

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

29th July 2022

**Subject CM2B – Financial Engineering and Loss
Reserving (Paper B)**

Time allowed: 2 Hour (14.30 – 16.30 Hours)

Total Marks: 100

Q. 1) A listed company, Pyarelal Inc., has just issued a zero coupon bond payable in 5 year time for a nominal amount of INR10,000 crores.

The risk neutral probability of defaulting in 5 years' time is 25%. Total current equity value is INR2,271 crores and the implied volatility of the company's share price is 44.66% p.a. The forward rates are given in the Excel sheet.

i) Calculate the single risk free rate applicable for the next 5 years. (4)

Using Merton Model for estimating Credit risk:

ii) Estimate the volatility of the Asset. (8)

iii)

a) Estimate the value of the Asset at time $t = 0$ assuming volatility of asset as 12.68% p.a. (10)

b) Estimate the value of debt at time $t = 0$ (2)

A credit rating agency models credit rating using a Markov chain model with the transition probabilities given in the Excel sheet. The rating agency has given a rating of B for the company's Debt.

iv) Validate the given rating by estimating the probability that the company would be in default state (D) at the end of 5 years. (Hint: Rating is validated if the probability of default is close to the risk neutral probability of 25%) (10)

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Q. 2) A financial institution offers three saving plans to its customers:

A. Fixed deposit paying guaranteed interest of 4% per annum

B. Mutual Fund managed passively that tracks an equity index

C. Fusion product that gives a guaranteed minimum return of 2.5% p.a. plus 75% of any excess return on the index above 2.5% p.a.

[For example, if the return on the equity index is 6% then product C will give a return of $2.5\% + 0.75 \times (6\% - 2.5\%) = 5.125\%$. If the return on the index is below 2.5% then product C will give a return of 2.5%.]

You have been given a distribution of the 1-year return on the equity index given in the Excel sheet.

An investor Mohit plans to invest Rs.1,00,000 in each of products A, B and C at time $t = 0$ to be withdrawn after 1 year (at time $t = 1$). He uses a utility function $U(w) = w - 2 \times 10^{-6} w^2$ where w is the level of wealth.

i) Calculate the expected utility of each investment at time $t = 1$. (12)

ii) Plot a chart of the utility at time $t = 1$ of product B for a range of equity index returns from -25% to +25%. (5)

iii) Comment on how your chart in part (ii) would differ if the investor's utility function:

a) exhibited constant relative risk-aversion. (2)

b) was risk-neutral. (3)

Another investor Rohan plans to invest Rs.2,00,000 in each of products A, B and C at time $t = 0$ to be withdrawn after 1 year (at time $t = 1$). He uses a utility function

$$U(w) = 5000 * \ln(w)$$

iv) Calculate the expected utility of each investment at time $t = 1$. (9)

v) Plot a chart of the utility at time $t = 1$ of product C for a range of equity index returns from -25% to $+25\%$. (5)

vi) Comment on absolute risk aversion and relative risk aversion in part (v). (4)

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Q. 3) i) A non-dividend paying share has a current value of ₹52 and an annual volatility of 0.15. An investor who has ₹1000 to invest has a choice between investing in either a one-year zero coupon bond (redeemable at par) with a current market value of ₹93.70 or in one-year put options with a strike price of ₹47.5. If an investor chooses to allocate all of his/her money to the options, how many put options can he/she buy? (10)

ii) The price S_t of a particular share follows a geometric random walk:

$$S_t = S_{(t-1)} * Z_t$$

Where Z_t is a sequence of independent, identically distributed random variables:

$$Z_t = \begin{cases} 1.2 & \text{with probability } 0.55 \\ 0.8 & \text{with probability } 0.45 \end{cases}$$

t denotes time in months

A 1 month European call option is available on the share with a strike price of ₹11.5. The current market price of the share is ₹10. No dividends are to be paid over the next 6 months. An annualised risk free rate of interest of 5% is available.

Find the expected pay-off of the call option. (3)

iii) The market price of a security can be modelled by assuming that it will either increase by 2% or decrease by 1% each day, independent of the price movement on other days. No dividends are payable during the next 1 month. The continuously compounded monthly risk-free rate of interest is 0.1%. The current market price is ₹600.

Demonstrate the expected daily price of the security for 1 month using a binomial tree. (Assume that the month has 30 days) (4)

- iv) Use a binomial model to calculate the value of a 1 month European put option on the security with a strike price of ₹595. (4)
- v) Calculate the value of a 1 month American put option on the same security with the same strike price as given in (iv). (3)
- vi) Calculate the value of a 1 month American Call Option on the same security with the same strike price as given in (iv). (2)
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