INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

15th March 2017

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) **i)** Explain what is meant by a zero sum two person game? Explain what is meant by Minimax solution? (2)

ii) A man is standing at the bus stop and there are two bus routes, A and B. Buses of routes A and B travel 10 kms to and fro journey each but to the different destinations and the man does not know which bus goes to the correct destination that he wants to go. It is two times more likely that the bus of route A goes to the correct destination than that of route B. The man wants to minimize his expected travel time for the entire trip. Buses of route A are expected to take 20 minutes and of route B are expected to take 15 minutes for the journey. The return journey times from the destinations are same as the original journey times. Find out which Bus route the man chooses first and the expected travel time. You may assume that there is no waiting time at the bus stop. (4)

[6]

Q. 2) Y_t follows a time series model defined as below:

$Y_t = (1+\lambda)Y_{t-1} - \lambda Y_{t-2} + e_t$, where e_t follows $N(0,\sigma^2)$ and $Y_0 = 0$

i) Determine mle of λ and σ^2. (5)

ii) Using Yule-Walker equations, determine estimate of λ and σ^2 and comment for λ. (5)

[10]

Q. 3) A health insurance company is planning to reinsure its claims portfolio. The size of the claims follow log normal distribution with parameters $\mu = 9$, $\sigma = 0.9$. The insurer has the option of two reinsurance treaties as provided below:

1) Individual excess of loss with retention 30000
2) Proportional reinsurance with retained proportion x%

i) Calculate the expected cost of claims for the insurer without any treaty, and under treaty 1. (6)

ii) Calculate the value of x if the expected amount paid by the reinsurer in case of treaty 2 is same as in case of treaty 1. (All the figures are in INR) (4)

[10]

Q. 4) **i)** Describe following methods of random number generation:

a) Inverse transform method (2)

b) Acceptance rejection method (3)

ii) Describe the steps clearly to generate random variable (X) from Bin(n,0.4) distribution using a sequence of random numbers U(0,1) from uniform distribution. Write the complete expressions for X = 0,1,2,3,4 & n along with the generalised formula. (3)

iii) Give examples of cases where same set of random numbers must be used. (3)

[11]

Q. 5) In a General Insurance Company, cumulative claim amounts and projected ultimate claims are given the table below where ultimate claims are estimated using Basic Chain Ladder method with development year 3 as ultimate year and the development ratios for first two development years are equal.

Accident Year	**Development Year**				**Ultimate**
	0	**1**	**2**	**3**	
1	A	900	B	C	1600
2	700	D	1800		2100
3	850	1450			E
4	1000				2800

i) Calculate the missing values A, B, C, D & E. (5)

ii) If earned premium for accident year 3 is 3,000 and expected loss ratio is 80% then re-estimate 'E' using 'Bornhuetter-Ferguson' method. (1)

iii) If risk free discount rate is 6.0% per annum then calculate outstanding claim reserve on the final estimated table. (5)

[11]

Q. 6) A General Insurance Company sells various insurance and the details of number of policies sold and claim amounts are summarized in the table below:

Insurance	**Policies Sold**			**Claims (in Cr)**		
	Year 1	**Year 2**	**Year 3**	**Year 1**	**Year 2**	**Year 3**
Two Wheeler	500	750	650	400	500	550
Car	450	550	600	300	450	400
Truck	300	420	380	350	400	300

i) Estimate expected claim payments in next two years under Car Insurance using EBCT Model 1. (5)

ii) If we assume that the number of policies sold in each year is increasing at 5% per annum then explain how the above approach can be adjusted to derive revised estimates. (2)

iii) If the projected sales of truck insurance policies in Year 4 & Year 5 are 500 & 600 respectively, then estimate the expected claim under truck insurance for next two years using EBCT Model 2. (8)

[15]

Q. 7) Independent claims ($X_1, X_2, ..., X_n$) follow Exponential (λ_i) distribution where

$$\lambda_i = \begin{cases} e^a, i \le m \\ e^b, others \end{cases}$$

i) Derive m.l.e. of a & b and scaled deviance in simplest form (11)

ii) Calculate deviance residual for $X_1 = 10$ where

$$\sum_{1}^{m=10} X_i = 150, \sum_{1}^{n=30} X_i = 400$$

(3)

[14]

Q. 8) **i)** Aggregate claims for a group of polices follows compound Poisson process with parameter λ.

a) State the conditions that must be satisfied by the claim number process $\{N(t)\}_{t>=0}$ (3)

The rate of premium income is 1200 per year, initial surplus of 1000. Value of λ is 25 per year. Assuming aggregate claims distribution may be approximated by Normal distribution, find out

b) Individual claim amount has lognormal distribution with parameters $\mu = 3$, $\sigma^2 = 1.04$. Calculate the probability that the surplus at the end of year 2 will be negative. (3)

c) The regulatory changes may result in the insurer changing the premium rates. Calculate the premium rates such that the probability of ruin at end of year 1 is 5%. (3)

d) Initial surplus falls with a rate of 100 per year & the company incurs expense of 500 per year, calculate the loading to the premium such that the probability of ruin at the end of year 2 is 5%. (3)

ii) In marine insurance, probability of claims in any year is 5% and claim amount follows exponential distribution with mean 100 crore. If the company charges annual premium of Rs 6 crore payable in advance then calculate the minimum required capital so that the ruin probability on first claim is less than 5%. (you may ignore any interest income) (6)

[18]

Q.9) **i)** Define the components of a generalised linear model. (2)

ii) Show that the Normal distribution belongs to exponential family of distributions. Write down expressions for the variance function and canonical link function. (3)

[5]

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