

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

22nd July 2022

Subject CM1A – Actuarial Mathematics
(Paper A)

Time allowed: 3 Hours 30 Minutes (9.30 - 13.00 Hours)

Total Marks: 100

Q. 1) The liabilities and assets of an insurance company are given below in terms of x and y :

Time point (t)	Liability Cash-flow at the end of time t	Asset Cash-flow at the end of time t
1	x	10
2	$2x$	10
3	$3.5x$	0
4	$4.5x$	0
5	$5x$	20
6	100	30
7	0	y

The risk-free rate is 6% p.a. effective.

- i) Calculate the values of x and y such that first and second conditions of Redington's immunization theory are fulfilled. (10)
- ii) Calculate the convexity of assets and liabilities and explain whether the portfolio is immunized. (5)
- iii) State the limitations and practical difficulties of Redington's immunisation theory. (4)
- [19]**

Q. 2) While analysing the model output, which of the following test involves comparing real world data and model output by the real-world system experts without being told which are which? Select the correct option:

- A. Scenario Testing
 B. Monte Carlo Simulation
 C. Sensitivity Testing
 D. Turing Test
 E. Hypothesis Testing

[2]

Q. 3) Government of a country is concerned with the citizens' preference towards investment in physical Gold as an asset class. In order to improve current account deficit and prevent valuable capital being locked into non-productive asset i.e. physical gold, Government proposes to introduce fixed- interest bearing Gold bonds in denomination of 1 gram of gold equivalent with tenure of 8 years to be backed by sovereign guarantee. Government proposes to raise public debt vide Gold bonds on fixed semi-annual coupon payments and maturity repayment linked to Gold prices prevailing at the maturity.

- i) Which type of model shall be suitable to estimate the total cost of borrowing from Gold bonds. Your response should be supported by appropriate rationale. (3)
- ii) Identify the steps involved in the modelling process. (5)
- iii) Explain the difference between "short-term" and "long-term" properties of a model. (2)

[10]

Q. 4) Calculate the value of ${}_{1.75}p_{40.75}$ using AM92 Ultimate mortality and assuming that:

- i) Deaths are uniformly distributed between integer ages. (2)

ii) Force of mortality is constant between integer ages. (2)

[4]

Q. 5) Amanda aged exactly 45 years buys a 20-year regular premium paying policy that pays INR50,000 on maturity. A premium of INR1,577 is payable at the start of each year throughout the 20-year policy term, or until death if it happens earlier. In case of death, all premiums paid till date are returned without interest at the end of the year of death. From the options provided below, select the value which best represents the expected present value of the benefits payable under this policy assuming AM92 Select mortality and an interest rate of 4% p.a. (workings are not required to be provided)

- A. INR18,000
- B. INR19,500
- C. INR8,500
- D. INR21,000

[4]

Q. 6) Mr. Khurana purchases INR100,000 nominal of a bond on 1 January 2022 which is redeemable at 105 in four years' time and pays coupons of 4% per annum at the end of each year. He wishes to invest the coupon payments in deposit until the bond is redeemed.

Calculate the mean value of the total accumulated investment on 31 December 2025 if the annual effective rate of interest has an expected value of 5.5% in 2022 and 2023, 6% in 2024 and 4.5% in 2025 assuming that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year.

Select the option from the choices provided below which best represents the accumulated value (workings are not required to be provided).

- A. INR122,300
- B. INR122,500
- C. INR118,300
- D. INR118,500

[4]

Q. 7) How many days does one need to hold a 364 days Government Bond redeemable at INR100 if he buys at INR96.5 and sells at INR98.0 after achieving a return of 4% per annum effective.

Select the correct option from the choices provided below (workings are not required to be provided).

- A. 182 days
- B. 144 days
- C. 156 days
- D. 142 days

[2]

Q. 8) Three bonds redeemable at INR103 per 100 nominal can be redeemed in 1, 2 & 3 years and pay annual coupons in arrears at 6% p.a. Calculate the implied one-year and two-year spot rates if price of each bond is INR97 per 100 nominal.

[3]

Q. 9) A loan was taken out on 1 September 2008 and was repayable by the following increasing annuity.

The first repayment of INR1,000 was made on 1 July 2009. Thereafter, payments were made on 1 November, 1 March and 1 July each year until 1 March 2014, inclusive. Each payment was 5% greater than its predecessor. The effective rate of interest throughout the period was 6% per annum.

- i) Calculate the initial loan amount. (3)
 - ii) Calculate the capital repaid in first instalment. (2)
 - iii) Calculate both the capital component and the interest component of the seventh repayment. (5)
- [10]**

Q. 10) A joint life annuity is issued to a male life now aged 60 years exact and a female life now aged 55 years exact. The annuity is payable monthly in arrears and is subject to following conditions:

- The amount of annuity INR200,000 per annum is payable while both lives survive.
- If the male life dies first leaving the female life surviving the annuity reduces to INR100,000 per annum payable until she dies.
- If the female life dies first leaving the male life surviving the annuity reduces to INR150,000 per annum payable until he dies.
- In addition if either life is alive at the 10th and 20th anniversaries of the policy a cash lump sum of INR50,000 is paid at each date.

Calculate the present value of this annuity assuming the basis below:

Mortality	PMA92C20 and PFA92C20
Interest	4% per annum
Expenses	Nil

[10]

Q. 11) Briefly describe the concept of net premium & net premium valuation and how they relate to gross premium & gross premium valuation respectively. (5)

Q. 12) A life insurance company has launched an education-based endowment plan. Under this policy, the parent is the life insured while the child is the beneficiary. The policy matures when the child attains age of 18 years exact. Upon death of the life insured during the term of the policy, all future premiums due are waived and the policy continues till maturity with no changes to the maturity benefit. However, upon voluntary non-payment of premiums by the life insured, the policy terminates. The premium paying term of the policy is the same as the policy term.

The policy is bought by life insured aged 35 years exact with the beneficiary aged 5 years exact. The annual premium is INR50,000 and the maturity benefit is INR900,000.

- i) The life insured would like the overall returns to be in excess of 4% p.a. to invest in this plan. State whether the proposed policy meets the requirements of the policyholder and produce supporting analysis. (4)
- ii) Calculate the gross premium reserve just after the life insured attains the age of 43 years exact. Reserving basis is provided below:

Mortality for life insured: AM92 Ultimate

Interest Rate: 4% p.a.

Commission: 2.5% of annual premium, payable at the time of premium collection.

Renewal expense of INR500 p.a. is incurred till the policy remains in-force. Assume these occur at the end of policy year.

Surrenders: nil

Expense inflation: nil

Assume negligible mortality for beneficiary (11)

iii) From part (ii) above, calculate the death strain at risk. (3)

iv) If the policyholder were allowed to change premium frequency to monthly, with all other terms remaining the same, explain whether the reserves will increase or decrease? (2)

v) The Company intends to include a surrender option. The surrender value is broadly equal to maturity value less premiums due, discounted to the surrender date at 9% p.a. Assuming a surrender rate of 5% p.a., describe how the reserves in part (ii) will get impacted following this change? (3)

vi) What is the difference between terms “lapse” and “surrender”. (2)

vii) Define a multiple decrement model. (2)

[27]
