INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20th June 2019

Subject CS1A – Actuarial Statistics (Paper A)

Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

Q. 1) **i)** In a game, a player pulls three cards at random from a full deck of 52 cards, and earns as many points as the number of red cards among the three. Assume 2 people each play this game once with their own decks, and let X be the sum of their combined points. Derive the moment generating function of X. (5)

ii) Hence prove that he mean of total points earned by the players is $\frac{12}{\binom{52}{3}}\left(\binom{26}{3} + \binom{26}{1}\binom{26}{2} \right)^2$ (3)

[8]

Q. 2) A study into the average claim (in Rs. '000) per health insurance policy was performed for the claims incurred in public and private hospitals. Data for some cities is given below:

	City 1	City 2	City 3	City 4	City 5	City 6	City 7	City 8	City 9
Public	24	45	29	33	20	40	26.5	25	27.5
Private	30	54.5	30	40	28.5	36	30.5	30.5	35.5

i) Determine the sample mean and sample variance of average claim size in both the type of hospitals. (6)

ii) State the primary condition that needs to be true for testing equal mean and verify whether that condition is satisfied in the above example (You may assume that the samples come from a normal population). (4)

iii) Test whether the treatments in private hospitals result in higher claim size at 95% level. (5)

[15]

Q. 3) An academician proposes that the joint distribution of a number of weeks of study leave (X) and the proportion of questions answered correctly (Y) is given by:

$$f_{XY}(x,y) = \frac{9}{10}xy^2 + \frac{1}{5} \; for \; 0 \le x \le 2, 0 \le y \le 1$$

i) Determine the Marginal distributions of X and Y. (3)

ii) A student is selected at random, what would be the expected number of weeks of study leave and expected proportion of questions answered correctly. (4)

iii) Compute the covariance of X and Y (2)

[9]

Q. 4) You are an actuarial analyst working at a life insurance company in India. Your actuary has asked you to analyze the claims data to aid her in setting pricing assumptions for a new product. You have obtained the following information from the claims department:

Age band	Number of claims per 10,000 policies
0-10	112
11-20	122
21-30	133
31-40	187
41-50	258
51-60	400
61-70	522

The actuary has asked you to perform a regression analysis on this data to identify the relationship between age and number of claims.

i)   Perform a linear regression of the claims as a function of the age   (6)

Note: You can assume that the rate of claims in a particular age band is constant and hence perform a regression on the middle age of each age band.

ii)   Calculate the R-squared for the regression model   (2)

iii)   Calculate a two-sided 95% confidence interval for the slope parameter   (3)

iv)   Carry out an F test to determine whether the slope parameter is zero   (3)

v)   Given below is the scatter plot of residuals against the values of the independent variable. Please Comment on the plot.   (3)
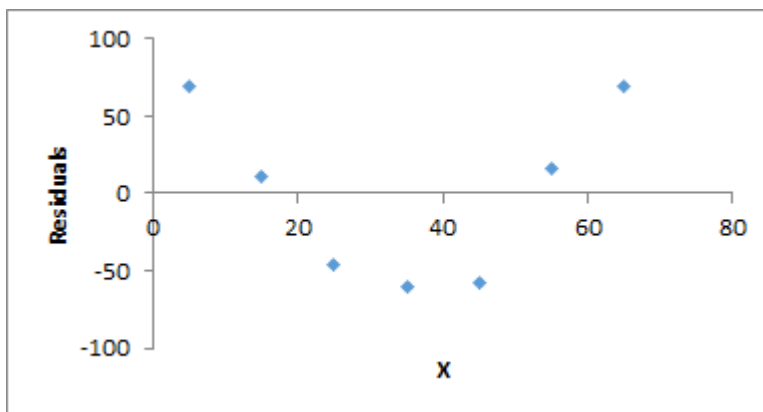
[17]

Q. 5) If $\hat{\theta}$ is an estimator of parameter θ, answer the following:

i)   Define unbiased estimator   (1)

ii)   Define 'bias'.   (1)

iii)   Define Mean Square Error (MSE) of this estimator $\hat{\theta}$ (1)

There exist another estimator $\tilde{\theta}$ of the same parameter θ, such that $\hat{\theta}$ has no bias but higher MSE than $\tilde{\theta}$ while $\tilde{\theta}$ has a positive bias.

iv)   State giving reason, which estimator is 'efficient'?   (1)

v)   When would either of the two estimators be termed as consistent?   (1)

vi)   Outline (in one sentence each) any two methods of estimating θ (2)

[7]

Q. 6) Define

    i)   Pearson residuals   (2)

    ii)   Deviance residuals   (2)

    Clearly describe all notations used.

                                                 [4]

Q. 7) State and derive the Bayes' theorem   **[4]**

Q. 8) **i)**   Wickets taken by a cricket team 'A' follows Poisson process with rate of 1 wicket per 100 balls bowled. How many wickets will the team take with 95% confidence after bowling 500 balls?   (3)

    ii)   A cricket team 'B' has batsmen for the last wicket that can score 1 run per ball with a probability of 40% and 0 runs with probability of 60%. The team B (playing against the above team A and having only 1 wicket in hand) needs to score 26 runs in 50 balls. Team A wins if it takes the 1 wicket in these 50 balls and team B wins in case it scores the required runs in 50 balls. Determine which team has a higher probability of win. State any assumptions you make.   (7)

    iii)   Hence determine the probability that team B will bat for at least 30 balls   (3)

                                                 [13]

Q. 9) The annual distribution of claims arising from a portfolio of motor insurance policies follows a Poisson distribution with mean μ. The prior distribution for μ has a gamma distribution with parameters $\alpha = 4$ and $\lambda = 7$.

    Claim figures over the last n years are $x_1, x_2, x_3 \ldots x_n$.

    i)   Show that the posterior distribution is gamma and determine its parameters   (3)

    ii)   Given that n = 10, $\sum_{i=1}^{10} x_i = 15$ determine the Bayesian estimate for μ under

        a) squared error loss   (2)
        b) 'all-or-nothing' loss   (3)
        c) absolute error loss   (3)

                                                 [11]

Q. 10) A life insurance company sells Group Term Insurance Policies to different private companies. Each company will have a different degree of risk with the difference arising because of a difference in the nature of business, the proportion of blue and white collared employees etc.

    The Life Insurance Company has collated claims data across 2 such group policies. Claim payments (in INR '000) and the number of employees covered in each company are as given below:

Claim Payments: INR '000	Year1	Year2	Year3	Year4
Company A	4084	4387	4550	3456
Company B	4634	3203	2073	4485

Number of employees	Year1	Year2	Year3	Year4
Company A	121	119	120	110
Company B	150	135	122	145

You are an actuarial analyst in the life insurance company. You have been asked to compute the expected claims payouts for both the companies over the coming year assuming that the number of employees covered will be 135 and 155 for company A and company B.

Analyze the data using Empirical Bayes Credibility theory – EBCT model 2 and calculate the expected claims payout for companies A and B. **[12]**

Note: A group term insurance policy is an insurance product sold to corporates under which all employees of the company are provided life insurance.

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