# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 24th March 2017

## Subject ST6 – Finance and Investment B

## Time allowed: Three Hours (10.15* – 13.30 Hours)

## Total Marks: 100

### INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. ** You have 15 minutes at the start of the examination in which you are required to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*
3. *You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*
4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
5. *Attempt all questions, beginning your answer to each question on a separate sheet.*
6. *Mark allocations are shown in brackets.*
7. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

| AT THE END OF THE EXAMINATION |
|---|
| **Please return your answer booklet and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.** |

**Q. 1)** An insurance company has approached an investment bank to structure its assets and liabilities book to be sold as a single asset to a group of investors who have large cash flows coming in 1 year's time. The insurance company has the following assets.

| Assets | Current Market Value | Redemption Value | Time to Maturity (In Years) | Coupon Rate | Yield to Maturity |
|---|---|---|---|---|---|
| Corporate bonds | 1102 | 1000 | 4 | 10% | 7% |
| Risk free government bonds | 1305 | 1000 | 4 | 10% | 2% |

All the yields and coupon rates are quoted with annual compounding. Coupons on both the bonds are paid once in a year.

The company also has a direct property investment of market value of 2000 and annual rental yield of 5%.

The liability cash flows are shown in the table below

| Liability | Cash Flows |
|---|---|
| 1 | 0 |
| 2 | (1500) |
| 3 | 0 |
| 4 | (2700) |

All the market value and redemption value figures are in Crores.

The investment consultant suggested first stage approach: Convert property into bond type instruments with all properties sold at the end of year 4 by converting into asset backed securities.

**i)** Explain what asset backed securities (ABS) are and how they can be issued? (4)

**ii)** Construct the cash flows for the ABS created from the property. Use the following assumptions: annual rental income of 120, annual maintenance expenses of 20 and annual growth rate in the market value of property of 5%. State the key risks of the structure (4)

The second stage of the process is to construct an asset of the overall company by looking at future cash flows of the company by including assets redemptions, liability payouts, bank interest earned on cash or overdraft charges (if liability cash flows exceed assets), expected default of assets, expected void experiences.

**iii)** Construct the cash flows for all assets using the following assumptions (4)

| | |
|---|---|
| Interest rate | 2% |
| Expected Corporate bond default | 2% |
| Property void or property ABS expected default | 4% |
| Reinvestment rate | 5% |
| Overdraft rate | 10% |
| Investor profit | 8% |

All the rates mentioned above are annual rates.

**iv)** Using Liability cashflows, determine asset shortfall or excess each year. (2)

**v)** Using the discount rate of 10% per annum (compounded annually), value the single asset representing the insurance company (6)

**vi)** State what risks and issues the insurance company needs to be aware of before accepting the transaction. (3)

**[23]**

**Q. 2)** An insurance company has exposure which pays out benefit every six months starting 5 years from now and continuing for a further 3 years. This exposure has a risk of falling future interest rates. The risk appetite of the company allows the risk of (-) 50 basis points movement but beyond that the company needs to hedge the risk. The investment department has suggested the use of swaptions to hedge the risk. The board has accepted recommendation of swaption purchase. Suppose that the MIBOR yield curve is flat at 6% per annum with continuous compounding and the volatility of the forward swap rate is 20%.

**i)** Discuss how swaptions can be used to hedge falling interest rates and assess any basis or residual risk. (3)

**ii)** Calculate the cost of hedging the current value of exposure of Rs. 100 Crores. (5)

**iii)** Now assume that the board had reviewed the proposal and suggested the hedging cost is prohibitive for the business. The investment analyst suggested using a swaption collar instead of a simple swaption. Discuss how a swaption collar (with equal and opposite strike) would help to reduce the cost of hedging and cover risk. Use a cash flow diagram to demonstrate hedging and residual risks. Also calculate the price of the swaption collar using the above mentioned information. (8)

**iv)** Now assume that the company's risk appetite also prohibits interest rate exposure above 150 basis points. Suggest a new hedging structure (assuming a collar has already been purchased) which would ensure the risk appetite is not breached and demonstrate hedging by a cashflow diagram. Calculate the price of the overall hedging

(7)

**[23]**

**Q. 3)** Consider two random variables $s_1$ and $s_2$ which follow the following stochastic process

$$ds_i = \mu_i s_i dt + \sigma_i s_i dz_i \;\; for\ i = 1, 2$$

Find the process followed by $s_1 s_2$ , if the correlation coefficient between $dz_1$ and $dz_2$ is $\rho_{12}$ **[4]**

**Q. 4)** You are observing bonds and swaps in a LIBOR based interest rate market. Column A gives the set of zero coupon bond prices that apply currently (each price is for a nominal face value of 100). Column B gives the prices of the same bonds after the yield curve has instantly shifted up by 10 basis points (0.10%) for all maturities.

| | A | B |
|---|---|---|
| Maturity (Years) | Current Zero Coupon Bond Price | Shifted Zero Coupon Bond Price |
| 1 | 95.238 | 95.147 |
| 2 | 90.488 | 90.316 |
| 3 | 85.770 | 85.526 |
| 4 | 81.492 | 81.183 |
| 5 | 77.796 | 77.428 |

The modified duration of a bond priced at par is 4.299 on current rates.

Using this information,

**i)** **a)** Calculate the value of a 5-year 10% annual coupon bond at current rates. (2)

**b)** Hence derive the value of 5-year annual fixed-floating swap with 10% fixed coupon. (2)

Let the "absolute yield sensitivity" of an instrument be defined as the instrument's value now less its value when rates have risen by a small increment ($\Delta$r), all divided by the small increment ($\Delta$r).

**ii)** **a)** Show how absolute yield sensitivity for a bond is related to its modified duration. (2)

**b)** Show numerically that the absolute yield sensitivities of the bond and swap in i) are the same. (3)

[For the purpose of this computation, assume that a 10 basis point shift is a small increment.]

A "reverse floater" is a bond that pays a coupon equal to a fixed value X less the value of LIBOR at each fixing. So, for example, if LIBOR is 5.75% at the next fixing, the coupon becomes X – 5.75%.

Consider a 5-year annual coupon reverse floater where X = 10%.

**iii)** **a)** Show how the cashflows of the reverse floater can be decomposed into those of a fixed-floating swap and a zero coupon bond. (3)

**b)** Hence demonstrate numerically that the modified duration of the reverse floater is approximately twice that of a bond priced at par. (4)

**iv)** Explain how you would risk manage, in terms of duration, a portfolio of bonds that includes a reverse floater. (3)

**[19]**

**Q. 5)** A fund manager has a well – diversified portfolio that tracks the performance of a certain index and is worth INR 275million. The current value of the index is 1,100. The manager would like to buy portfolio insurance against a reduction of more than 5% in the value of the portfolio over the next year. The risk free rate is 5% per annum with continuous compounding; the dividend yield on both the portfolio and the index is 3% per annum with continuous compounding while the market implied volatility for the index is currently 25% per annum.

**i)** Calculate the cost of hedging the portfolio using European put options. (4)

**ii)** Describe alternative strategies involving European call options which would have the same effect as the options in i). (4)

**iii)** Calculate the initial (delta) position if the manager sought to replicate the effect of the put options by investing part of the portfolio in risk-free securities. (3)

**iv)** Calculate the initial number of futures contracts required if, instead of risk-free securities in iii), the manager decided to use 9-month index futures contract. (3)

**[14]**

**Q. 6)** The price of a non-dividend paying equity worth 500 today has four possible values in a year's time: 300, 450, 550 or 700. The risk neutral probabilities with which the future prices would be reached are $p_1$, $p_2$, $p_3$ and $p_4$ respectively.

The problem in question is to calculate the appropriate risk-neutral probabilities for this equity using a single step quadrinomial (four branch) tree. The approach to be used is to calibrate the tree by calculating the four risk-neutral probabilities $p_1$, $p_2$, $p_3$ and$p_4$ so that the tree correctly prices:

- the equity
- one-year risk-free bonds
- a Call option on the equity with a strike of 600
- a Put option on the equity with a strike of 400

Assume that risk-free interest rates are 5% continuously compounded and market implied volatilities are 22.5% for both options.

**i)** **a)** Calculate the prices of the Call and Put options using the Black Scholes formula (5)
**b)** Calculate the four risk-neutral probabilities (5)

Now assume that risk-free interest rates remain at 5% continuously compounded but that market implied volatilities are such that a volatility of 20% applies to options with a strike of 600 and 25% applies to options with a strike of 400. This is called a "volatility skew".

**ii)** Explain (without calculating any of the values once again) how the Black-Scholes values of the Call and Put, and hence the four risk-neutral probabilities, will differ relatively from those found in i). (4)

**iii)** Sketch the risk neutral probability distribution against equity prices for both the scenarios; i.e., in the case of the flat volatility and the scenario when there is a volatility skew. What can you say about the tail behavior of the two curves at the low equity price end and the high equity price end? Also, what can be said about the mean of the distribution under volatility skew with respect to the distribution without the skew? (3)

**[17]**

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