Institute of Actuaries of India

Subject CM2-Paper A – Financial Engineering and Loss Reserving

June 2019 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i)

For Government Bonds:

Mean = 6.64% [½]

Variance = $\frac{\sum(x-Mean)2}{n}$ = 0.14 %% [1]

For Corporate Bonds:

Mean = 5.14% [½]

Variance = $\frac{\sum(x-Mean)2}{n}$ = 2.65 %% [1]

For Equities:

Mean = 13.61% [½]

Variance = $\frac{\sum(x-Mean)2}{n}$ = 433.68 %% [1]

For the Portfolio:

Mean = 7.43% [½]

Variance = $\frac{\sum(x-Mean)2}{n}$ = 15.76 %% [1]

[6]

ii)

Variance of return

Merits:

- Variance is mathematically tractable.
- Variance fits neatly with a mean-variance portfolio construction framework.

De-merits:

- Variance is a symmetric measure of risk. The problem of investors is really the downside part of the distribution.
- Not suitable for returns with asymmetric distribution or fat tails.
- Neither skewness or kurtosis of returns is captured by a variance measure.

[½ for each point]

[2]

Shortfall probability

Merits:

- It gives an indication of the possibility of loss below a certain level.
- It allows a manager to manage risk where returns are not normally distributed.

De-merits:

- The choice of benchmark level is arbitrary.
- For a portfolio of Equities, the shortfall probability will not give any information on:
  - upside returns above the benchmark level
  - nor the potential downside of returns when the benchmark level is exceeded.

[½ for each point]

[2]

Value at Risk (VaR)

Merits:

- VaR generalises the likelihood of underperformance by providing a statistical measure of downside risk.

De-merits:

- Equity Portfolios may exhibit non-normal distributions. The usefulness of VaR in the above situation depends on modelling skewed or fat-tailed distributions of returns. The further one gets out into the "tails" of the distributions, the more lacking the data and, hence, the more arbitrary the choice of the underlying probability distribution becomes.

- No attention is paid to the distribution of outperformance above the benchmark.

[½ for each point]

[1]

Tail Value at Risk (TailVaR)

Merits:

- Relative to VaR, TailVaR provides much more information on how bad returns can be when the benchmark level is exceeded.

De-merits:

- It has the same modelling issues as VaR in terms of sparse data, but captures more information on tail of the non-normal distribution.
- Like VaR, no attention is paid to the distribution of outperformance above the benchmark

[½ for each point]

[1]

[6]

iii)

Using the confidence level of 95%, Risk Metrics is 1.645 as the z-score for 95%.

For Equities:

Var at 95% = 13.61% - 1.645 * $\sqrt{0.04337}$ = - 20.65 % [1]

The employee is 95% confident that she would not lose more than 20.65% in Equity in the following year. [½]

For the portfolio:

Var at 95% = 7.43% - 1.645 * $\sqrt{0.00158}$ = 0.904 % [1]

The employee is 95% confident that her return would not be less than 0.904% in the following year.

[½]

[3]

iv)

a)

Herd mentality bias - refers to investors' tendency to follow and copy what most other investors are doing. Many employees would have followed other employees to shift due to herd mentality.

Hindsight bias – events that happen will be thought of as having been predictable prior to the event; events that do not happen will be thought of as having been unlikely prior to the event. If Equities were thought to give higher return, they have actually given higher return now by hindsight. In reality, it is just a co-incidence that Equities have given higher return over the past 3-year period.

Confirmation bias – people will tend to look for evidence that confirms their point of view (and will tend to dismiss evidence that does not justify it). Employees would have looked for evidence of higher return thereby ignoring negative return in 2015. **[3]**

b) **Equity premium puzzle** is one of the most famous quandaries in Finance. The puzzle relates to the fact that returns from risk bearing instruments such as Equity exceeds the returns from other instruments such as bonds and bills by more than is predicted by risk aversion alone. [1]

It is explained by Myopic loss aversion, which suggests that investors are much more concerned by losses than by equivalent gains, and so tend to focus on very short-term returns and volatility rather than long-run earnings. Investors thus need to earn additional on equities to overcome their aversion to the short-term losses. [1]

[2]

[20 Marks]

Solution 2:

a. *Mutual funds are good for investors as they can consistently give higher returns than Nifty Index returns due to their expertise in analysing companies' earnings, Balance sheet, etc.*

   The market is either Weak form or inefficient. Under Semi-strong & Strong market, it's difficult to outperform the market using fundamental analysis in the long run.

b. *A stock trader has made super normal profit for past 20 years using Bollinger-band technique (a well- known technical tool)*

   The market is not even Weak form (hence, Inefficient) as technical analysis has given super normal profits.

c. *A trust-worthy newspaper reports that XYZ Mining Corp has larger than expected coal reserve which would increase the Market capitalization by 5% and the share price increased by 5%.*

   The market is Semi-strong as it has immediately reacted to the information coming to public.

d. *A Television reporter traded in a stock and made money on the basis of information in his 'Interview' with the management (before it was broadcast)*

   The market is not strong form and can be any of the other form as the reporter was able to make money on the basis of insider information.

   [½ mark each for identifying correct form and ½ mark for explanation. Students are expected to understand the hierarchy of EMH. Just mentioning a particular EMH form is not sufficient. Student should also know whether other forms [lower/higher in hierarchy] are satisfied.]

[4 Marks]

Solution 3:

Let S_n = Accumulated value at time n of Rs.1 invested at time 0

$$S_n = (1+ i_1)(1+ i_2)....(1+ i_n)$$

Therefore $E[S_n] = E [(1+ i_1)(1+ i_2)....(1+ i_n)]$

$= E(1+ i_1). E(1+ i_2).... .E(1+ i_n)$ by independence [½]

and $E(1+ i_t) = 1+ E(i_t) = 1+ j$

Hence $\boldsymbol{E[S_n] = (1+j)^n}$ [1]

Now

$Var[S_n] = E[S_n^2] - E[S_n]^2$

$E[S_n^2] = E [(1+ i_1)^2 (1+ i_2)^2....(1+ i_n)^2]$ [1]

$= E(1+ i_1)^2. E(1+ i_2)^2.... .E(1+ i_n)^2$ by independence [½]

and

$E(1+ i_t)^2 = E(1+2i_t+ i_t^2)$

$= 1 + 2E[i_t] + E(i_t^2)$

and $Var[i_t] = s^2 = E[i_t^2] - E[i_t]^2$

implies $E[i_t^2] = s^2 + E[i_t]^2$

$= s^2 + j^2$ [1]

Hence

$E(1+i_t)^2 = 1 + 2j + s^2 + j^2$

& $E[S_n^2] = (1 + 2j + s^2 + j^2)^n$

Therefore

$Var[S_n] = (1 + 2j + s^2 + j^2)^n - (1+j)^{2n}$ [1]

Hence mean accumulation = 8,00,000$E(S_5)$ [½]

= 8,00,000(1.055)5

= Rs.10, 45,568 [½]

Standard deviation of accumulation = 8,00,000 √Var (S_5) [½]

$Var(S_5) = (1 + 2j + s^2 + j^2)^5 - (1+j)^{10}$

$= (1+2*.055+.055^2+.04^2)^5 - (1.055)^{10}$

= 1.7204573-1.7081445

= .0123128

√Var (S_5)= .0110963 [1]

Standard deviation of accumulation = 8,00,000√Var (S_5)

= 8,00,000 *0.0110963

= Rs. 88,770 [½]

Alternative Solution

i)$(1+i_t) \sim$ lognormal (μ,σ^2)

$\ln(1+i_t) \sim N(\mu,\sigma^2)$

$\ln(1+i_t)^5 = \ln(1+i_t) + \ln(1+i_t) + \ldots + \ln(1+i_t) \sim N(5\mu,5\sigma^2)$

Given assumption that they are independent and identically distributed

$(1+i_t)^5 \sim$ lognormal $(5\mu,5\sigma^2)$ [2]

$E(1+i_t) = \exp(\mu + \sigma^2/2) = 1.055$ [1]

$Var(1+i_t) = \exp(2\mu + \sigma^2) * \exp(\sigma^2 - 1) = 0.04^2$ [1]

Solving the above two equations

$.04^2 = (1.055)^2 * \exp(\sigma^2 - 1)$

Exp $\sigma^2 = 1.001437524$

$\sigma^2 = .0014365$

$\sigma = .0379$ [1]

From

$\exp(\mu + \sigma^2/2) = 1.055$

we have $\exp(\mu + (.0014365/2)) = 1.055$

$\mu = .052823$ [1]

Therefore

$5\mu = 0.264113$

$5\sigma^2 = 0.0071825$

Let S_5 be the accumulation of one unit after five years, then $S_5 \sim$ lognormal $(5\mu, 5\sigma^2)$ i.e lognormal(0.264113, 0.0071825)

$E[S_5] = \exp(0.264113 + 0.0071825/2) = 1.30696$ [½]

$Var(S_5) = \exp(2*5\mu + 5\sigma^2/2) * \exp(5\sigma^2 - 1)$

$= \exp(2 \times 0.264113 + 0.0071825) * (\exp 0.0071825 - 1)$

$= \exp 0.53541 * (\exp 0.0071825 - 1)$

$= 0.01231284$

$\sqrt{Var(S_5)} = .110963$ [½]

Mean value of the accumulation of premiums is

8,00,000 × 1.30696 = Rs.10,45,568 [½]

Standard deviation of the accumulated value of the premiums is

8,00,000 × 0.110963 = Rs.88,770.4 [½]

[8]

ii) By definition, the accumulated amount 800,000 S_5 will exceed the upper quartile u with probability 25% i.e

P (800,000 S_5 ≥ u)= 0.25 [1]

⇨ 1- P (800,000 $S_5 \leq$ u)= .25

⇨ P (800,000 $S_5 \leq$ u)= .75

⇨ P ($S_5 \leq$ u/800,000)= .75

Since S_5 ~ lognormal (5μ, 5σ^2)

$$P\left(\frac{lnS_5-5\mu}{\sqrt{5\sigma^2}} \leq \frac{(\ln(\frac{u}{800,000})-5\mu)}{\sqrt{5\sigma^2}}\right) = .75$$ [1]

$$\Rightarrow \Phi\left(\frac{(\ln(\frac{u}{800,000})-5\mu)}{\sqrt{5\sigma^2}}\right) = .75$$

From table we have Φ(0.6745)=0.75

Therefore $\frac{(\ln(\frac{u}{800,000})-5\mu)}{\sqrt{5\sigma^2}}) = 0.6745$

Solving this we get *u= 800,000exp (5μ+.6745σ√5) = 11,03,109.8 ≈ 11,03,110* [1]

Similarly lower quartile *l= 800,000exp (5μ-.6745σ√5)= 9,83,940* [1]

[4]

[12 Marks]

Solution 4:

i)

Expected price at Time 1 from the tree

$E(S_1)$ = 0.53 × 1302.6 + 0.47 × 767.7 = **1051.2**

[½]

Check: the expectation should be

S_0 exp(0.05) =1000 exp(0.05) = **1051.3**

[½]

These two expected values are very close. So the drift has been calibrated correctly.

Variance of S_1 from the tree is:

$Var[S_1] = E[S_1^2] - E[S_1]^2$

$= .53 * 1302.6^2 + .47 * 767.7^2 - 1051.2^2$

$= 71,272.0$

Standard deviation of price at Time 1 from the tree is √71,272.0 = **267.0** [1]

Check: We expect the Standard deviation to be 1000*√[exp(2μ + σ²) *(exp(σ²) – 1)],
where
volatility σ = 25% and
drift μ = *r* – ½ σ² = .01875

Var[S_1]= S_0^2exp(2μ+σ^2)*(exp(σ^2)-1)

= 1000²* exp(2*.01875+$.25^2$)*(exp($.25^2$)-1)

= 1000²* exp(.1)(exp(.0625)-1)

= 71,277.4
And the theoretical standard deviation is √71,277.4 = **267** [½]
Hence the tree has been calibrated appropriately **[3]**

ii)
(a) price of a three-year European Put option with a strike of 1,000
Maturity payoffs are max(1000 – *S*, 0).

Working backwards through the tree with *V* = exp(-0.05)[0.53*V*up + 0.47*V*down]:

Time 0	Time 1	Time 2	Time 3
			0
		0	
	46.45		0
119.2		103.9	
	214.2		232.3
		361.9	
			547.6

Altenatively
V_{Tree} (Euro) = exp(-0.05)*3[0+ 0 + 232.3* 3q(1-q)² + 547.6(1-q)³]= 119.2

Hence tree value of European option = 119.2 [3]

(b) price of a three-year American Put option with a strike of 1,000

Payoffs at maturity are still max(1000 – *S*,0)
But in working backwards through the tree, we need to allow for early exercise option with *V*

= max{exp(–0.05)[0.53*V*up + 0.47*V*down], 1000 – *S*}.

We get

Time 0	Time 1	Time 2	Time 3
			0
		0	
	46.45		0
129		103.9	
	236> 232.3		232.3
		361.93<410.7	
			547.6

Hence tree value of American option = 129.0 [3]

[6]

[9 Marks]

Solution 5:

i) Taking the sentence from the question "…the patient wants to live more" (He prefers more to less). This implies u '(x) >0.

Taking the sentence from the question "…wants to reduce the risk of death during the Surgery" (He is risk-averse) . This implies u ''(x) < 0. **[1]**

ii) Pdf of Poisson distribution is given by:

$$p\ (x) = e^{-\lambda}\ \frac{\lambda^x}{x!}$$

Checking for first order dominance & second order (if first fails)

	Pdf	pdf	cdf	cdf	Cumulative cdf	Cumulative cdf
X	Procedure A	Procedure B	Procedure A	Procedure B	Procedure A	Procedure B
0	0.3000	0.4000	0.3000	0.4000	0.3000	0.4000
1	0.0149	0.0027	0.3149	0.4027	0.6149	0.8027
2	0.0446	0.0107	0.3595	0.4134	0.9744	1.2161
3	0.0892	0.0286	0.4487	0.4420	1.4231	1.6581
4	0.1339	0.0573	0.5826	0.4993	2.0057	2.1574

[3]

The first order dominance fails as cdf A(x = 2) < cdf B(x = 2) but cdf A(x = 3 or 4) > cdf B(x = 3 or 4). The Second order dominance of A succeeds as Cumulative cdf of A < Cumulative cdf of B for all points given in the question. [1]

Further, it has been already proved that u '(x) >0 and u " (x) <0 . [½]

Hence, the patient would choose Procedure A based on second-order dominance. [½]

[5]

[6 Marks]

Solution 6:

i) **IBNR reserve**: It is the reserve required in respect of claims that have been incurred but not reported to the insurer ie. the claim event has occurred, but the claim has not yet been reported to the insurer.
Outstanding claim reserve: It is the reserve required in respect of claims that have been reported, but not yet been closed by the insurer. **[2]**

ii) Assumptions underlying the following methods:
Average cost per claim method:

- The first accident year is fully run off.
- The average cost per claim in each development year is a constant proportion in monetary terms of ultimate average cost per claim for each accident year.
- The number of claims in each development year is a constant proportion of the ultimate number of claims for each development year.
- Inflation is not allowed explicitly, rather it is allowed implicitly as a weighted average of past inflation. [1½]

Bornhuetter-Ferguson method:

- The first accident year is fully run off.
- The loss ratio is correct.
- Claims in each development year are constant proportion in monetary terms of total claims for each accident year.
- Inflation is not allowed explicitly, rather it is allowed implicitly as a weighted average of past inflation. [1½]

[3]

iii) The cumulative number of claims is given in the below Table:

	Development year			
Accident year	0	1	2	3
2011	1300	2400	4000	6300
2012	1700	3300	5200	7700
2013	2200	4100	6600	
2014	2300	3800		
2015	2500			

Development factors:

DV1 = (2400+3300+4100+3800)/(1300+1700+2200+2300) =1.81

DV2 = (4000+5200+6600)/(2400+3300+4100) = 1.6

DV3 = (6300+7700)/(4000+5200) = 1.52 **[3]**

[8 Marks]

Solution 7:

i)

Given that

$dS_t = (\mu+\frac{1}{2}\sigma^2)\, S_t\, dt+ \sigma S_t\, dW_t$

Let $f(S_t)= ln(S_t)$

Applying the Taylor's series formula to the above function, we get

$$df(S_t)=d(ln(S_t))=\frac{1}{S_t}\, dS_t+\frac{1}{2}(-\frac{1}{S_t^2})(dS_t)^2$$

$$= \frac{1}{S_t}((\mu+\frac{1}{2}\sigma^2)\, S_t\, dt+ \sigma S_t\, dW_t) - \frac{1}{S_t^2}((\mu+\frac{1}{2}\sigma^2)\, S_t\, dt+ \sigma S_t\, dW_t)^2$$

$$= (\mu+\frac{1}{2}\sigma^2)\, dt+ \sigma\, dW_t -\frac{1}{2}\sigma^2\, dt$$

$$= \mu dt+ \sigma\, dW_t$$

Integrating we get

$ln(S_t)- ln(S_0)= \mu t+ \sigma W_t$

Therefore

$\boldsymbol{S_t= S_0 exp\,(\mu t+ \sigma W_t\,)}$

Now $\boldsymbol{D_t= B_t^{-1}\, S_t = e^{-rt}\, S_t = S_0 exp\,((\mu-r)t+ \sigma W_t\,)}$ ---------(A)

Comparing the S_t ***and*** D_t it is evident that

$$d\,D_t = (\mu - r + \tfrac{1}{2}\sigma^2)\, D_t\, dt + \sigma\, D_t\, dW_t$$ **[5]**

Alternate Solution

Given that

$$dS_t = (\mu + \tfrac{1}{2}\sigma^2)\, S_t\, dt + \sigma S_t\, dW_t$$

and $D_t = B_t^{-1} S_t = e^{-rt} S_t$

We have

$$\frac{\partial D_t}{\partial t} = -re^{-rt} S_t = -r D_t$$

$$\frac{\partial D_t}{\partial S_t} = e^{-rt}$$

$$\frac{\partial^2 D_t}{\partial S_t^2} = 0$$

Applying Ito's Lemma we get

$$d\,D_t = \frac{\partial D_t}{\partial t}\, dt + \frac{\partial D_t}{\partial S_t} dS_t + \frac{\partial^2 D_t}{\partial S_t^2} dS_t^2$$

$$= -rD_t\, dt + e^{-rt}\left((\mu + \tfrac{1}{2}\sigma^2)\, S_t\, dt + \sigma S_t\, dW_t\right)$$

$$= -rD_t\, dt + D_t\left((\mu + \tfrac{1}{2}\sigma^2)\, dt + \sigma\, dW_t\right)$$

$$= D_t\left((\mu - r + \tfrac{1}{2}\sigma^2)\, dt + \sigma\, dW_t\right)$$

ii)

Consider a dynamic portfolio (φ_t, ψ_t) consisting of φ_t units of St and ψ_t units of Bt.

A portfolio is ***self-financing*** if and only if changes in its value depend only on changes in the prices of the assets constituting the portfolio.

Mathematically, if V_t denotes the value of the portfolio (φ_t, ψ_t), then the portfolio is self- financing if and only if

$dV_t = \phi_t\, dSt + \psi_t\, dBt$

where ϕ_t and ψ_t are previsible. [1]

A ***replicating strategy*** for X is a strategy which involves investing in previsible quantities (φ_t and ψ_t) of stock and risk-free bonds, such that:

- the portfolio (φ_t, ψ_t) of stocks and bonds will be self-financing
- the portfolio (φ_t, ψ_t) will have terminal value equal to the magnitude of the claim, i.e.
  $V_T = \varphi_T S_T + \psi_T B_T = X$,

which means that the portfolio cash flows at claim exercise date match the cash flows under the claim.

[2]

[3]

iii) Steps are as follows.

(1) Apply the Cameron-Martin-Girsanov theorem to D_t .This states that there exists a new probability measure, say Q, equivalent to the current measure, such that $D_t = e^{-rt} S_t$ is a Q-artingale,

$\boldsymbol{d\, D_t = \sigma\, D_t\, d\hat{W}_t}$

where $\boldsymbol{\hat{W}_t}$ is a standard Brownian motion under Q.

(2) Define:

$V_t = e^{-r(T-t)} E_Q [X/ F_t]$

We propose that this is the value of the claim.

(3) Form the discounted expected claim process E_t under measure Q:

$E_t = \mathrm{E}[B_T^{-1} X \mid \mathbf{F}_t] = e^{-rT} E_Q [X/ F_t] = e^{-rt} V_t$

where $\mathbf{F}_t$ is the history of the process up to time t.

This process is a Q-Martingale, which can be demonstrated using the properties of Martingale and the Tower Law of conditional probabilities.

(4) Invoke the Martingale Representation Theorem (MRT) which states that there is a previsible function φt such that:
$dE_t = \varphi_t\, dD_t$

(5) Consider a replication strategy of holding φ_t units of stock, where φ_t is chosen based on the MRT, and

Let $\psi_t = E_t - \varphi_t D_t$ of risk free bonds.

Firstly, we show that this portfolio replicates the value of the claim.

The value of the portfolio at any time t can be written:

$V_t = \varphi_t S_t + \psi_t B_t = B_t E_t$ (substituting from the definition of ψ_t)

so $V_T = B_T E_T = \mathrm{E}[X \mid \mathbf{F}t] = X$.

Secondly, differentiating V_t gives:

$dV_t = d(B_t E_t) = B_t dE_t + E_t dB_t$ (using the product rule, as B_t is non-stochastic)

$\Rightarrow dVt = \varphi_t B_t dD_t + (\varphi_t D_t + \psi_t) dB_t$ (substituting dEt and Et from the above)

$\Rightarrow dVt = \varphi_t (B_t dD_t + D_t dB_t) + \psi_t\, dB_t = \varphi t dS_t + \psi_t\, dBt$

(substituting $dSt = d(B_t D_t) = B_t dD_t + D_t\, dB_t$, as B_t is non-stochastic),

hence the portfolio is self-financing.

Since the portfolio replicates the claim without any additional funds (generated or required), the arbitrage-free condition requires that the value of the claim equals the value of the replicating strategy.

[6]

[14 Marks]

Solution 8:

i) The set of efficient portfolios is called as efficient frontier. A portfolio is efficient if the investor can't find a better portfolio in the sense that it has either a higher expected return and the same (or lower) variance or a lower variance and the same (or higher) expected return. [2]

The efficient frontier is a straight line which is tangent to the efficient frontier (of risky assets) and passes through the point in (S.D., return) space corresponding to the risk-free asset.
Initially, we need to find the portfolio using A and B that maximises

(expected return – 3%)/standard deviation

Assume proportion x of assets in A and (1 – x) in B

Expected return of risky portfolio is 7% *x + 4% *(1 – x)

Standard deviation of the risky portfolio is

$[(15\%*x)^2 + (7\%*(1-x))^2]^{0.5}$ [2]

Comment: Examiners shall provide 2 Marks, in case the candidates follow the correct approach and describe the problem mathematically.

We need to find x that maximises the function

$[7\% *x + 4\% *(1-x) - 3\%] / [(15\%*x)^2 + (7\%*(1-x))^2]^{0.5}$

Taking log, we get

$Ln[0.07x + 0.04 - 0.04x - 0.03] - 0.5*Ln[(0.15x)^2 + (0.07(1-x))^2]$

$=Ln[0.03x + 0.01] - 0.5*Ln[0.0225x^2 + 0.0049(1 - 2x + x^2)]$

$= Ln[0.03x + 0.01] - 0.5*Ln[0.0274x^2 - 0.0098x + 0.0049]$

Differentiate and set to zero,

$0.03/[0.03x + 0.01] - 0.5*[0.0548x - 0.0098] / [0.0274x^2 - 0.0098x + 0.0049] = 0$

$0.03*[0.0274x^2 - 0.0098x + 0.0049] - 0.5*[0.0548x - 0.0098]*[0.03x + 0.01] = 0$

$[0.000822 x^2 - 0.000294x + 0.000147] - [0.000822 x^2 + 0.000274 x - 0.000147 x$
$- 0.00049] = 0$

Solving we get x = 0.46556

When x = 0.46556,

Expected return of the risky portfolio = 0.054
Standard deviation of the risky portfolio = 0.0792 [3]

Thus, the efficient frontier is the straight line passing through (0.03,0) and (0.054,0.0792) [1]

[8]

ii) The portfolio would be corresponding to the point where the utility indifference curve of the investor touched the efficient frontier. **[1]**

[9 Marks]

Solution 9:

i) The SDE for r(t) under the risk neutral measure Q is given as

Hull & White Model:

$dr(t) = \alpha(\mu(t) - r(t))dt + \sigma dW(t)$

where $\mu(t)$ is a deterministic function of t. $\mu(t)$ has the natural interpretation of being the local mean-reversion level for $r(t)$. [2]

2-factor Vasicek Model:

It models two processes: r(t), as before, and m(t), the local mean-reversion level for r(t). Thus

$dr(t) = \alpha_r(m(t) - r(t))dt + \sigma_{r1}dW1(t) + \sigma_{r2}dW2(t)$

$dm(t) = \alpha_m(\mu - m(t))dt + \sigma_{m1}dW1(t)$

where $W1(t)$ and $W2(t)$ are independent, standard Brownian motions under the risk neutral measure Q. [2]

[4]

ii) The formula for Zero Coupon Bond price is given as

$e^{-rT}(1 - (1 - \delta)(1 - e^{-\lambda(i)T}))$, where δ is the recovery rate and $\lambda(i)$ is the constant default rate for the bond i and T is the redemption time.

Thus, we can write as

$1.6 = 2\, e^{-0.0\,7}(1 - 0.5(1 - e^{-2\lambda(A)}))$

$2.2 = 3\, e^{-0.0\,7}\,(1 - 0.5(1 - e^{-2\lambda(B)}))$

Solving the above equation,

$\lambda(A) = 0.167$

$\lambda(B) = 0.278$ **[5]**

[9 Marks]

Solution 10:

i) Surplus Process: Suppose at time 0 the insurer has an amount of money (U) set aside for the portfolio. The insurer surplus at any future time t (>0) is a random variable since its value depends on the claim experience up to time t. [1]

The insurer surplus at time t is denoted by U(t). This can be represented by

U(t) = U + ct – S(t), c is the premium income per unit time & S(t) is the aggregate claim process

For a given value of t, U(t) is a random variable because S(t) is a random variable. Hence, $\{U(t)\}_{t>=0}$ is a stochastic process, which is known as Surplus process. [1]

[2]

ii) Let the random variable T1 denote the time to first claim. Then, for a fixed value of t, if no claims have occurred by time t, T1 > t. Hence,

P(T1 >t) = P(N(t)=0)=exp(-λt)

And

P(T1<=t) = 1- exp(-λt)

Hence, T1 has an exponential distribution with parameter λ. **[2]**

iii) We need to find the mean and variance of aggregate claims over the two-year period. The expected number of claims over the two-year period will be 50. So the mean and variance of the aggregate claim S(2), using the Lognormal distribution is given by

E[S(2)] = 50*Mean(X)
=50*3
=150 [1]

Var[S(2)] = 50*E(X^2)
= 50*(Var(X) +(E(X))^2)
=50*(1.1 + 3^2)
=50*10.1
=505 [2]

Ruin (Insurer surplus will be negative) will occur if S(2) is greater than initial surplus plus premium received over the two years.

P[S(2) > 1200+2*1000] = P[N(0,1) > (3200 – 150)/sqrt(505)]
= 0%

The probability of ruin is approximately 0%. [2]

[5]

[9 Marks]

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