constant nominal rate of interest of 8% per annum in its premium basis, find the constant rate of inflation assumed.   **[5]**

Q. 4) Mr Allen took a loan of Rs 25,00,000 from a bank to build his house. The loan is to be repaid by regular annual payments in arrears for 25 years and calculated on the basis of an interest rate of 10% per annum.

    i) Calculate the level annual repayments.   (2)

    ii) The capital repayment and interest paid at the end of
      a) Tenth year and   (2)
      b) Fifteenth year.   (2)

    iii) After which repayment, the outstanding loan will be first less than Rs 12,00,000   (3)

    iv) For which repayment, the capital content will first exceed the interest content.   (4)

                                                           [13]

Q. 5) The ABC company has just got an order for exploration of oil in Arabian sea for next ten years. The company estimates the following cashflows over the period of the exploration:

Outgoes :
- Initial outlay (building of plant) Rs 100 crore payable immediately.

- Cost of exploration of oil is Rs 80 crore per annum payable annually for ten years, the first payment being made in one year's time.
- Disposal of wastes and demolition of plant: Rs 80 crore per annum payable annually for three years, the first payment being made in eleven years' time.

Income :
- Income from sale of oil is expected to be Rs 120 crore per annum payable annually for ten years, the first income being received in one year's time.

The company proposes to finance the project by borrowing the entire initial cost from a bank, which charges interest on loans at 15% per annum effective. The partial repayment of loan will be allowed at any time and the loan will be repaid in installments as soon as possible based on the profits earned.

i)   Calculate how many years will be required to pay-off the bank loan.   (4)

After repayment of the bank loan, the company will accumulate its profits in a current account at the bank, which pays an effective rate of interest of 12% per annum. The costs of disposal of wastes and demolition of plant will be met from this account.

ii)   Calculate the anticipated balance in the company's current account at the end of thirteenth year.   (6)

iii) Define Payback and discounted payback period of the project and why both are not considered to be good measures for the evaluation of the project.   (3)

[13]

Q. 6) A Government issued an index linked bond on 1st January 2016 which will be redeemed on 31st December 2018. The bond had a nominal coupon rate of 5% per annum payable half-yearly in arrear and had a nominal redemption payment of Rs 100. The coupon and redemption proceeds are index linked and are indexed according to the increase in the retail price index (RPI) between 6 months before the bond issue date and 6 months before the coupon or redemption payment dates.

The value of the RPI in the relevant month are :

Date	RPI
1st June, 2015	100
1st December, 2015	102
1st June, 2016	104
1st December, 2016	108
1st June, 2017	111
1st December, 2017	113
1st June, 2018	115
1st December, 2018 (projected)	118

An investor purchased Rs 1,00,000 nominal of the bond at the issue date, and held it until it was redeemed. The issue price was Rs 92 per Rs 100 nominal.

i)   Calculate all the cashflows to the investor before tax.   (4)

ii)   The investor is subject to

- Income tax at the rate of 20% and
- Capital gain tax at a rate of 30%.

When calculating the amount of capital gain which is subject to tax, the price paid for the investment is indexed in line with the increase in RPI between the month in which the investment was purchased and the month in which it was redeemed.

a)   Calculate the investor's capital gain tax liability in respect of this investment   (2)
b)   Calculate the effective yield per annum to the investor.   (4)

[10]

Q. 7)   An investor had three options of investment as follows

Option 1: To invest in 500 shares in a small education company, ex-dividend. Dividends are paid annually and the next dividend is due in one month's time. The last dividend was Rs 8 per share and dividends are expected to rise by 4% pa. The price to be paid is Rs 120 per share.

Option 2: To invest in a loan of Rs 10,000 bearing interest of 6% per annum payable yearly and will be redeemed at par after ten years, the price to be paid is Rs 8,000.

Option 3: To invest in a fixed interest security for a period of 10 years, redeemable at the rate of 110% with 6% annual coupons. The term structure of interest rates applicable under this option is given below:

Term (in Year)	Rate
1	7%
2	7%
3	8%
4	8%
5	9%
6	9%
7	10%
8	10%
9	11%
10	11%

Which of the above 3 options should he select in order to maximize his IRR? Ignore taxation.   **[10]**

Q. 8)   On every 1ˢᵗ of January the Government issues long term fixed interest bonds redeemable at par and bearing interest at 3.125% per annum payable annually in arrears.

On 1ˢᵗ January 2015 when the market price of the stock was consistent with a yield of 5% per annum effective, a company issued a ten year annual premium paying bond with maturity value of Rs 1,00,000. The annual premium was calculated on the basis of an interest rate of 5% per annum effective.

i)   Calculate the annual premium.   (2)

ii) The office invested the first premium in the above Government bonds for which the discounted mean term, at 5% p.a. interest was closest to 10 years

Calculate
a) The outstanding term of this stock (4)
b) The nominal amount purchased (2)

[8]

Q. 9) The insurance company after adopting a particular investment strategy expects that the annual yield on their fund will be 8% per annum. The company adopts a high risk investment policy and it is anticipated that the standard deviation of the annual yield will be 7%. The yield in different year may be assumed to be independently distributed.

i) Find the expected value and the standard deviation of the accumulated amount of a single premium of Rs 1,000 over a period of 15 years. (4)

ii) Assuming further that each year $(1+i)$ has a log-normal distribution with parameters $\mu = 0.075$ and $\sigma^2 = 0.065$. Here i is the annual yield on the fund.

Calculate the probability that a single premium accumulation will be

a) Less than 60% of its expected value (2)
b) More than 150% of the expected value (2)

[8]

Q. 10) **i)** State the two alternative conditions required for the existence of an arbitrage opportunity. (2)

ii) A 7-month forward contract was issued on 1st January, 2000 on a stock with a price of Rs 60 per share. Dividends of Rs 2 per share are expected after 3 and 6 months from now.

Assuming a risk-free force of interest of 7% per annum and no-arbitrage, calculate the forward price. (4)

[6]

Q. 11) A company is liable to make four payments at five yearly intervals. The first payment being due five years from now. The amount of the tth payment is $(1000 + 100\,t)$. The liabilities are valued at rate of interest of 5% per annum effective.

i) Find the present value and discounted mean term of the liability (4)

An amount equal to total value of the liability is invested in two loans each redeemable at par at the end of 10 years and 30 years respectively. Each bears interest rate of 5% per annum effective payable annually in arrears.

ii) Given that at the interest rate of 5% per annum effective and the discounted mean term of the assets is the same that of the liability, determine how much is invested in each loan. (6)

[10]

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