Institute of Actuaries of India

Subject CT6 – Statistical Methods

December 2018 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) A pair of strategies will be in equilibrium if and only if the element corresponding to the equilibrium is both the largest in its column and the smallest in its row. Such equilibrium is called a saddle point. [1]

ii)

Strategy	*Minimum Profit*
Flydigo	*4*
Superjet	*6*
Kristara	*8*

Hence, the minimax strategy is to sell on Kristara [1]

iii)

Strategy	*Maximum Profit*
Flydigo	*14*
Superjet	*50*
Kristara	*20*

Hence the maximmax strategy is Superjet [1]

iv)

Strategy	*Expected profit*
Flydigo	$\frac{1}{3}(10+14+4) = 9.33$
Superjet	$\frac{1}{3}(50+40+6) = 32$
Kristara	$\frac{1}{3}(20+15+8) = 14.33$

Hence the strategy selected by the Bayes criterion is Superjet [2]

[5 Marks]

Solution 2:

It is stated that in a given year the number of claims has a poisson distribution with parameter λ, Therefor as per Bayes approach the Posterior density is proportional to λ times the number of policies,

$$= \frac{e^{-600\lambda}(600\lambda)^{75}}{75!} \times \frac{e^{-900\lambda}(900\lambda)^{210}}{210!} \times \frac{e^{-500\lambda}(500\lambda)^{50}}{\lambda\Gamma(50)} \propto \text{ constant } \times \lambda^{335} e^{-2000\lambda}$$

Looking at the density it is observed that this is a gamma function with parameters as 335 and 1/2000.

Therefore, the expected no. of claims per policy is 335/2000 = 0.1675 and so the expected no of claims in next year is = 1100 × 0.1675 = 184.25

[4 Marks]

Solution 3:

i) The characteristics equation is given by:

$(1 - \frac{12}{35}\lambda + \frac{1}{35}\lambda^2) = (1-\frac{1}{5}\lambda)(1-\frac{1}{7}\lambda) = 0$

Which has roots = 5 and 7. They both are greater than 1. Hence subject to the initial values having appropriate distributions, this implies (weak) stationarity. [2]

ii)

a) Firstly, note that $Cov(X_t, Z_t) = 1$ and $Cov(X_t, Z_{t-1}) = \frac{12}{35} - \frac{1}{7} = \frac{1}{5}$

We need to generate 3 distinct equation linking γ_0, γ_1 and γ_2

This can be done as follows:

(A)

$\gamma_0 = Cov(X_t, X_t) = Cov(1 + \frac{12}{35} X_{t-1} - \frac{1}{35} X_{t-2} + Z_t - \frac{1}{7} Z_{t-1}, X_t)$

$= 12/35\ \gamma_1 - 1/35\ \gamma_2 + 1 - 1/7 \times 1/5$

$= 12/35\ \gamma_1 - 1/35\ \gamma_2 + 34/35$

(B)

$\gamma_1 = Cov(X_t, X_{t-1}) = Cov(1 + \frac{12}{35} X_{t-1} - \frac{1}{35} X_{t-2} + Z_t - \frac{1}{7} Z_{t-1}, X_{t-1})$

$= 12/35\ \gamma_0 - 1/35\ \gamma_1 - 1/7$

(C)

$\gamma_2 = Cov(X_t, X_{t-2}) = Cov(1 + \frac{12}{35} X_{t-1} - \frac{1}{35} X_{t-2} + Z_t - \frac{1}{7} Z_{t-1}, X_{t-2})$

$= 12/35\ \gamma_1 - 1/35\ \gamma_0$

Finally, solving these equations

Substituting (C) into (A) gives

$12/35\ \gamma_1 - 1/35(12/35 - 1/35\ \gamma_0) + 34/35$

$\gamma_0 = (420\ \gamma_1 - 12\ \gamma_1)/1225 + \gamma_0/1225 + 34/35$

So,

$\gamma_0 = \gamma_1/3 + 35/36$

Now substituting into (B), we have

$\gamma_1 = 12/35 \times (\gamma_1/3 + 35/36) – 1/35\, \gamma_1 – 1/7$

Solving this we have,

$\gamma_1 = 3\, \gamma_1/35 + 4/21$

$\gamma_1 = 0.2083$

And

$\gamma_0 = 1/3 \times 0.2083 + 35/36 = 1.04165$

$\gamma_2 = 12/35 \times 0.2083 - 1/35 \times 1.04165 = 0.04165$

Finally, we have
$\rho_0 = 1$, $\rho_1 = \frac{\gamma 1}{\gamma 0} = 0.19997$, $\rho_2 = \frac{\gamma 2}{\gamma 0} = 0.03999$

[4]

b) $\rho_k = \frac{12}{35}\rho_{k-1} - \frac{1}{35}\rho_{k-2}$ for k>=2

We need to show that the solution has the form :
$\rho_k = A\left(\frac{1}{5}\right)^k + B\left(\frac{1}{7}\right)^K$

Substituting the proposed solution into the recurrence relation gives

$\frac{12}{35}\rho_{k-1} - \frac{1}{35}\rho_{k-2} = \frac{12}{35}\left(A\left(\frac{1}{5}\right)^{k-1} + B\left(\frac{1}{7}\right)^{k-1}\right) - \frac{1}{35}\left(A\left(\frac{1}{5}\right)^{k-2} + B\left(\frac{1}{7}\right)^{k-2}\right)$

$= A\left(\frac{1}{5}\right)^k\left(\frac{12}{35}\times 5 - \frac{1}{35}\times 25\right) + B\left(\frac{1}{7}\right)^k\left(\frac{12}{35}\times 7 - \frac{1}{35}\times 49\right)$

$= A\left(\frac{1}{5}\right)^k\left(\frac{60-25}{35}\right) + B\left(\frac{1}{7}\right)^k\left(\frac{84-49}{35}\right)$

$= A\left(\frac{1}{5}\right)^k + B\left(\frac{1}{7}\right)^k$

$= \rho_k$

So the solution does have this form.

The values of *A* and *B* are fixed by $\rho 0 = 1$, $\rho 1 = 0.19997$

$\therefore A + B = 1$

$= \frac{1}{5}A + \frac{1}{7}B = 0.19997$

$= \frac{1}{5}A + \frac{1}{7}(1 - A) = 0.19997$

$= 0.9995$

$\therefore B = 1\text{-}A = 0.00047$

$\rho_k = 0.19997\,(\frac{1}{5})^k + 0.00047\,(\frac{1}{7})^K$ [4]

iii) We require mean and variance of X_t which must be normally distributed since Z is normally distributed.

Variance is γ_0 = 1.04165 from above

$E(X_t) = 1 + \frac{12}{35}E(X_t) - \frac{1}{35}E(X_t)$

Therefore,

$E(X_t) = \frac{35}{24}$ [2]

iv) The autocovariance function is measured in squared units, so that the values obtained depend on the absolute size of the measurements. Thus to make this quantity independent of the absolute sizes by defining a dimensionless quantity, known as autocorrelation function. [1]

v)

a) $Cov(Y_2, Z_3) = 0$ [1]

b) $Cov(Y_3, Y_3) = \alpha_0$ i.e. autocorrelation with lag of 0 [1]

[**15 Marks]**

Solution 4:

i)

- The policyholder must have an interest in the risk being insured
- The risk must be of a financial and reasonably quantifiable nature
- Individual risks should be independent of one another
- The probability that the insured event will occur should be small.
- Large numbers of similar risks should be pooled in order to reduce the variance and achieve greater certainty.
- The insurer's liability should be limited

- Moral hazards should be eliminated as far as possible since these are difficult to quantify, result in selection against the insurer and lead to unfairness in the treatment of some policyholders.

[3]

ii)

Calculation of parameters of Gamma distribution:

$\lambda = \frac{E(X)}{Var\ (X)} = \frac{40}{20} = 2$

$\alpha = E(X) \times \lambda = 80$

The posterior distribution is given by:

$f(\theta 1|x) \propto f(x|\theta 1)\, f(\theta 1)$

$\propto \left(\prod_{j=1}^{5} e^{-\theta_1}\, {\theta_1}^{n_{1j}}\right) \times \theta_1^{\alpha-1} \times e^{-\lambda\theta_1}$

$\propto e^{-(\lambda+5)\theta_1} \theta_1^{\alpha+\sum_{j=1}^{5} n_{1j}-1}$

Which is the pdf of gamma distribution with paramters

$\alpha + \sum_{j=1}^{5} n_{1j} - 1 = 80 + 107 = 187$

$\lambda + 5 = 7$

Under quadratic loss the Bayes estimate is the mean of the posterior distribution i.e =187/7 = 26.71

[4]

iii)

a) The original data for the total claims is Y_{ij} and table for number of policies gives the value of P_{ij}

The claims per unit volume is $X_{ij} = Y_{ij}/P_{ij}$ and is shown in the table below

Insurer (i)	Total claim per unit volume (X_{ij})			
	Year (j)			
	1	2	3	4
A	3.118	2.9	2.611	2.619
B	2.63	3.714	2.448	3.636
C	6.714	3.706	5.667	3.524
D	1.667	1.37	1.951	2.444

Further we calculate $\bar{P}i$, $\bar{P}$ and $P*$

Furthermore the corresponding figures are given below,

Insurer (i)	$\bar{P}i$	$\bar{P}i(1 - \frac{\bar{P}i}{\bar{P}})$
A	76	61.152
B	99	73.805
C	67	55.460
D	147	91.450
	$\bar{P}$ = 389	P* = 18.791

Furthermore, $\overline{X\iota}$ and $\bar{X}$ can be calculated as $\overline{X\iota} = \sum_{j=1}^{n} Yij / \overline{P\iota}$ and $\bar{X} = \sum_{i=1}^{N}\sum_{j=1}^{n} Yij / \overline{P\iota}$

Insurer (i)	$\overline{X\iota}$	$\sum Pij(Xij - \overline{X\iota})^2$	$\sum Pij(Xij - \bar{X})^2$
A	2.803	3.245	3.248
B	3.030	32.064	36.879
C	4.716	116.654	360.220
D	1.796	21.258	172.358
	$\bar{X}$ = 2.810		

So this gives E[$m(\vartheta)$] ≈ 2.810

From the other columns in the table we get:

$E[S^2(\vartheta)] \approx 1/4\sum_{i=1}^{4} 1/3 \sum_{j=1}^{4} Pij(Xij - \overline{X\iota})^2$
= (3.245 + 32.064 + 116.654 + 21.258)/12
= 14.435

Also,
$Var[m(\vartheta)] \approx 1/ P^* \left\{ \frac{1}{4 \times 4-1} \sum_{i=1}^{4}\sum_{j=1}^{4} Pij(Xij - \bar{X})^2 - 14.435 \right\}$

$= \frac{1}{18.791}[(3.248+36.879+360.220+172.358)/15-14.435]$

= 1.264

The credibility factor for all the insurer A, B, C and D is

$Z_A = \frac{\sum_{j=1}^{n} PAj}{\sum_{j=1}^{n} PAj + \frac{E[s^2(\theta)]}{var[m(\theta)]}} = \frac{76}{76+\frac{14.435}{1.264}} = 0.8693$

Similarly,

Z_B = 0.8966 , Z_C = 0.8543 and Z_D = 0.9279

Therefor risk volume for respective insurer is,
$Z_A\overline{X1} + (1- Z_A)E[m(\vartheta)]$ = 2.8036 for Insurer A
$Z_B\overline{X1} + (1- Z_B)E[m(\vartheta)]$ = 3.0075 for Insurer B
$Z_C\overline{X1} + (1- Z_C)E[m(\vartheta)]$ = 4.4387 for Insurer C
$Z_D\overline{X1} + (1- Z_D)E[m(\vartheta)]$ = 1.8690 for Insurer D [10]

b) The Pj 's specify the relative weightings to be put on the claims for each year.

Since the definition $E[S^2(\vartheta)]$ = Pj var(Xj/ϑ) includes a Pj factor, but var[$m(\vartheta)$]doesn't, the Ratio $E[S^2(\vartheta)]$/ var[$m(\vartheta)$] varies in proportion to the Pj 's. So any extra factor incorporated in the Pj 's would cancel out, leaving Z unchanged.

There for we don't expect a uniform increase applied to all the weightings to affect the credibility factor.
Also,
Changing the unit of currency should not affect the credibility factor.
Since the quantities E[S2(θ)] and var[m(θ)] are measured in units of Rs , their ratio is dimensionless. So changing the unit of currency to pound would not affect Z . [2]

c) If all the Pj 's are equal to 1, then EBCT Model 2 is exactly the same as EBCT Model 1 and hence would give the same creditability factors. [1]

[20 Marks]

Solution 5:

i) Likelihood ratio – This is defined as twice the difference between the log-likelihood of the model under consideration and the saturated model. .

Saturated Model - A saturated model is defined to be a model in which there are as many parameters as observations, so that the fitted values are equal to the observed values. [2]

ii) The Pearson residuals are defined as (y - μ) / sqrt(var(μ), while the deviance residuals are defined as the product of the sign of (y - μ) and the square root of the contribution of y to the scaled deviance. Thus, the deviance residual is sign(y - μ)di , where the scaled deviance is SUMMATION(d_i^2).

The Pearson residual, which is often used for normally distributed data, has the disadvantage that its distribution is often skewed for non-normal data. This makes the interpretation of residuals plots difficult. Deviance residuals are usually more likely to be symmetrically distributed and to have approximately normal distributions, and are preferred for actuarial applications. [3]

iii) For normally distributed data, the Pearson and deviance residuals are identical.

If Y_i follows N(μ_i, σ^2), then Pearson residuals are (yi - μi)/ σi.

The scaled deviance is $\sum_{i=1}^{n}\frac{(y_i-\mu_i)^2}{\sigma^2}=\sum_{i=1}^{n}d_i$

The deviance residuals are given by

$$sign(y_i-\mu_i)\sqrt{d_i}=sign(y_i-\mu_i)\left|\frac{y_i-\mu_i}{\sigma}\right|=\frac{y_i-\mu_i}{\sigma}$$

Hence the Pearson residuals and the deviance residuals are the same. [2]

[7 Marks]

Solution 6:

i) The distribution of *X* given λ is a binomial with n = 2 and p = λ

P(X=0) = $\int_0^1 f(0,\lambda)d\lambda$ = $\int_0^1 P(X=0|\lambda)f(\lambda)d\lambda$ = $\int_0^1 (1-\lambda)^2\alpha\lambda^{\alpha-1}\ d\lambda$

= $\alpha \int_0^1 (1-2\lambda+\lambda^2)\lambda^{\alpha-1} d\lambda$ = α * [$\frac{1}{\alpha}$ - $\frac{2}{\alpha+1}$ + $\frac{1}{\alpha+2}$] = $\frac{2}{(\alpha+1)(\alpha+2)}$

P(X=1) = $\int_0^1 f(1,\lambda)d\lambda$ = $\int_0^1 P(X=1|\lambda)f(\lambda)d\lambda$ = $\int_0^1 2\lambda(1-\lambda)\alpha\lambda^{\alpha-1}\ d\lambda$
= $2\alpha \int_0^1 (\lambda^{\alpha} - \lambda^{\alpha+1})d\lambda$ = 2α * [$\frac{1}{\alpha+1}$ − $\frac{1}{\alpha+2}$] = $\frac{2\alpha}{(\alpha+1)(\alpha+2)}$

P(X=2) = $\int_0^1 f(2,\lambda)d\lambda$ = $\int_0^1 P(X=2|\lambda)f(\lambda)d\lambda$ = $\int_0^1 \lambda^2\alpha\lambda^{\alpha-1}\ d\lambda$
= $\alpha \int_0^1 (\lambda^{\alpha+1})d\lambda$ = $\frac{\alpha}{\alpha+2}$

[3]

ii) From the given information we observe that prior is a beta distribution with a = α and b = 1, and since the model distribution of X is a binomial with n = 2 and p = λ, and since we have observed x = 1,

The posterior distribution of λ is also beta with
a´ = a + x = a +1 and b´ = b + n – x = 1 + 2 -1 = 2, with pdf

f(λ|x =1) = $\frac{\Gamma(a' + b')}{\Gamma(a')\Gamma(b')} \times \lambda^{a'-1}(1-\lambda)^{b'-1}$ = $\frac{\Gamma(a+3)}{\Gamma(a+1)\Gamma(2)} \times \lambda^{\alpha}(1-\lambda)$

= $(\alpha+1)(\alpha+2)\lambda^{\alpha}(1-\lambda)$
This probability is the mean of the posterior and the mean of the posterior is $\frac{a'}{a'+b'}$ = $\frac{\alpha+1}{\alpha+3}$

[3]
[6 Marks]

Solution 7:

i)

a) Calculation of capital required by Insurer in Financial year 2016-17

A	Gross Premium	900 Crores
B	Reinsurance %	0%
C	Reinsurance Premium C = AXB	0 Crores
D	Net Premium D = A – C	900 Crores
E	Capital Required basis value 1	=1.5 X 20% X Max(900 Crores, 50% of 900 Crores) =270 Crores
F	Gross Loss Ratio	75%
G	Gross Loss	= 75% X 900 Crores =675 Crores
H	Net Loss Ratio	75%
I	Net Loss	=75% X 900 Crores =675 Crores
J	Capital Required basis value 2	= 1.5 X 30% X Max(675, 50% of 675) =303.75 Crores

K	Final Capital Required Maximum of E or J	**303.75 Crores**

[3]

b) Calculation of capital required by Insurer in Financial year 2017-18

A	Gross Premium	1000 Crores
B	Reinsurance %	70%
C	Reinsurance Premium C = AXB	700 Crores
D	Net Premium D = A – C	300 Crores
E	Capital Required basis value 1	=1.5 X 20% X Max(300, 50% of 1000) =150 Crores
F	Gross Loss Ratio	90%
G	Gross Loss	900 Crores
H	Net Loss Ratio	90%
I	Net Loss	=300 Crores X 90% = 270 Crores
J	Capital Required basis value 2	=1.5X30%XMax(270, 50% of 900) =202.50 Crores
K	Final Capital Required Maximum of E or J	**202.50 Crores**

[3]

ii) Following are the return on capital employed for both years:-

a) 2016-17

A	Total Capital Employed	303.75 Crores
B	Total Net Premium	900 Crores
C	Total Net Loss	675 Crores
D	Profit	=900 – 675 =225 Crores
E	Return on Capital	= 225/303.75 =74%

[2]

b) 2017-18

A	Total Capital Employed	202.50 Crores
B	Total Net Premium	300 Crores
C	Total Net Loss	270 Crores
D	Profit	=300 – 270 =30 Crores
E	Return on Capital	=30/202.50 =15%

[2]

iii) Following could have been the reasons for Regulator to give restricted benefit of Reinsurance in calculation of solvency capital:-

- To discourage insurers to act line fronting companies for Reinsurers
- Encourage underwriting discipline by retaining more premium
- Discourage too much outflow of reinsurance premium outside of the country
- To have adequate solvency capital in case of default by Reinsurers in paying claims

[2]

iv)

A	Gross Premium	1200 Crores
B	Retention	20%
C	Net Premium	=20% X 1200= 240 Crores
D	Reinsurance premium Rate	15%
E	Reinsurance premium	=15% X 240 =36 Crores
F	Maximum Net Loss ratio for Insurer	120%
G	**Maximum downside for Insurer**	**=(120%-100%)X240 + Reinsurance premium paid = 20%X240 + 36 Crores = 84 Crores**
H	Maximum Loss or Limit for stop loss Reinsurer	80%
I	Maximum Loss to be borne by Reinsurer	=80% X 240 = 192 Crores
J	**Maximum downside for Reinsurer**	**Max Possible Loss - Premium received =192 - 36 = 156 crores**

[2+2=4]

v) Following could have been the reasons for Insurer to take Reinsurance:-

- To provide financial capacity to write business due to limited available shareholder's capital
- To provide limited downside on net account
- To provide expertise in underwriting

[2]

[18 Marks]

Solution 8:

i) The stored table of random numbers generated by a physical process may be too short a combination of linear congruential generators (LCG) can produce a sequence which is infinite for practical purposes.

It might not be possible to reproduce exactly the same series of random numbers again with a truly random number generator unless these are stored. A LCG will generate the same sequence of numbers with the same seed.

Truly random numbers would require either a lengthy table or hardware enhancement compared with a single routine for pseudo random numbers.

The methods to generate random variates are,

- Inverse Transform method.
- Acceptance-Rejection Method
- Box-Muller algorithm (from the standard normal distribution)
- Polar algorithm (from the standard normal distribution) [3]

ii) Advantage - Generates a sample of every pair of u1 and u2 – no possibility of rejection
Disadvantage - requires calculation of sin and cos functions which is more computationally intensive [2]

iii) By central limit theorem, $p - \hat{\hat{p}} \approx N\left(0, \frac{\tau^2}{n}\right)$ where τ^2 cab be approximated by 0.10

$$P\left[-1.96 \leq \frac{p-\hat{\hat{p}}}{\sqrt{\frac{0.10}{n}}} \leq 1.96\right] = 0.95$$

And we require

$$1.96 \times \sqrt{\frac{0.10}{n}} \leq 0.01$$

i.e. $n \geq \frac{1.96^2 \times 0.10}{0.01^2}$

hence, n must be at least 3841.6 [3]

[8 Marks]

Solution 9:

i) Following are the key limitations of chain ladder method:-

- The chain-ladder technique is only accurate when patterns of loss development in the past can be assumed to continue in the future.
- In contrast to other loss reserving methods such as the Bornhuetter-Ferguson method, it relies only on past experience to arrive at an incurred but not reported claims estimate.
- When there are changes to an insurer's operations, such as a change in claims settlement times, changes in claims staffing, or changes to case reserve practices, the chain-ladder method will not produce an accurate estimate without adjustments
- The chain-ladder method is also very responsive to changes in experience, and as a result, it may be unsuitable for very volatile lines of business.

[2]

ii)

a) Inflation factors for each development year

Policy year	Development Year			
	0	1	2	3
2014	1.1910	1.1236	1.06	1

2015	1.1236	1.06	1	
2016	1.06	1		
2017	1			

Inflation adjusted claim payments in mid-2017 prices

Policy year	Development Year (Amounts in '000)			
	0	1	2	3
2014	712	506	318	180
2015	770	541	344	
2016	779	540		
2017	705			

Inflation adjusted cumulative claim payments in mid-2017 prices

Policy year	Development Year (Amounts in '000)			
	0	1	2	3
2014	712	1218	1536	1716
2015	770	1310	1654	
2016	779	1319		
2017	705			
Development factors	1.701	1.262	1.117	

Outstanding amounts arising from 2017 policies

Cumulative Claims at mid-2017 prices	705	1,199.53	1,513.65	1,691.03
Dis-accumulated claims	705	494.53	314.12	177.38
Inflation factor	1	1.06	1.1236	1.1910
Claims after adjusted for inflation	705	524.20	352.94	211.26

Total outstanding from 2017 = 524.20 + 352.94 + 211.26 = 1088.41 ('000) [4]

b) Ultimate amount of claims for 2017 policies ('000) = 2500 X 80%= 2000

2017	DY0	DY 1	DY2	DY3
Development factors	1.701	1.262	1.117	

Total Claims as per ULR	833.81	1418.70	1790.21	2000
Difference		584.89	371.51	209.79
Inflation factor		1.06	1.1236	1.1910
Adjusted amount		619.98	417.43	249.86

Hence, total outstanding = 1287.27 (‘000) [4]

[10 Marks]

Solution 10:

i) $P(Y = y) = \binom{n}{ny} \mu^{ny}(1-\mu)^{n-ny}$ [1]

ii)

$$P(Y{=}y) = \exp\left[ny\log\mu + n(1-y)\log(1-\mu) + \log\binom{n}{ny}\right]$$

$$= \exp\left\{ n\left(y\log\frac{\mu}{1-\mu} + \log(1-\mu)\right) + \log\binom{n}{ny}\right\}$$

which is in the form of an exponential family.

The natural parameter is $\log\frac{\mu}{1-\mu}$

The dispersion parameter is

either φ = n and a(φ) = $\frac{1}{\varphi}$

or $\varphi = \frac{1}{n}$ and a(φ) = φ

[2]

iii) $V(\mu) = b''(\theta)$

$b(\theta) = -\log(1-\mu) = \log\frac{1}{1-\mu} = \log(1+e^{\theta})$

$b'(\theta) = \frac{e^{\theta}}{1+e^{\theta}}$

$b''(\theta) = \frac{(1+e^{\theta})e^{\theta} - e^{\theta}e^{\theta}}{(1+e^{\theta})^2} = \frac{e^{\theta}}{(1+e^{\theta})^2}$

$= \mu(1-\mu)$

[2]

iv) scaled deviance is $-2(l_c - l_f)$

$$l_c = \Sigma\left\{n\left(y_i \log\frac{\mu i}{1-\mu i} - \log\frac{1}{1-\mu i}\right) + \log\binom{n}{nyi}\right\}$$

$$l_f = \Sigma\left\{n\left(y_i \log\frac{yi}{1-yi} - \log\frac{1}{1-yi}\right) + \log\binom{n}{nyi}\right\}$$

Hence the scaled deviance is

$$-2(l_c - l_f) = -2\ \Sigma\ n\left(y_i \log\left(\frac{\mu i(1-yi)}{yi(1-\mu i)}\right) - \log\left(\frac{1-yi}{1-\mu i}\right)\right)$$

[2]

[**7 Marks]**

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