INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st July 2022

Subject SP6 – Financial Derivatives

Time allowed: 3 Hours 30 Minutes (14.30 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***

3. ***Mark allocations are shown in brackets.***

Q. 1) The following table gives the prices of zero – coupon bonds with a face value of Rs. 100. The maturity of the bonds are given in the table. Column A provides the current prices, while Column B provides the prices of the same bonds after the underlying yield curve has instantly moved up by 10 basis points (0.10%) for all maturities.

	A	B
Maturity (Years)	Current Bond Price	Bond Price when rates go up by 10bps
1	95.238	95.147
2	90.488	90.316
3	85.770	85.526
4	81.492	81.183
5	77.796	77.428

Based on current rates, the modified duration of a bond priced at par is 4.299.

Using current rates:

i) Calculate the value of a bond with a maturity period of 5 years and a 10% coupon payable annually. Hence derive the value of 5-year annual fixed-floating swap with 10% fixed coupon. (3)

"Absolute Yield Sensitivity" of an instrument is defined as the ratio of the difference between the instrument's value currently and the value of the instrument when the underlying interest change by a small amount Δr, to the incremental change in the interest rate, Δr. Given this,

ii) Show how absolute yield sensitivity for a bond is related to its modified duration. Show numerically that the absolute yield sensitivities of the bond and the swap in (i) are the same. (3)

[You may assume that a 10-basis point shift is a small increment.]
An "Inverse Floater" is a bond that pays a coupon equal to a fixed value X less the value of a floating rate benchmark at each fixing. For example, if floating rate benchmark is 5.75% at the next fixing, the coupon becomes X – 5.75%.

Consider a 5-year annual coupon Inverse Floater where X = 10%.

iii) Show how the cashflows of the Inverse Floater can be decomposed into those of a fixed-floating swap and a zero-coupon bond. Hence demonstrate numerically that the modified duration of the Inverse Floater is approximately twice that of a bond priced at par. (7)

iv) Explain how you would risk manage, in terms of duration, of a portfolio of bonds that includes an Inverse Floater. (2)

[15]

Q. 2) Consider a perpetual bond with a continuously compounded yield of "y". Further, the bond pays interest @ Rs. 5 per annum. Assume that interest is paid continuously on the bond, and that "y" follows the process:

$$\mathbf{dy = \mu\,(\alpha - y)dt + \sigma y\, dZ}$$

where μ, α and σ are positive constants and dZ is a standard Brownian motion.

Based on this information, answer the following two questions:

i) What is the process followed by the bond price? (6)

ii) What is the expected return to the bond holder expressed as a percentage of the bond price? (4)

[10]

Q. 3) Assume that a stock S goes ex-dividend at time t with a dividend D due at that time. An investor holds a plain vanilla American call option on S with strike price X which expires at T > t. The stock will not pay any other dividends before time T.

i) State the times at which it may be optimal to exercise this option. (1)

ii) Derive the inequality that D must satisfy for it to be not optimal to exercise early. (3)

Consider another stock A. This stock goes ex-dividend at times $t_1, t_2, \ldots, t_n$ with $t_1 < t_2 < \ldots < t_n$. The dividends corresponding to these times are denoted $D_1, D_2, \ldots, D_n$.

iii) State the condition for it to be not optimal to exercise immediately prior to t_i for any $i < n$. (1)

The price of a stock is ₹60. The stock is expected to pay dividends of ₹1 in 2- months' and 5-months' time, and these are also the ex-dividend dates. The time to maturity of an American call option is six months, the volatility is 25% per annum, and the continuously compounded risk-free rate is 8% per annum.

iv) Using Black's approximation, calculate the value of the at the money (ATM) American call option. (10)

[15]

Q. 4) You are a consultant and have been approached by the clearing corporation of a country in its effort to initiate a process in which the initial margin requirements for interest rate derivatives will be set by modelling the behaviour of interest rates in the future. In this context, it has been decided to carry out scenario analysis and use the Vasicek model for interest rates.

i) Describe why you should carry out both real-world and risk-neutral projections of interest rates. (2)

ii) Write down the Stochastic Differential Equation (SDE) of the Vasicek model in the real world, defining all terms used. (1)

iii) Describe how you would calibrate the parameters of the real world SDE in part (ii). (2)

[5]

Q. 5) You are an Investment Actuary and a part of ABLUX insurance company, which sells participating product with guaranteed maturity value. There are 2 funds A and B into which the initial premium is split. In Fund A, the investment returns are passed on to policyholder via policyholder pay-outs. If there is a 10% return in fund A, policyholder pay-out would go up by 10% even though actual amount increase/decrease is different between fund A and policyholder pay-out due to different initial values.

Fund B which acts as a buffer, is used to support the guarantees and excess policyholder pay-out which is currently completely invested in Fixed income securities. Government regulation allows up to 40% exposure of participating product in equities for matching fund (Fund A in this case) which ABLUX maintains.

The available capital is excess of any asset over liabilities. The current value of Fund A is 800cr, and Fund B is 400cr. Policyholder pay-out is 1000cr and Guarantee cost is 100cr.

i) Explain the steps and assumptions to be used to value the guarantees using Monte-Carlo simulation. Describe the choice of risk driver and simulations to be used - Risk Neutral or real world. (5)

ii) Using Monte-Carlo simulation it has been established that for a 1% change in policyholder pay-out we have -0.15% impact on guarantees. Estimate using above information, change in the available capital for immediate 50% fall and 50% rise in the equity value. (5)

iii) Write a note to explain the direction of the movement of the available capital to ABLUX board and recommend fund balancing & hedging strategy to ensure stable available capital. (5)

[15]

Q. 6) **i)** Describe how the value of a call and put option in a bond with options can be expressed as the value of Straight bond and Callable/Puttable bond. (2)

ii) Describe Option-Adjusted Spread (OAS) and the approach for calculating the OAS. Explain how this is different from Z-spread. (3)

iii) Explain how the volatility of interest rate would impact the value of Call option, Put option, value of a Callable bond, Puttable bond and the corresponding OAS. Explain the movement of each of the above in the following two scenarios –

a. High interest rate volatility
b. Low interest rate volatility

(5)

[10]

Q. 7) Derive arbitrage free model to calculate the following (Use 30/360 convention for valuation, assume all time is in days and continuous interest rate)

i) Forward price for stock with current value S_0, no dividend & holding cost, continuous risk-free rate is r, and duration of forward is T. (2)

ii) Forward on a stock paying dividend D at time Td1 and Td2 and no cost of carry with term T (Td1<T and Td2>T). (2)

iii) Forward on a stock paying dividend at time Td1 and Td2 as above. The stock also has holding fees H to be paid at Thd1 <T and Thd2>T. (3)

iv) Value of future at time Tf such that Td2>T>Td1 >Tf and Thd2>T>Tf>Thd1. (4)

v) We have a future contract on a stock with below information:

T	120
S_0	100
r	5%
Td1	90
Td2	150
Thd1	30
Thd2	210
Tf	70
S_{tf}	105
Dividend	5
Holding	2

Assume continuous compounding and 30/360 days convention. Estimate the forward price at T=0 and forward value at Tf= 100. (4)

[15]

Q. 8) A lending company offers a 100 Cr loan for 6 months at an interest rate of 8% annually compounded determined on total capital. This overall payment amount is split equally into 6 instalments each payable at the end of each month. Estimate the annually compounded return by the company at the end of the 6 months. Assume that the company can earn risk free rate of 5% annually compounded. **[5]**

Q. 9) FHDC is a leading housing finance company in an emerging market economy. In order to fund its growth, the company has decided to raise 5000 Cr of capital by issuing non-convertible debentures and 5000 Cr of capital by issuing fresh equity through a Qualified institutional placement. FHDC trades at 1800 per share currently. The company has also issued warrants to its investors at 180 per warrant and raised 300 Cr. The warrants are convertible into equity shares at 2150 per share and the conversion can happen after 3 years. The warrants would be listed in stock exchanges.

i) What are the similarities and differences between the warrant and exchange traded call option on the stock? (3)

ii) What are the advantages to FHDC of issuing the warrants? (4)

iii) What are the advantages and disadvantages to investors who subscribe to the warrants? (3)

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