Institute of Actuaries of India

Subject CT3 – Probability & Mathematical Statistics

March 2017 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) Ordering the marks given, stem and leaf diagram is:

1	7, 9
2	1, 1, 2, 2, 6, 6, 7, 7, 7, 7
3	0, 0, 1, 2, 3, 3, 3, 4, 6
4	6
5	3
6	0

The stems are 10s and leaves are units. **(2)**

ii) Median: $\left(\frac{1}{2}n + \frac{1}{2}\right)^{th}$ value =12.5th value =(27+30)/2 =28.5.

Mode: 27. (27 appears the maximum number of times-four times) **(1)**

iii) Interquartile Range (IQR) = $Q_{3-}Q_1$

Now Q_1 = $\frac{n+2}{4}$ th value counting from below = 6.5th value

= (22+26)/2 = 24

Q_1 = $\frac{n+2}{4}$ th value counting from above =33

Hence, $IQR = Q_{3-}Q_1$ =33-24 =9

[*Alternatively,*

Q_1 = $\frac{n+1}{4}$ *th value counting from below = 6.25th value =23 and*

Q_3= $\frac{n+1}{4}$ *th value counting from above= 33*

Hence IQR = 33-23=10] **(2)**

[5 Marks]

Solution 2:

The binomial distribution (n, p) has probability function.

$$P\,(X = x) = \frac{n\,!}{(n-x)!\,x\,!}\,p^x\,(1-p)^{(n-x)}\;;\quad x=0,1,2,\ldots;\ 0<p<1.$$

and $P\,(X = x - 1) = \frac{n\,!}{(n-x+1)!(x-1)\,!}\,p^{(x-1)}\,(1-p)^{(n-x+1)}$; *x=1,2,3,… ;0<p<1*

Now, $\frac{P\ (X = x)}{P\ (X = x - 1)} = \frac{n-x+1}{x} \frac{p}{1-p}$

Therefore, $P\ (X = x) = \frac{n-x+1}{x} \frac{p}{1-p} P\ (X = x - 1)$; $x=1,2,3,\ldots$

[3 Marks]

Solution 3:

Given that $P(A) = 0.20, P(B) = 0.10 \text{ and } P(C) = 0.70$

Let *R* be the event that he reaches office on time.

$P(R^c/A)$= 0.15, $P(R^c/B)$=0.20, $P(R^c/C)$= 0.05. [R^c: $complement$ of *R*]

Using Bayes' Theorem:

$$P\ (B \mid R^c) = \frac{P\ (R^c|B)\ P\ (B)}{P\ (R^c|A)\ P\ (A)+P\ (R^c|B)\ P\ (B)+P\ (R^c|C)\ P\ (C)}$$

$$= \frac{(0.20)\ (0.10)}{(0.15)\ (0.20)+\ (0.20)\ (0.10)+\ (0.05)\ (0.70)} = 0.235$$

[3 Marks]

Solution 4:

i) $X_i \sim Exp\ (0.002)$, so $E\ [X_i] = 500$; $Var\ [X_i] = 250000$
$N \sim Bin\ (n, p)$, so $E\ [N] = np$; $Var\ [N] = np(1-p)$

Using the formulae given on page 16 of the *Tables*:

$E\ [S] = (np)\ (500) = 500np$

$Var\ [S] = (np)(500)^2 + (np(1-p))(500)^2 = (2np)(500)^2 - np^2(500)^2$

$=(np)(500)^2(2-p)$

$SD\ [S] = 500 * \sqrt{np * (2-p)}$ **(3)**

ii) Using Normal approximation, we have:

$S \sim N\ (500np, (np)(500)^2(2-p))$,

Hence, $P\ [S > 60000\ ] = P\left(Z > \frac{60000-500np}{500*\sqrt{np*(2-p)}}\right)$

$= 1\text{-}\emptyset\ ([\frac{120-np}{\sqrt{np*(2-p)}}]$ **(3)**

[6 Marks]

Solution 5:

i) Given that $f_{X,Y}(x,y) = \frac{12}{5}(x^2y + xy)\ ; 0 < x, y < 1.$

The marginal pdf of X : $h(x) = \int_0^1 \frac{12}{5}(x^2y + xy)\,\mathrm{d}y = \left[\frac{12}{5}\left(\frac{1}{2}x^2y^2 + \frac{1}{2}\mathrm{x}y^2\right)\right]_0^1$

$= \frac{12}{5}\left(\frac{1}{2}x^2 + \frac{1}{2}x\right)\ ;\quad 0 < x < 1.$

$= \frac{6}{5}\ (x^2 + x);\ \ 0 < x < 1$

The marginal pdf of Y : $g(y) = \int_0^1 \frac{12}{5}(x^2y + xy)\,\mathrm{d}x = \left[\frac{12}{5}\left(\frac{1}{3}x^3\mathrm{y} + \frac{1}{2}x^2y\right)\right]_0^1$

$= \frac{12}{5}\left(\frac{1}{3}\mathrm{y} + \frac{1}{2}y\right);\ \ 0 < y < 1.$

$= 2y\ \ ;\quad 0 < y < 1$ **(3)**

(ii) Clearly $f_{X,Y}(x,y) = h(x)g(y)$. The random variables are statistically independent. **(1)**

(iii) $E[X]$ $= \int_0^1 \frac{12}{5}\left(\frac{1}{2}x^3 + \frac{1}{2}x^2\right)\mathrm{d}x = \left[\frac{12}{5}\left(\frac{1}{8}x^4 + \frac{1}{6}x^3\right)\right]_{x=0}^1$

$= \frac{12}{5}\left(\frac{1}{8} + \frac{1}{6}\right) = 0.7$

$E[Y]$ $= \int_0^1 \frac{12}{5}\left(\frac{1}{3}y^2 + \frac{1}{2}y^2\right)\mathrm{d}y = \left[\frac{12}{5}\left(\frac{1}{9}y^3 + \frac{1}{6}y^3\right)\right]_{y=0}^1$

$= \frac{12}{5}\left(\frac{1}{9} + \frac{1}{6}\right) = 0.67.$ **(3)**

(iv) We know that $E(X/Y) = \int_0^1 x\,f(x|y)dx = \int_0^1 x\,\frac{f(xy)dx}{f(y)} = \int_0^1 \frac{\frac{12}{5}(x^3y + x^2y)}{2y}dx$

$= (6/5)\int_0^1 (x^3 + x^2)\,dx = (6/5)\left[\left(\frac{1}{4}x^4 + \frac{1}{3}x^3\right)\right]_{x=0}^1$

$= \frac{6}{5} \times \frac{7}{12} = \frac{7}{10}$

$$E(E(X/Y)) = \int_0^1 \left(\frac{7}{10}\right) f(x)dx = \int_0^1 \left(\frac{7}{10}\right) f(x)dx$$

$$= \int_0^1 \left(\frac{7}{10}\right) \frac{12}{5} [\tfrac{1}{2} (x^2 + x)]dx$$

$$= \frac{7}{10} \times \frac{12}{10} \left[\left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\right]_{x=0}^{1} = \frac{7}{10} \times \frac{12}{10} \times \frac{5}{6} = 0.7 = E(X).$$

$$E(X^2/Y) = \int_0^1 x^2 f(x|y)dx = \int_0^1 x^2 \frac{f(xy)dx}{f(y)} = \int_0^1 \frac{\frac{12}{5}(x^4y + x^3y)}{2y}dx$$

$$= (6/5)\int_0^1 (x^4 + x^3)\, dx$$

$$= (6/5) \left[\left(\frac{1}{5}x^5 + \frac{1}{4}x^4\right)\right]_{x=0}^{1} = \frac{6}{5} \times \frac{9}{20} = \frac{54}{100}.$$

$$V(X/Y) = E(X^2/Y) - (E(X/Y))^2 = \frac{54}{100} - (\frac{7}{10})^2 = \frac{1}{20}.$$

[*If the candidate has answered using* $E[X/Y] = E[X]$ and $V[X/Y] = V[X]$ *on computation of* $E[X^2]$ *and* $V[X]$ *full credit is to be given* **(5)**

[12 Marks]

Solution 6:

i) By definition, the moment generating function of Gamma(α, λ):

$$M_X(t) = E\,(e^{tX}) = \int_0^\infty \left(e^{tx} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-\lambda x}\right) dx$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty \left(x^{\alpha-1}e^{-(\lambda-t)x}\right) dx$$

$$M_X(t) = \frac{\lambda^\alpha}{(\lambda-t)^\alpha} \int_0^\infty \left(\frac{(\lambda-t)^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-(\lambda-t)x}\right) dx$$

The integrand is pdf of *Gamma* $(\alpha, \lambda - t)$ and the value of integral is 1.

Therefore, $M_X(t) = \left(\frac{\lambda}{\lambda-t}\right)^\alpha$, provided $t < \lambda$.

Dividing the numerator and denominator by λ gives:

$$M_X(t) = \left(\frac{1}{1-\frac{t}{\lambda}}\right)^\alpha = \left(1 - \frac{t}{\lambda}\right)^{-\alpha} \qquad t < \lambda$$

The cumulant generating function is:

$$C_X(t) = log\, M_X(t) = -\alpha\, log\left(1 - \frac{t}{\lambda}\right)$$ **(4)**

ii) The coefficient of skewness $= \frac{Skew(X)}{Var(X)^{1.5}}$

We know that $Var(X) = C_X''(0)$ and $Skew(X) = C_X'''(0)$

$$C_X'(t) = \frac{\alpha}{\lambda}\left(1 - \frac{t}{\lambda}\right)^{-1}$$

$$C_X''(t) = \frac{\alpha}{\lambda^2}\left(1 - \frac{t}{\lambda}\right)^{-2} \Rightarrow \quad Var(X) = C_X''(0) = \frac{\alpha}{\lambda^2}$$

$$C_X'''(t) = \frac{2\,\alpha}{\lambda^3}\left(1 - \frac{t}{\lambda}\right)^{-3} \Rightarrow \quad Skew(X) = C_X'''(0) = \frac{2\,\alpha}{\lambda^3}$$

Hence, the coefficient of skewness $= \frac{2\,\alpha/_{\lambda^3}}{\left(\alpha/_{\lambda^2}\right)^{1.5}} = \frac{2\,\alpha}{\alpha^{1.5}} = \frac{2}{\sqrt{\alpha}}$ **(4)**

[8 Marks]

Solution 7:

Let *X* denote the number of shares bought (or sold if *X* is negative) during each trading session.

Based on the strategy adopted, we have the following distribution of *X*.

x	-1	0	1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

In order to apply normal approximation, we need to calculate *E* [*X*] and *Var* [*X*].

$$E\,[X] = \sum x\, P\,(X = x) = (-1)\frac{2}{6} + (0)\frac{1}{6} + (1)\frac{2}{6} + (2)\frac{1}{6} = \frac{1}{3}$$

$$E\,[X^2] = \sum x^2\, P\,(X = x) = (-1^2)\frac{2}{6} + (0^2)\frac{1}{6} + (1^2)\frac{2}{6} + (2^2)\frac{1}{6} = \frac{4}{3}$$

$$Var\,[X] = E\,[X^2] - (E\,[X])^2 = \frac{4}{3} - \frac{1}{9} = \frac{11}{9}$$

Let Y denote the total number of shares after 18 trading sessions:
i.e., $Y = 100 + \sum X$

It is clear the $E(Y) = 100 + (18)\frac{1}{3}$ $and\ Var(Y) = (18)\frac{11}{9}$

Using Central Limit Theorem, we have

$P(Y > 110) = P(Y > 110.5)$ using continuity correction.

$= P\left(Z > \frac{110.5 - 106}{\sqrt{22}}\right)$

$= P(Z > 0.9594) = 1 - P(Z < 0.9594)$

$= 1 - 0.8313 = 0.1687$ **[5 Marks]**

Solution 8:

i) The survival times follow Exponential distribution with mean $\frac{1}{\lambda}$.

Let $X_1, X_2, \dots, X_n$ be a random sample

The method of Moments estimator is obtained by equating Population Mean and sample mean.

That is, $\frac{1}{\lambda} = \bar{x}$

For the given data, the MM estimate is $\frac{1}{5}$ **(2)**

ii) Then the likelihood function of the random sample is

$$L = L(\lambda / x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} \lambda\, e^{-\lambda x_i}$$

$$= \lambda^n e^{-\lambda \sum x_i}$$

$$\log L = n \log \lambda + \lambda \sum_{i=1}^{n} x_i$$

$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n} x_i$ implies that $\widehat{\lambda} = \frac{1}{5}$

Checking for maximum $\frac{\partial^2 \log L}{\partial \lambda^2} = -\frac{n}{\lambda^2} < 0$.

For the given data, $\bar{x} = 5$. Therefor the ML estimate is: $\frac{1}{5}$. **(3)**

iii) Estimate for $P(X > x) = e^{-\widehat{\lambda x}}$

(By invariance property of MLE)

Hence, the ML estimate for $P(X > 50\, hours) = P(X > \frac{5}{72} months)$ is $e^{-\widehat{\lambda x}} = e^{-\frac{1}{5} * (\frac{5}{72})} = e^{-(\frac{1}{72})} =$ 0.986207

(Assuming 30 days is 1 month) **(2)**

iv) The CRLB for the variance of an unbiased estimator for is λ is computed as

$$-\frac{1}{E\left[\frac{\partial^2 \log L}{\partial \lambda^2}\right]} = \frac{\lambda^2}{n}$$

For the given data, the lower bound for the variance of an unbiased estimator is $\frac{1}{5^2 * 10} = \frac{1}{250}$= 0.004.

(1)

[8 Marks]

Solution 9:

i) Let true mean weight be μ. Then for the sample $X_1, X_2, \dots, X_n$ from store *A*

$$\frac{\bar{\bar{X}}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$

Hence, $\bar{X} - 2.262\frac{S}{\sqrt{n}} < \mu < \bar{X} + 2.262\frac{S}{\sqrt{n}}$

From the given data, $\sum x_i = 660 \; and \; \sum x_i^2 = 44{,}650.$

Hence, $\bar{x} = 66$ and $s^2 = \frac{1}{9}\left(44650 - 10(66)^2\right) = 121.11$

and $s = 11.005$

Therefore, $\bar{x} - 2.262\frac{S}{\sqrt{n}}$ = 66- (2.262× 11.005/√10)= 58.128 and

$\bar{x} + 2.262\frac{S}{\sqrt{n}}$ = 66+ (2.262 × 11.005/√10)=73.872

Therefore, a 95% confidence interval for the true mean is (58.128, 73.872) **(4)**

(ii) It is required that $2\,\frac{S}{\sqrt{n}}\,t_{n-1;0.025} \le 10$

Using trial method when *n*=22,

$2\,\frac{11.005}{\sqrt{n}}\,t_{n-1;0.025}$ = 4.692548 ×2.08 =9.7605 ≅ 10

Therefore, a minimum sample size of at least 22 is required. **(2)**

(iii) Let the random sample from store *B* be $Y_1, Y_2, \dots, Y_m$,

Then from the data,

$\bar{y} = \frac{816}{12}$ = 68 and $s_y^2 = \frac{1}{11}$(56,644 - 12×68^2) = 105.0909

The pooled variance s_p^2 = [(9× 121.1111)+(11×105.0909)]/(10+12-2)

= 112.30

Let μ_X and μ_y be the true mean weights of adults who visited shops *A* and *B* respectively.

Then $\frac{(\bar{X}-\bar{Y})-(\mu_X - \mu_y)}{\sqrt{s_p^2\,(\frac{1}{n}+\frac{1}{m})}} \sim t_{n+m-2}$

Now, From the *t* table,

$$-2.086 < \frac{(\bar{X}-\bar{Y})-(\mu_X - \mu_y)}{\sqrt{s_p^2\,(\frac{1}{n}+\frac{1}{m})}} < 2.086$$

$$-2.086 < \frac{(-2)-(\mu_X - \mu_y)}{\sqrt{112.30*(\frac{1}{10}+\frac{1}{12})}} < 2.086$$

$$-2.086 < \frac{(-2)-(\mu_X - \mu_y)}{4.5374} < 2.086$$

$$-11.465 < \mu_Y - \mu_X < 7.465$$

As this interval contains 0, there is evidence at 5% level to suggest no significant difference in the weights measured at two different shops. **(4)**

(iv) Here n=10 and m=12

Using $\frac{s_x^2}{S_y^2} / \frac{\sigma_x^2}{\sigma_y^2} \sim F_{n-1,\ m-1}$

i.e $\frac{s_x^2}{S_y^2} / \frac{\sigma_x^2}{\sigma_y^2} \sim F_{9,\ 11}$

$$\frac{s_x^2}{S_y^2}\frac{1}{F9{,}11} < \frac{\sigma_x^2}{\sigma_y^2} < \frac{s_x^2}{S_y^2} F_{11,9}$$

$$1.152441 * \frac{1}{2.896} < \frac{\sigma_x^2}{\sigma_y^2} < 1.152441 * 3.105$$

$0.397942 < \frac{\sigma_x^2}{\sigma_y^2} < 3.578329$

Since this confidence interval contains 1, variances of weights of adults in both shops may be assumed to be equal. **(3)**

[13 Marks]

Solution 10:

i) Let X have pdf : $f_X(x) = \theta x^{\theta-1}$; $0 < x < 1;\ \theta > 0$

The desired test is: The Most Powerful test of level α = 0.05 for testing the null simple hypothesis H_0: $\vartheta = 5$ against the simple alternative H_1: $\vartheta = 4$, based on a random sample of size 1, x (say) .

By Neyman Pearson lemma, we have

Reject H_0: $\vartheta = 5$ if $\frac{f(\theta_1,x)}{f(\theta_0,x} > k$, where k is such that

$P_{H_0}(\text{Reject } H_0) = \alpha$.

That is, Reject H_0: $\vartheta = 5$, if $\frac{4x^3}{5x^4} > k$ or $\frac{4}{5x} > k$.

This means that reject H_0 if $x < \frac{4}{5}k$ (=k^*) where k^* is such that

$P_{\theta=5}(x < k^*) = 0.05$.

That is $\int_0^{k^*} 5\,x^4\, dx = 0.05$

This implies $k^* = \sqrt[5]{0.05}$ = 0.54928 which means reject H_0: $\vartheta = 5$ if

$x < 0.54928$. **(4)**

ii) Power = P (Reject H_0 when H_o is false) = P ($x < 0.54928$ when $\vartheta = 4$)

$= \int_0^{0.54928} 4\,x^3\, dx = 0.54928^4 = 0.09103$ **(2)**

[6 Marks]

Solution 11:

i) The Least Squares Estimates for :

μ = $\sum_i \sum_j yij\ /n$ =138/19 = 7.2632

$\tau_1 = \frac{\sum_j y_{ij}}{ni} - \frac{\sum_{ij} y_{ij}}{n}$

τ_1= 33/4 -7.2632 = 0.9868

τ_2= 30/4 -7.2632 = 0.2368

τ_3 = 40/6 -7.2632 =-0.5965

τ_4= 35/5 -7.2632 =-0.2632 **(4)**

ii) Assumption:

Observations are from normal populations with same variance.

Hypotheses:

H_0: Each brand has same average lifetime of devices.

H_1: There are differences between the average lifetimes of devices manufactured by different brands.

We have

SST = $\sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{n}$ = 1100 - $\frac{138^2}{19}$=97.68421

SSB = $\sum \frac{y_{i.}^2}{ni} - \frac{y_{..}^2}{n}$ =($\frac{33^2}{4} + \frac{30^2}{4} + \frac{40^2}{6} + \frac{35^2}{5}$)- $\frac{138^2}{19}$ = 6.60088

SSR= SST-SSB =97.68421-6.60088 =91.0833

ANOVA table:

Source of Variation	*DF*	*Sum of Squares*	*Mean Squares*
Between Treatments	3	6.60088	2.20029
Residuals	15	91.08333	6.07222
Total	18	97.68421	

The variance ratio $F = \frac{2.20029}{6.07222}$ =0.3623

Under *Ho,* this has an $F_{3,15}$ distribution. The 5% critical point for $F_{3,15}$ is 3.287, so we have no evidence to reject H_0 and hence we conclude that there is no difference between average lifetimes of devices manufactured by different brands. **(7)**

iii) The unbiased estimate of σ^2 = SSR/ *(n-k)* = 91.083333/15 =6.07222

We know that SSR/ $\sigma^2 \sim \chi^2_{n-k}$

We have χ^2_{15}, and hence P(6.262< SSR/ σ^2 <27.49)

So, 95% CI for σ^2 is $\left(\frac{91.08333}{27.49}, \frac{91.08333}{6.262}\right)$ = (3.3133, 14.5454) **(3)**

iv) We have $\bar{y}_1$ = 8.25; $\bar{y}_2$ =7.50; $\bar{y}_3$ =6.67 $\bar{y}_4$ =7.00;

Hence $\bar{y}_3 < \bar{y}_4 < \bar{y}_2 < \bar{y}_1$

Considering the highest and the lowest sample means

$t_{(0.025,n\text{-}k)}\, \hat{\sigma} \left(\frac{1}{n1} + \frac{1}{n3}\right)^{0.5}$ =2.131× 2.4642 × $\left(\frac{1}{6} + \frac{1}{4}\right)^{0.5}$ =3.39

The least significant difference 3.39 is more than the difference of 1.58 (= 8.25 - 6.67) between the sample average lifetimes of the devices selected from brand 1 and brand 3. Hence we would be indifferent to select brand 1 and brand 3. **(3)**

[17 Marks]

Solution 12:

i) Given that $\bar{x}$ = 30.667 $\bar{y}$ =69.167

$S_{xx} = \sum x^2 - n\,\bar{x}^2$ =13854-12× $\left(\frac{368}{12}\right)^2$ = 13854-11285.33 =2568.67

$S_{yy} = \sum y^2 - n\,\bar{y}^2$ =60,900-12× $\left(\frac{830}{12}\right)^2$ = 60,900-57408.33=3491.67

$S_{xy} = \sum xy - n\bar{x}\,\bar{y}$ =28,180-12×30.667×69.167 = 28180-25453.733=2726.27

$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{2726.27}{2568.67}$ =1.0615

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ =69.167 -1.0615×30.667 =36.6140

$\hat{y}$ =36.6140 + 1.0615 *x* **(4)**

ii) We are carrying out a test of

H_0: β=1 *vs* H_1: β >1

`We know that $\frac{\hat{\beta}-\beta}{\sqrt{\frac{\sigma^2}{S_{xx}}}} \sim t_{n-2}$

$\sigma^2 = \frac{1}{(n-2)} \times \left(S_{yy} - \frac{S_{xy}^2}{S_{xx}}\right) = \frac{1}{10} \times \left(3491.67 - \frac{2726.27^2}{2568.67}\right)$ =59.813

The value of test statistic is

$$\frac{1.0615-1}{\sqrt{\frac{59.813}{2568.67}}} = 0.403$$

At 5%, the critical value for t_{10} from table is 1.812. The critical value is greater than 0.403. Hence, we have insufficient evidence to reject H_0. Hence, it is reasonable to assume that the β, gradient parameter is 1. **(4)**

iii) The proportion of variability explained by this model is

$$R^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}} = \frac{2726.27^2}{2568.67*3491.67} = 82.87\%$$

This tells us that 82.87% of the variation in the data can be explained by the model and indicated the good fit of the model. **(2)**

iv) We know from part (i) that

$\hat{y}$ =36.614 + 1.0615 x

$\hat{y}$=36.614 + (1.0615×34) =72.705

Standard error of the estimate

$$\sqrt{\sigma^2\{1+\frac{1}{n}+\frac{(x_i-\bar{x})^2)}{S_{xx}}\}}$$

$$=\sqrt{59.813\ \times\ (1+\frac{1}{12}+\frac{(34-30.667)^2}{2568.67})}$$

=8.066

95% confidence interval for daily maintenance cost $\hat{y}$ *is* 72.705 $\pm\, t_{0.025,n-2}$ SE($\hat{y}$)

=72.705 $\pm$ 2.228×8.066 = (54.734, 90.676) **(4)**

[14 Marks]

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