Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

March 2017 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) ${}_{10|1}q_{[50]}$
$= d_{60}/l_{[50]}$
$= 74.5020/9,706.0977$
$= 0.00768$

ii) $(IA)^{1}_{50:\overline{10}|}$
$= (IA)_{50} - v^{10} * (l_{60} / l_{50}) * ((IA)_{60} + 10\ A_{60})$
$= 4.84555 - 1.06^\wedge(-10) * (9287.2164 / 9712.0728) * (5.46572 + 10 * 0.32692)$
$= 0.181385$

iii) $\ddot{a}^{(12)}_{[45]:\overline{15}|}$
$= \ddot{a}^{(12)}_{[45]} - v^{15} * (l_{60}/l_{[45]}) * \ddot{a}^{(12)}_{60}$
$= (\ddot{a}_{[45]} - 11/24) - v^{15} * (l_{60}/l_{[45]}) * (\ddot{a}_{60} - 11/24)$
$= (14.855 – 11/24) – (1.06^\wedge(-15)) * (9287.2164/9798.0837) * (11.891 – 11/24)$
$= 9.874943$

[5 Marks]

Solution 2:

i) UDD method is based on the assumption that for integer x and $0 \le t \le 1$, the function ${}_tp_x.\mu_{x+t}$ is a constant. i.e. for an individual exactly aged x(integer) the probability of dying on any one particular day over the next year is same as that of dying on any other day over the next year.
Hence, UDD implicitly assumes an increasing force of mortality over a given year.
Whereas, constant force of mortality method is based on the assumption of constant force of mortality, which means that for integer x and $0 \le t \le 1$, the function μ_{x+t} is a constant i.e. $\mu_{x+t} = \mu$= constant.

(2)

ii) Calculate ${}_{1.75}q_{45.75}$ using Constant Force of Mortality and ELT15 Male

${}_{1.75}q_{45.75}$

$= 1 – {}_{1.75}p_{45.75}$
[0.5]
$= 1- {}_{0.25}p_{45.75} * {}_1p_{46} * {}_{0.5}p_{47}$
[0.5]
$= 1 - (p_{45})^\wedge(0.25) * p_{46} * (p_{47})^\wedge(0.5)$
[0.5]
$= 1 - (1-0.00266)^\wedge(0.25) * (1-0.00297) * (1-0.00332)^\wedge(0.5)$
$= 0.005289$

(2)
[4 Marks]

Solution 3:

A continuous whole life annuity is issued to a Life aged X.

i) Continuous whole life annuity issued to life aged X is denoted by $\bar{a}_{\overline{Tx|}}$

It's expected present value is denoted by $\bar{a}_x$

(1)

ii) Given the force of mortality is constant we could say that:

$\bar{a}_x = \int_0^\infty v^t * tpx * dt$

$= \int_0^\infty e^{-\delta t} * e^{-\mu t} * dt$

$= \int_0^\infty e^{-(\delta+\mu)t} * dt$= 16 (Given in Question)

= (1/(μ+δ)) = 16

= (μ+δ) = 0.0625

Give δ = 0.04 gives μ = 0.0225

Now:

$\bar{A}_x = \int_0^\infty v^t * tpx * \mu_{x+t} * dt$

$\bar{A}_x = \int_0^\infty e^{-\delta t} * e^{-\mu t} * \mu * dt$

$\bar{A}_x$= μ / (μ+δ)

= 0.0225 * 16

= 0.36

Also

${}^2\bar{A}_x$ = $\bar{A}_x$ with twice force of interest. Hence

${}^2\bar{A}_x$ = μ / (μ+2δ)

= 0.0225/(0.0225 + 2*0.04)

= 0.219512

Now Var($\bar{a}_{\overline{Tx|}}$) = (${}^2\bar{A}_x$ - ($\bar{A}_x$)^2) / δ^2

= (0.219512 – 0.36^2)/0.04^2

= 56.195 ➔ SD = 7.496332 **(5)**

[6 Marks]

Solution 4:

PV of Net CF = PV of Premium – PV of benefits – PV of Expenses – PV of Commission

Let maximum initial commission be I

Now

PV of Premium = 30,000 *$\ddot{a}_{[45]:20}$τ

= 30,000 * 11.888

= 356,640

PV of Benefits = 1,000,000 * $A_{[45]:20}$τ

= 1,000,000 * 0.32711
= 327,110

PV of Expenses = 5,000 + 2.5% * 30,000 * ($\ddot{a}_{[45]:20}\tau$ - 1)
+ 500 * $p_{[45]}$ * 1.06^(-1) * (1 + 1.019231*v*$_{1}p_{[46]}$ + 1.019231^2 * v^2*$_{2}p_{[46]}$....
+ 1.019231^18 * v^18*$_{18}p_{[46]}$)
= 5,000 + 2.5% * 30,000 * ($\ddot{a}_{[45]:20}\tau$ - 1)
+ 500 * $p_{[45]}$ * 1.06^(-1) * $\ddot{a}_{[46]:19}\tau$ (@4%)
= 5,000 + 2.5% * 30,000 * (11.888 - 1)
+ 500 * (9783.3371/9798.0837) * 1.06^(-1) * 13.316
= 19,438

PV of Renewal commission= 2% * 30,000 * ($\ddot{a}_{[45]:20}\tau$ - 1)
= 2% * 30,000 * (11.888 - 1)
= 6,533

PV of Net CF = 356,640 – 327,110 – 19,438 – 6,533 – I
= 3,559 – I

Margin = PV of Net CF / Annual Premium
= (3,559 –I) / Annual Premium
= (3,559 – I)/ 30,000 = 10%

I = Rs. 559 **[7 Marks]**

Solution 5:

Let the benefit be broken into following 3 components:

A: Guaranteed annuity of Rs. 10 Lac PA for first 15 years, and post that for the life time of X

B: Annuity of Rs. 5 Lac PA paid to Y, commencing post completion of 15years, for the life time of Y

C: Annuity of Rs. 5 Lac PA paid while both X and Y are alive, commencing post completion of 15years.

Now S (Single Premium) = A + B – C

Now:

A = 1,000,000 * ($\ddot{a}_{\overline{15|}}$+ 1.04^(-15) * ($l_{80}$(m)/$l_{65}$(m)) * $\ddot{a}_{80}$(m))
= 1,000,000 * (11.56312 + 1.04^(-15) * (6953.536/9647.797) * 7.506)
= 14,567,024

B = 500,000 * 1.04^(-15) * (l_{75}(f)/l_{60}(f)) * $\ddot{a}_{75}$(f)
= 500,000 * 1.04^(-15) * (8784.955/9848.431) * 10.933
= 2,707,583

C = 500,000 * 1.04^(-15) * (l_{80}(m)/l_{65}(m)) * (l_{75}(f)/l_{60}(f)) * $\ddot{a}_{80(m):75(f)}$
= 500,000 * 1.04^(-15) * (6953.536/9647.797) *(8784.955/9848.431) * 6.441
= 1,149,670

Hence Single premium = A+B-C = Rs. 16,124,937

[10 Marks]

Solution 6:

i) Calculate the annual premium:

Let the annual premium be P

Present value of premiums

= P * $\ddot{a}_{[50]:\overline{10|}}$ = P * 7.698

Present value of Annual Guaranteed benefits:

= 250,000 * 1.06^(-10) * ($l_{60}/l_{[50]}$) * $\ddot{a}_{60:\overline{5|}}$

= 250,000 * 1.06^(-10) * (9287.2164/9706.0977) * 4.390

= 586,390

Present value of death benefit in first 10 years:

= P * $(IA)^{1}_{[50]:\overline{10|}}$

= P * $(IA)_{[50]}$ - 1.06^(-10) * ($l_{60}/l_{[50]}$) * (P * $(IA)_{60}$ + 10* P * A_{60})

= P * 4.84789 - 1.06^(-10) * (9287.2164/ 9706.0977) * (5.46572 * P + 10*P*0.32692)

= 0.180854 * P

Present value of Death benefit in last 5 years:

= 1.06^(-10)*($l_{60}/l_{[50]}$)*(1,000,000 * $A^{1}_{60:\overline{5|}}$ +5*250,000* $A^{1}_{60:\overline{5|}}$ -250,000 *$(IA)^{1}_{60:\overline{5|}}$)

= 1.06^(-10)*($l_{60}/l_{[50]}$)*(1,000,000 * $A^{1}_{60:\overline{5|}}$ +1,250,000* $A^{1}_{60:\overline{5|}}$ -250,000 *$(IA)^{1}_{60:\overline{5|}}$)

Now

$A^{1}_{60:\overline{5|}}$ = A_{60} - 1.06^(-5) * (l_{65}/l_{60}) * A_{65}

= 0.32692 – 1.06^(-5) * (8821.2612/9287.2164)*0.40177

= 0.041757

$(IA)^{1}_{60:\overline{5|}}$ = $(IA)_{60}$- 1.06^(-5) * (l_{65}/l_{60}) * ($(IA)_{65}$ + 5*A_{65})

= 5.46572 - 1.06^(-5) * (8821.2612/9287.2164) * (5.50985 + 5*0.40177)

= 0.12920

So PV of death benefit in last 5 years is:

= 1.06^(-10)* (9287.2164/ 9706.0977) *(1,000,000 *0.041757 + 1,250,000* 0.041757 - 250,000 *0.12920)

= 32,941

Equating the PV of premium to PV of benefits:

P * 7.698 = 586,390 + 0.180854 * P + 32,941

7.51715 * P = 619,331

P = Rs. 82,389 **(10)**

ii) ${}_{59}V$ = (${}_{60}V$ * (1- q_{59}) + q_{59} * 10* P)/ (1+i) - P

= (1,200,000 * (1 – 0.007140) + 10*82,389*0.007140)/ (1.04) - 82,389

= Rs. 1,068,875 **(2)**

[12 Marks]

Solution 7:

i) Importance of Expense Investigation:

Expense investigation is an important exercise carried out by Insurance companies. It helps in:

- Deciding the amount of expense loadings to be included in the premium.
- Helps in allocation of expenses across different lines of business and hence being fair to respective class of policyholders.
- It helps in understanding the expense dynamics of the company, i.e. to understand the nature of expenses into Initial and renewal. Also it helps in understanding the costs of various distribution channels.
- Helps the management to strategize the future business and expenses.

(2)

ii) Outline the key difference between the direct and overhead expenses, giving suitable examples:

Direct expenses are those which vary with amount of business written

Whereas, overhead expenses are those, which over the **short term,** do not vary with the amount of business written.

However in the long term, large changes in the amount of business written will result in all expenses being direct.

For example, the cost of head office premise is an overhead expense. If the amount of business written in any one month is substantially more than expected, it will not increase the cost under this head. However if the increased amount of New business, is consistent for few years, it would lead to increase in head office premise, as more people would be required hence their seating as well.

Other examples of overhead expenses could be Board directors' remuneration, salary cost of corporate functions like, Finance, Actuarial and Investments could also be termed as overhead expense.

Examples for direct expenses include:

Commission payments to agents, Underwriting expenses for new business sales, salary cost of New business administration team, etc.

Overhead expenses are usually allocated on a per policy basis, whereas direct expenses are allocated according to their drivers.

(4)

[6 Marks]

Solution 8:

i) The assumption of Independence of Decrements is used to derive single decrement table from a multiple decrement table.

Mathematically:

$(a\mu)_x^j = \mu_x^j$ for all j and all x

When we look at transition intensities (forces of decrement), we are looking at infinitesimally small time interval in which there is only time for one decrement. Thus it is reasonable to assume that the independent and dependent forces of decrement are equal. **(3)**

ii) In case of term assurance, it is likely that people who lapse their policies have a lower than average mortality **(1)**

iii)

a) $1000\int_0^5 e^{-\delta t}({}_tp_{45}^{PI}\mu_{45+t} + {}_tp_{45}^{PP}v_{45+t})\,dt$ **(1)**

b) $250\int_0^5 e^{-\delta t}\ {}_tp_{45}^{IP}v_{45+t}dt$ **(1)**

c) $150\int_0^5 e^{-\delta t}\ {}_tp_{45}^{II}dt$ **(1)**

d) $250\int_0^5 e^{-\delta t}\ tp_{45}^{PP}\ \vartheta_{45+t}\,dt$ **(1)**

e)

iv) Lets assume that decrements are independent. Hence the given independent forces of decrement can be assumed to apply when both decrements occur together in the same population.

$(aq)_{45}^{C} = \frac{0.05}{0.18}(1 - e^{-0.18}) = 0.045758$
$(aq)_{45}^{S} = \frac{0.13}{0.18}(1 - e^{-0.18}) = 0.118972$
$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$

$(aq)_{46}^{C} = \frac{0.06}{0.16}(1 - e^{-0.16}) = 0.055446$
$(aq)_{46}^{S} = \frac{0.1}{0.16}(1 - e^{-0.16}) = 0.092410$
$(ap)_{46} = 1 - 0.055446 - 0.092410 = 0.852144$

The probability of being in state Inforce at the beginning of age 47 =
0.835270 × 0.852144 = 0.711770 **(4)**

[12 Marks]

Solution 9:

Define a service table:

l_{x+t} = Number of members aged x + t last birthday
e_{x+t} = Number of members who retire early aged x + t last birthday
s_{x+t} / s_x = Ratio of earnings in the year of age x + t to x + t + 1 to the earnings in the year of age x to x + 1
Define $z_{x+t} = \frac{1}{5}(s_{x-5} + s_{x-4} + s_{x-3} + s_{x-2} + s_{x-1})$

$\bar{a}_x^e$= Value of annuity of 1 p.a. to an early retiree aged exactly x + t.
Let (AS) be the member's expected salary earnings in the year of age 52 to 53.
Assume that early retirements take place uniformly over the year of age.

Consider early retirement between ages 52 + t and 52 + t + 1, t < 11.

The present value of the retirement benefits related to future service:

$$\frac{(t+{}^1\!/_2)\,(AS)}{50}\frac{z_{52+t+{}^1\!/_2}}{s_{52}}\frac{v^{52+t+{}^1\!/_2}}{v^{52}}\frac{e_{52+t}}{l_{52}}\bar{a}^e_{52+t+{}^1\!/_2} = \frac{(t+{}^1\!/_2)\,(AS)}{50}\frac{{}^zC^{ea}_{52+t}}{{}^sD_{52}}$$

where ${}^zC^{ea}_{52+t} = z_{52+t+{}^1\!/_2}v^{52+t+{}^1\!/_2}e_{52+t}\bar{a}^e_{52+t+{}^1\!/_2}$

and ${}^sD_{52} = s_{52}v^{52}l_{52}$

Similarly it may be shown that the present value of the benefits is, in total:

$$\frac{(AS)}{50\ {}^sD_{52}}\left[{}^1\!/_2\ {}^zC^{ea}_{52} + 1\,{}^1\!/_2\ {}^zC^{ea}_{53} + \ldots + 10\,{}^1\!/_2\ {}^zC^{ea}_{62} + 11\ {}^zC^{ea}_{63} + 11\ {}^zC^{ea}_{64}\right]$$

$$= \frac{(AS)}{50\ {}^sD_{52}}\left[{}^1\!/_2\ {}^zC^{ea}_{52} + 1\,{}^1\!/_2\ {}^zC^{ea}_{53} + \ldots + 10\,{}^1\!/_2\ {}^zC^{ea}_{62} + 11\,{}^1\!/_2\ {}^zC^{ea}_{63} + 12\,{}^1\!/_2\ {}^zC^{ea}_{64} - \left({}^1\!/_2\ {}^zC^{ea}_{63} + 1\,{}^1\!/_2\ {}^zC^{ea}_{64}\right)\right]$$

$$= \frac{(AS)}{50\ {}^sD_{52}}\left[{}^z\bar{M}^{ea}_{52} + {}^z\bar{M}^{ea}_{53} + \ldots + {}^z\bar{M}^{ea}_{64} - \left({}^z\bar{M}^{ea}_{63} + {}^z\bar{M}^{ea}_{64}\right)\right]$$

where ${}^z\bar{M}^{ea}_x = \sum_{t=0}^{64-x} {}^zC^{ea}_{x+t} - {}^1\!/_2\ {}^zC^{ea}_x$

$$= \frac{(AS)}{50\ {}^sD_{52}}\left[{}^z\bar{R}^{ea}_{52} - {}^z\bar{R}^{ea}_{63}\right]$$

where ${}^z\bar{R}^{ea}_x = \sum_{t=0}^{64-x} {}^z\bar{M}^{ea}_{x+t}$

Similarly it may be shown that the present value of benefits related to past service is:

$$\frac{19(AS)}{50\ {}^sD_{52}}\ {}^zM^{ea}_{52}$$

where ${}^zM^{ea}_{52} = \sum_{t=0}^{12} {}^zC^{ea}_{52+t}$

[8 Marks]

Solution 10:

i) The vector of balancing items in the projected revenue accounts for each policy year is called the profit vector. Profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year. **(1)**

ii) The objective specified for expected level of profit is termed as "profit criterion".
E.g. NPV = 40% of Initial Sales Commission
Profit Margin = 3% of the EPV of Premium Income **(2)**

iii) Risk discount rate = risk free rate + margin for risk
= 10% + 3%
= 13%

EPV profit $_{(@13\%)}$ = $-250v + 150v^2 + 200v^3 + 225v^4$, where $v = {}^{1}/_{1.13}$
= 172.84

Let the first year premium be P.

EPV premiums $_{(@13\%,\ AM92\ Ultimate)}$ = $P\,(1 + 1.04\,{}_1p_{42}v + 1.04^2\,{}_2p_{42}v^2 + 1.04^3\,{}_3p_{42}v^3)$
= P (1 + 0.919338 + 0.845094 + 0.776754)
= 3.541186 P

Profit margin = (EPV profit / EPV premiums)

$5\% = \frac{172.84}{3.541186\ \mathrm{P}}$

$\mathrm{P} = 976.17$

The company must charge a premium of Rs 976.17 in the first year of the contract. **(5)**

[8 Marks]

Solution 11:

i) *Experience basis* is used to calculate the expected future profits of a contract for comparison with a stated profit criterion. It represents the best estimate of expected future experience.

Pricing basis is used while setting premium. Assuming that the risk discount rate reflects fully the uncertainties in the assumptions, the pricing basis represents the insurer's realistic expected outlook.

Valuation (or reserving) basis is used to calculate the reserves to be held by an Insurer. It represents prudent assumptions (pessimistic as compared to best estimate) of expected future experience. **(3)**

ii) While the student actuary is correct that reserves are held such that there is an acceptably low probability of insolvency occurring in the future, it is incorrect to assume that everyone will die on the day after the valuation date. This is because:

- A company whose reserving basis is extremely pessimistic will be holding extremely high reserves leading to large capital requirement. This will require higher profits resulting in higher premiums/charges for customer.
- The reserving basis chosen has to satisfy any local legislation and professional guidance which exists to protect the interests of the policyholders. **(2)**

iii) We need the profit vector of the policy in order to calculate the non unit reserves. It can be derived by dividing the given profit signature with the probability of surviving to the start of that year.

$$= \left(-20, {}^{-35.20}/_{{}_1\mathrm{p}_{[70]}}, {}^{50}/_{{}_2\mathrm{p}_{[70]}}, {}^{-28.50}/_{{}_3\mathrm{p}_{[70]}}, {}^{-20}/_{{}_4\mathrm{p}_{[70]}}, {}^{90.21}/_{{}_5\mathrm{p}_{[70]}}\right)$$

$$= (-20, -35.79, 52.12, -30.65, -22.27, 104.40)$$

Let reserves required at the end of policy year n be denoted as ${}_nV$.
No reserve will be required at the end of the fifth year. ${}_5V = 0.$

${}_4V = \frac{22.27}{1.06} = 21.01$

After setting up ${}_4V$, the revised profit in year five will be 0.

${}_3V = \frac{(30.65 + 21.01\ {}_1p_{73})}{1.06} = 48.06$

After setting up ${}_3V$, the revised profit in year four will be 0.

The cashflow of year 3 is sufficient to set up this reserve. Hence ${}_2V$ = 0.
Allowing for this, the revised profit in year three will be
$= 52.12 - 48.06\ {}_1p_{72}$
$= 5.54$

${}_1V = \frac{35.79}{1.06} = 33.76$

After setting up ${}_1V$, the revised profit in year two will be 0.

The revised profit in year one will be
$= -20 - 33.76\ {}_1p_{[70]}$
$= -53.20$

The revised profit vector is
$(-53.20, 0, 5.54, 0, 0, 104.40)$

(7)

[12 Marks]

Solution 12:

i) The q_x curve is a rising curve, starting near zero and reaching one as age tends to the limiting age (usually 120). Its main feature is the rapid, in fact nearly exponential increase beyond middle age.

The d_x curve starts near zero and keeps slightly increasing. The increase becomes steep around middle age and the curve peaks at later ages. Subsequently it starts falling and reaches zero as age tends to the limiting age.

The l_x curve starts at the chosen radix and decreases very slightly until middle age, followed by a steep plunge. It reaches zero as age tends to the limiting age. **(3)**

ii)

a) Occupation
It determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances or potentially dangerous situations. Some occupations are more healthy by their very nature. Occupation also determines income which permits to adopt a particular standard of living.

b) Nutrition
Poor quality nutrition can increase risk of contracting diseases and hinder recovery from sickness which can influence mortality in the longer term. Similarly, excessive or inappropriate nutrition can lead to obesity and an increased risk of associated diseases leading to higher mortality.

c) Climate and Geographical location
Levels and patterns of rainfall and temperature lead to an environment that is amicable to certain types of diseases. Further, the following will also vary according to geographical location: Access to medical care and transport, Road accidents, Natural Disasters and Political unrest.

d) Education
Education influences the awareness of the components of a healthy lifestyle which lowers mortality rate. This effect manifests itself through many proximate determinants such as increased income, choice of a better diet, the taking of exercise, moderation in consumption of alcohol and smoking, awareness of dangers of drug abuse, awareness of a safe sexual lifestyle etc. **(4)**

[7 Marks]

Solution 13:

Actual number of deaths for Mumbai = 20 + 23 + 22 = 65

$$\text{Expected deaths for Mumbai} = \frac{\text{Deaths in India (Standard Population)}}{\text{Exposed in India (Standard Population)}} \times \text{Exposed in Mumbai}$$

Age 58: $\frac{400}{50000} \times 3500 = 28$

Age 59: $\frac{380}{40000} \times 2600 = 24.7$

Age 60: $\frac{330}{25000} \times 2000 = 26.4$

$$\text{Standardised mortality ratio} = \frac{65}{28 + 24.7 + 26.4} = 82.17\%$$

[3 Marks]

****