INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

14th March 2017

Subject CT3 – Probability & Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) The following are the marks scored by 24 students in a theory paper conducted for 60 marks:

22	31	26	22
19	21	33	27
53	27	34	27
46	30	21	30
17	60	36	32
26	33	27	33

i) Display the above data in a stem and leaf diagram. (2)

ii) Calculate the median and mode. (1)

iii) Calculate the interquartile range. (2)
[5]

Q. 2) Obtain the recursive relation for the binomial distribution (n, p) of the form

$$P(X = x) = g(x, n, p)P(X = x - 1); x = 1,2, \dots, n\,; 0 < p < 1.$$

where $g(x, n, p)$ is a general function of x, n and p. **[3]**

Q. 3) An employee of a company chooses one of the three routes $A, B$and C with respective probabilities 0.20, 0.10 and 0.70 to reach his office. The probabilities that he reaches office on time by $A, B$and C respectively are 0.85, 0.80 and 0.95.

On a particular day, he does not reach office on time. Calculate the probability that he chose route *B*. **[3]**

Q. 4) Let $X_1, X_2, \dots, X_N$ be iid exponential random variables with mean 500 and N be a binomial random variable with parameters (n, p) independent of X_i's.

i) By defining $S = X_1 + X_2 + \cdots + X_N$ with $S_0 = 0$ when $N = 0$, calculate the mean and variance of *S*. (3)

ii) Calculate the approximate probability that S exceeds the value 60,000. (3)
[6]

Q. 5) Let the random variables X and Y have joint probability density function (*pdf*)

$$f_{X,Y}(x, y) = \frac{12}{5}(x^2y + xy)\,; 0 < x, y < 1.$$

i) Find the marginal *pdf* s of X, Y. (3)

ii) Check for the independence of X and Y. (1)

iii) Compute $E(X)$and $E(Y)$. (3)

iv) Compute $E(X/Y)$ and $Var(X/Y)$.
Hence, verify $E(E(X/Y)) = E(X)$. (5)
[12]

Q. 6) Suppose X has a Gamma (α, λ) distribution.

i) Derive the Moment Generating Function of X, from first principles and hence obtain its Cumulant Generating Function. (4)

ii) Obtain an expression for the coefficient of skewness. (4)
[8]

Q. 7) A stock trader has 100 shares of a company and adopts a random strategy for buying/ selling of shares based on the outcomes of a die roll at each trading session. The strategy is:

Outcome of die	*Decision*
1	Do nothing
2 or 4	Buy one share
3 or 5	Sell one share
6	Buy two shares.

Calculate an approximate probability that after 18 independent trading sessions the trader will have more than 110 shares of the company. **[5]**

Q. 8) A random sample of 10 bulbs are selected and their survival times (in months) are recorded as 1,1,1,2,3,4,7,7,8,16 and suppose that the survival times X is as a random variable from exponential distribution with mean 1/λ .

i) Find the Method of Moments (MM) estimate of λ. (2)

ii) Find the Maximum Likelihood Estimate (MLE) of λ. (3)

iii) Obtain the Maximum Likelihood estimate of $P(X > 50 \text{ hours})$. (2)

iv) Obtain the Cramer- Rao Lower Bound (CRLB) for the variance of the estimator for λ. (1)
[8]

Q. 9) The weights (in *kgs*) of a random sample of 10 adults who visited a retail store A on a particular day are given below.

50, 55, 60, 60, 60, 70, 70, 70, 80, 85.

i) Calculate a 95% confidence interval for the true mean weight of the adults who visited the store. (4)

ii) Calculate the minimum sample size required in order to obtain a 95% confidence interval for the true mean to have a width of no more than 10 *kgs*. (2)

The summary measures of weights of a random sample of 12 adults who have visited retail store B on the same day are:

$$\sum y_i = 816 \text{ and } \sum y_i^2 = 56{,}644.$$

iii) Calculate 95% confidence interval for the difference in mean weights of the adults who visited these stores and comment on the significant difference in their mean weights. (4)

iv) Obtain a 90% confidence interval for ratio of the two variances of weights of adults who have visited these stores and comment on the equality of variances. (3)

(Assume that the random samples are from normal populations). **[13]**

Q. 10) A random sample of size 1 is taken from a population with probability density function

$$f(x) = \theta x^{\theta-1} \;; 0 < x < 1;\ \theta > 0.$$

i) Obtain the Most Powerful test of level $\alpha = 0.05$ for testing the null hypothesis H_0: $\theta = 5$ against the alternative H_1: $\theta = 4$. (4)

ii) Calculate the power of the test. (2)

[6]

Q. 11) A manufacturing consultant wishes to study the average life of the four brands of an electronic device and selects a random sample of devices from each of the brands. Data on the life (in hours) of the devices are given below:

Brand 1: 7, 13, 9, 4
Brand 2: 6, 8, 7, 9
Brand 3: 5, 7, 6, 9, 10, 3
Brand 4: 7, 9, 6, 8, 5

Assuming the model: $y_{ij} = \mu + \tau_i + e_{ij}$; i=1,2,3,4 ; j =1,2,…,n_i.

i) Calculate the least square estimate of μ and τ_i , i=1,2,3,4 (4)

ii) Perform an analysis of variance, stating the assumption and the hypothesis tested. (7)

iii) Calculate 95% confidence interval for the underlying common variance σ^2 of the lifetime of the devices. (3)

iv) Analyse the above data using the least significant difference approach based on the highest and the lowest sample averages for given brands and suggest between these two brands, which one you would likely prefer. (3)

[17]

Q. 12) A local transit authority for a metropolitan city wants to determine whether there is any relationship between the age of a bus and the daily maintenance cost. Data on a random sample of 12 buses resulted in the following.

Age of Bus (X) (in months)	10	11	18	20	22	26	30	38	42	46	50	55
Daily Maintenance Cost (in Rs) (Y)	50	40	60	50	70	60	80	70	80	90	80	100

$$\sum x = 368\ ; \sum y = 830\ ; \sum x^2 = 13{,}854; \ \sum y^2 = 60{,}900; \sum xy = 28{,}180$$

i) Fit the linear regression model $y = \alpha + \beta x + \epsilon$ (4)

ii) Carryout a test for $H_0: \beta = 1$ against $H_1: \beta > 1$ at 5% level of significance. (4)

iii) Calculate the proportion of variability explained by this model and comment on the result. (2)

iv) Calculate a 95% confidence interval for the daily maintenance cost for an individual bus aged 34 months. (4)

[14]

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