# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 21st June 2019

## Subject CM1A – Actuarial Mathematics (Paper A)

## Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)

## Total Marks: 100

### *INSTRUCTIONS TO THE CANDIDATES*

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

**AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.**

**Q. 1)** List any 6 criteria to be taken into account while assessing the suitability of a model for a particular exercise. **[3]**

**Q. 2)** Briefly describe how the events below are expected to affect interest rates?

**i)** Expectation of an increase in inflation (1)

**ii)** Increase in tax rates (1)

**iii)** Increase in interest rates of a major economy (1)

**iv)** Sharp increase in number of life insurance companies operating in the market (2)
**[5]**

**Q. 3)** **i)** Define the terms "duration" of an asset and the "effective duration" of an asset; and derive the relationship between the two. (4)

**ii)** A fixed interest asset has an effective duration of 2.6 and convexity of 9.7. Estimate the change in asset value due to a 0.5% increase in interest rates. (2)
**[6]**

**Q. 4)** An endowment assurance plan pays INR 75,000 at the end of the year of death during the term of 15 years or INR 50,000 on survival to the end of the term. Premiums are paid annually in advance throughout the policy term or on earlier death. Calculate the retrospective net premium reserve for this contract, issued to a man aged 45 at entry, immediately before the sixth premium payment is due.

Basis: AM92 Ultimate mortality, and 4% pa interest. **[6]**

**Q. 5)** **i)** Calculate the effective monthly rate of interest corresponding to:

a) Nominal rate of interest of 6% p.a. convertible quarterly.
b) Nominal rate of interest of 10% p.a. convertible six times a year. (2)

**ii)** The rate of interest at time t is given by

$$\delta(t) = \begin{cases} .05+.005\,t & 0<=t<5 \\ .08t - .01 & 5<=t<10 \\ 0.10 & 10<=t \end{cases}$$

Calculate the present value at time 2 of a payment of 5000 at time 15 years. (4)
**[6]**

**Q. 6)** **i)** An insurance contract has the following future loss random variable:

$$S * v^{\min(Tx,n)} + I + e * \bar{a}_{\overline{\min(Tx,n)}|} + f * v^{\min(Tx,n)} - G * \bar{a}_{\overline{\min(Tx,n)}|}$$

Where,
I = Initial expenses
e = Renewal expenses
S = Sum assured
G = Gross premium
f = Additional expenses when the contract terminates

From the expression above, what can you conclude about the nature of the contract and the timing of the various cash flows? (4)

**ii)** A 5-year term assurance policy is issued to a life aged 47 exact. Under this policy, annual premiums of INR 2,000 are payable annually in advance. Death benefit of INR 110,000 is payable at the end of the year of death. Calculate the reserve required to be held at the end of the second year using the assumptions below:

Force of mortality equals:
- 0.02 for ages 25 - 39
- 0.025 for age 40 – 49
- 0.03 for ages 50 - 59

Expenses: Nil

Interest rate: 6% p.a. effective. (4)

**[8]**

**Q. 7) i)** Describe the characteristics, including cash flows involved, of

a) An endowment assurance
b) A term assurance (4)

**ii)** Consider a life insurance company which is offering three types of contingent annuity for lives aged 40 years (say). In case of joint life assume that both the lives are of same age

a) Regular payment till the survival of annuitant.
b) Joint life annuity- payable till the second life survives
c) Reversionary annuity- commencing on the death of the first life and payable till the second life survives

Explain which arrangement will give highest annuity amount for an investment of INR 1,00,000. (2)

**iii)** The company offers different type of options to annuitants in terms of choosing the frequency of the annuity payments:

a) Annuity payable in arrear ($a_x$)
b) Annuity payable in advance ($\ddot{a}_x$)
c) Annuity payable in arrear in quarterly mode
d) Annuity payable in advance in quarterly mode
e) Annuity payable continuously

For interest rate, $i > 0$, arrange the present value of annuities of above options in ascending order. (2)

**[8]**

**Q. 8)** Priti and Rahul, aged 56 and 60 years exact respectively, take out a policy with the below benefits:

a) Lump sum of INR 1,00,000 payable at the end of the year of first death if it happens in first 5 years and INR 2,00,000 if the first death happens between $6^{th}$ and $10^{th}$ year.

b) After 10 years, an annuity benefit of INR 20,000 will be payable annually in advance until both are alive and INR 10,000 while only one of them is alive.

Level premiums are payable annually in advance for at most 5 years and will cease on the first death if this occurs earlier.

Calculate the level annual premium payable by the couple on the following basis:

- Interest: 4% p.a.
- Mortality: PMA92C20 for Rahul (male) and PFA92C20 for Priti (female)
- Initial expense: INR 500
- Renewal expense: 2% of each premium excluding the first premium **[8]**

**Q. 9)** A life insurance company sells health insurance policies to healthy individuals. The policy would pay an income of INR 20,000 per year during the period of permanent sickness (i.e. from which it is not possible to recover) and the income is halved to INR 10,000 per year in case of temporary sickness (from which it is possible to recover), with all the sickness benefits ceasing at age 65 years. Also in case of death before 65 years, a benefit of INR 1,00,000 is payable.

Level Annual premiums of INR "P" p.a. are payable continuously to age 65 or to earlier death, except that premium is waived during any period of sickness.

**i)** Draw and label a transition diagram suitable for modeling the pricing process. (5)

**ii)** Basis the transition rates in your diagram and probabilities in the below form:

${}_tp_x^{ij}$ = Probability that a person aged x in state i is in state j at time x+t

Construct a formula, using integrals, to calculate the level premium "P" for this contract (ignoring expenses), for a life aged exactly 50 at entry. (7)

**[12]**

**Q. 10)** An Insurance Company plans to explore replacing its existing leased IT system with a new policy administration system to gain better control on administration and enhance the operational efficiency. The cost of procuring the system has been estimated at INR 15,00,000 and its usual working lifetime is assumed to be 10 years. The vendor supplying the new system will buy it back at the end of 10 years at the price assuming depreciation of 2% p.a. at simple rate.

The annual maintenance cost incurred by the insurance company will be 5% p.a. of the procurement cost payable quarterly starting from the beginning of the second year till its usual working lifetime.

The Company will further leverage the system to one of its clients & receive a rent of INR 2,40,000/- per annum payable monthly in arrears starting from first month.

**i)** Calculate NPV of the project assuming rate of interest of 8% p.a. convertible half yearly. (6)

The Company instead of purchasing the system decides to continue with the lease arrangement and decides to invest the procurement cost in a 10-year zero-coupon bond giving a return of 10% p.a. compounding yearly. In addition, the Company pays the lessor an annual amount of INR 1,80,000/- payable monthly at the end of each month for using the leased asset for 10 years.

**ii)** Calculate NPV of the leased arrangement assuming rate of interest of 8% p.a. convertible half yearly and comment which of the arrangement will be profitable for the Company. (3)

**[9]**

**Q. 11) i)** List at least four areas that you expect to be included within the bonus philosophy of a company. (4)

**ii)** What do you understand by the term "pricing basis"? (1)

**iii)** A male life aged 65 exact purchases a single premium policy with the following benefits:

a) Annuity of amount INR 10,000 payable monthly in arrears as long as the policyholder is alive.
b) Return of single premium immediately upon death of the policyholder within 10 years of purchasing the annuity.

Calculate the single premium using the assumptions below assuming principle of equivalence holds.

Mortality table: PMA92C20
Interest: 4% p.a.
Initial expense of 2% of Single premium
Regular expenses of 0.5% of annual annuity amount
Claim expenses of 0.5% of death benefit payable (6)

**iv)** Without undertaking further calculations, briefly describe the impact on the Single Premium of the policy alterations below:

**a)** Death benefit is payable at the end of year of death (1)
**b)** Annuity is payable annually in arrears, with annual annuity amount remaining unchanged (1)

**[13]**

**Q. 12)** On 1st Jan 2010, an employee working with a bank, takes out a home loan of INR 35,00,000 from a housing finance company called *HomeFin* for a tenure of 20 years. The company charges a rate of interest of 10% p.a. convertible monthly.

**i)** Calculate the monthly instalment payable by the employee. (2)

At the end of 5 years, the employee makes a lump sum payment of INR 5,00,000 towards partial home loan repayment.

**ii)** Calculate the revised monthly instalment if the interest if outstanding duration of the loan remains unchanged. (3)

**iii)** Calculate the revised outstanding duration, if the employee plans to maintain the instalment as in **(i)** with no change in interest rate. (3)

The employee chooses to change the monthly instalment to that calculated in **(ii)** so as to keep the outstanding duration same as the outstanding term of the loan at the time of partial payment.

On 1st Jan 2019, takes a personal loan from its bank equal to the outstanding loan amount and repays the amount due to the housing finance company. The rate of interest charged by the bank on the personal loan is 7% p.a. convertible monthly.

**iv)** Calculate the duration of the personal loan amount, if the employee plans to maintain the same instalment as calculated in **(ii)**. (4)

**v)** Calculate the total amount of interest paid in the entire arrangement. (4)

**[16]**

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