INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st June 2019

Subject SP6 – Financial Derivatives
Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)
Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Mark allocations are shown in brackets.*

5. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

Q. 1) ALM actuary has been employed by a pension fund company to help reduce the volatility of the fund due to fluctuation in interest rates. He has been provided with the following cash flows for the company, which represents outflows (in case of liabilities) and inflows (in case of assets). All figures are in crores of rupees and assume that cash flows are accumulated till the end of each year.

Year	Liabilities	Assets
1	200	400
2	400	800
3	400	450
4	800	

   i) With interest rate of 10% per annum compounded annually, calculate the modified duration of assets and liabilities. (3)

   ii) If Interest rate is expected to fall by 3%, what is the current value of expected shortfall? (3)

   iii) ALM actuary has suggested usage of Interest rate swaps to mitigate the expected shortfall in the value. What type of interest rate swap should be used and why? Calculate the nominal amount of the swap to ensure no impact of 3% fall in future interest rates (assume all future floating rates are 7%). List additional risks those the company may be exposed due to this structure. (6)

   iv) Interest rate fall is expected to happen after one year time and it has been suggested to use the interest rate swaption after one year with 80% of the nominal amount of the swap (as computed in part (**iii**)). Compute the cost of this strategy and discuss its effectiveness. Interest rate volatility is 25%. (7)

[19]

Q. 2) i) Define the following options and write the formula for the capped rate.

     a) Ratchet Caps (1.5)
     b) Sticky Caps (1.5)
     c) Flexi cap (1.5)

   ii) If P is bond price and F is the forward rate, Use the relationship between them and forward rate volatility to construct one factor LMM (LIBOR Market Model) model. (6)

   iii) Describe why LMM model would be better suited to price the above derivatives (mentioned in part (**i**)) and highlight key limitations of LMM models. (4)

[14.5]

Q. 3) i) Define Gaussian copula and describe how it can be used to relate evolution of multiple risk factors. Describe approach to generate simulated default time for set of bonds. (6)

   ii) The following default correlation matrix is given between 3 bonds.

	A	B	C
A	1	0.75	0.3
B	0.75	1	0.5
C	0.3	0.5	1

Using the following format of cholesky decomposition, estimate the correlated random numbers.

$$\epsilon_1 = \alpha_{11} x_1$$

$$\epsilon_2 = \alpha_{21} x_1 + \alpha_{22} x_2$$

$$\epsilon_3 = \alpha_{31} x_1 + \alpha_{32} x_2 + \alpha_{33} x_3$$

Where xi are given random numbers and ϵ_i are correlated random variables. Initial random numbers are 0.35, 0.50 and 0.65.   (6)

iii)   Assume that the three bonds (mentioned in part **(ii)** follow exponential distribution [1-exp(-λ*T)]. The lambda values for the three bonds are 4, 8 and 16 respectively. Estimate the default time for the three bonds.   (4)

                                                                                                                                                            [16]

Q. 4)   Consider the following:
- Assume a principal of $1.
- Conditional on no earlier default, a reference entity has a (risk-neutral) probability of default of 3% in each of the next 5 years.
- Assume payments are made annually in arrears, which defaults always happen half way through a year.
- Expected recovery rate is 30%.
- Interest rate in the market is 8% per annum (with continuous compounding).
- Suppose that the breakeven CDS rate is *s* per dollar of notional principal.

Answer the following questions:

i)   What is the probability of default in year 4?   (2)

ii)   What is the probability of survival at the end of the 5 years?   (2)

iii)   As a function of "s", what is the present value of the payments to be made (both in the case of no default as well as in the accrual payment in the event of default)?   (4)

iv)   What is the present value of the expected payoff in the event of a default?   (2.5)

v)   What is the breakeven CDS spread in this case?   (3)

vi)   Assume that the value of a swap negotiated at an earlier point in time with a CDS spread of 323.98. What would be the value as a function of the principal?   (4)

                                                                                                                                                          [17.5]

Q. 5)   A stock is currently priced at $160 and pays no dividends. The price at time "i+1" is given by S_{i+1} and can be written as $S_{i+1} = S_i + 0.5\ S_i$ for the up move or $S_{i+1} = S_i - 0.5\ S_i$ for the down move. The risk – free interest rate of 18.232% per period and the strike price is $150.

Using the above information, answer the following.

i)   Draw a three period binomial tree.   (4)

ii)   Using the tree drawn in part **(i)**, price a European Call Option using the information given in the question (rounded to the nearest integer).   (4)

iii) Repeat part **(ii)** but now price the corresponding American Call Option.   (3)

iv)   Finally, using the tree drawn in part **(i)** and the information given in the question, what will be the price of the American Put Option?   (2)

                                                                                                                                            [13]

Q. 6) Answer the following:

i)   What happens to a "look back call" as we increase the frequency with which we observe the asset price in calculating the minimum.   (3)

ii)   What happens to a "down and out call" as we increase the frequency with which we observe the asset price in determining whether the barrier has been crossed.   (3)

iii) Compare the price of an Asian option to the price of a vanilla option with the same strike and maturity.   (3)

iv)   Construct an arbitrage portfolio when the price of a knock-in option plus the corresponding knockout option is not equal to the price of the corresponding vanilla option.   (3)

v)   An investor believes that there will be a big jump in a stock price, but is uncertain of which way the move will happen. What should the investor's strategy to remain protected from the large move?   (3)

                                                                                                                                            [15]

Q. 7) Consider an 11 month European Bond Option on a 10.25 year bonds. The face value of the bond is Rs. 1000 and the current price is Rs. 975. The strike price is Rs. 1000. Further, The 11 month interest rate is 10% per annum and the corresponding volatility is also 10% per annum. Also, note that the bond pays a coupon of Rs. 50 each in three and nine months' time. The three month risk free rate is 9.5% and the 9 month rate is 9.8%. You can assume that the current price and the strike price are both cash prices.

Using this information and the Black's formula, compute the value of the call option.   **[5]**

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