Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

December 2018 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) In addition to variation by age and sex, mortality and morbidity rates are observed to vary: between geographical areas, by social class and over time. None of these categories provide a direct (causal) explanation of the observed differences. Rather they are proxies for the real factors that cause the observed differences. Such factors are:

- occupation
- nutrition
- housing
- climate
- education
- genetics.

It is rare that observed differences in mortality can all be ascribed to a single factor. It is difficult to disentangle the effects of different factors as a result of confounding. Confounding is a result of correlation between these factors**.** [3]

ii) Directly standardised mortality rate (DSMR):
DSMR =

$$\frac{\int {}^{s}E^{c}_{x,t}\ M_{x,t}}{\int {}^{s}E^{c}_{x,t}}$$

It is a weighted average of $M_{x,t}$ using ${}^{s}E^{c}_{x,t}$ as weights.

$E^{c}_{x,t}$ =Central exposed to risk in population being studied between ages x and x + t .

$M_{x,t}$ = Central rate of mortality either observed or from a life table in population being studied for ages x to x + t .

${}^{s}E^{c}_{x,t}$ = Central exposed to risk for a standard population between x and x + t . ,

${}^{s}M_{x,t}$ = Central rate of mortality either observed or from a life table in standard population for ages x to x + t . [2]

iii) Actual number of death for City A
128 + 135+180 = 443

Mortality rates in standard population
Age 50 : $\frac{22532}{2200000}$ = 0.010242

Age 51: $\frac{28331}{2400000}$ = 0.011805

Age 52: - $\frac{25742}{2500000}$ = 0.010297

Expected number of Death for A

10500 X 0.010242 + 12090 X .011805 + 10012 X 0.010297 = 353.35

SMR = 443/353.35 [3]

[8 Marks]

Solution 2:

$\frac{r_{65}}{l_{35}}$ **X** $V^{(65-35)}$ **X 50000 X** $\frac{1.5}{12}$ **X (65-35) X** $\frac{(s_{63} + s_{64})/2}{s_{35}}$

$\frac{3757}{18866}$ **X** $V^{(65-35)}$ **X 50000 X** $\frac{1.5}{12}$ **X (65-35) X** $\frac{(11.151+11.328)/2}{6.655}$

= 19443.75

Assume value of Contribution is k% of salary

Value of contribution of k % of salary

50000 X K % X ${}^{s}N_{35}/{}^{s}D_{35}$

= 50000 k% (502836/31816)

= 790225 k%

K = 2.46%

[4 Marks]

Solution 3:

$=100000 \int_0^{20} v^t \, {}_tP_{40}^{h} \, (\mu_{40+t} + \sigma_{40+t}) \, dt$

$=100000 \int_0^{20} e^{-\ln(1.05)t} \, {}_tP_{40}^{hh} 0.006 dt$

${}_tP_{40}^{hh} = {}_tP_{40}^{hh}$ = exp $(-\int_{40}^{40+t} (\mu_s + \sigma_s) \, ds$

$e^{-0.006t}$

Therefore, value =

$=100000 \text{ X } .006 \int_0^{20} e^{-\ln(1.05)t} \, e^{-0.006t} dt$

$=600 \int_0^{20} e^{-\ln(1.05)t}\, e^{-0.006t} dt$

$=600 \int_0^{20} e^{--5479} tdt$

$=600 \times (-e^{--5479t}/.5479)_0^{20}$

=600 (-6.10097+18.25151)

=7290.3

[7 Marks]

Solution 4:

Direct expenses are those that vary with the amount of business written.
Direct expenses are divided into:

- Initial expenses
- Renewal expenses
- Termination expenses

Examples of each:

Initial expenses – those arising when the policy is issued e.g. initial commission

Renewal expenses – those arising regularly during the policy term e.g. renewal commission

Termination expenses – those arising when the policy terminates as a result of an insured contingency (e.g. death claim for a temporary life insurance policy)

[5 Marks]

Solution 5:

i) P $\ddot{a}_{30:35}$ =500600 $A_{30:35}$ – 400 $A_{30:35}1$ + 0.02 P $\ddot{a}_{30:35}$ – 0.02 P + 300 + 0.5P

Expected Present Value of premium

P $\ddot{a}_{30:35}$ = 15.150 P

EPV of benefits and claim expenses

$A_{30:35}$ =0.14246

$A_{30:35}1$ = v^{35} X ${}_{35}P_{\ 40}$ = 0.13011X0.88877 = 0.11563

EPV of benefits

500600 X .14246 – 400 X 0.11563

= 71269.22

EVP of remaining Expense

0.02 X 15.150 P – 0.02 P +0.5P + 300 = .783 P +300

Equation of value

15.150 P = 71269.22 + 300 +.783 P

14.367 P = 71569.22

P = 4981.5 pa [6]

ii) Retrospective reserve

${}_{25}V^{Retro} = \frac{1}{v^{25} \text{ X } {}_{25}P_{30}}$ X 0.98 P $\ddot{a}_{30:25}$ – 0.48P-300 -500600 $A^{\ 1}_{30:35}$

v^{25} X ${}_{25}P_{30}$ = .37512 X 0.96298 = 0.36123

$\ddot{a}_{30:25}$=$\ddot{a}_{30}$ - $v^{25}{}_{25}P_{30}\ddot{a}_{55}$ =21.834 - .36123 X 15.873 = 16.1

$A^{\ 1}_{30:35}$= A_{30} -$v^{25}{}_{25}P_{30}A_{55}$ =.16023 - .36123X.38950 = 0.01953

${}_{25}V^{Retro}$= (1/.36123) [4981.5((0.98)*(16.100)-(0.48)]-300-(500600X0.01953)

=1,83,069.7 [4]

[10 Marks]

Solution 6:

Let P be the annual premium payable

P $\ddot{a}_{[61]:4}$ =500000 $A_{[61]:4}$ + (100+.025P) ($\ddot{a}_{[61]:4}$-1) + 800

P * 3.730 = 500000* .85654 +(100+.025 P*(3.730-1) + 800

Solving for P

P (0.975 X 3.730+ 0.025) = 500000*.85654 + 100*2.730 +500

P = 429343/3.662

P =117250

Reserve required on the policy per unit sum assured

$_1V_{61:4}$=1 - $\frac{\ddot{a}_{62:3}}{\ddot{a}_{61:4}}$ = 1-$\frac{2.857}{3.722}$ = .23240

$_2V_{61:4}$=1 - $\frac{\ddot{a}_{63:2}}{\ddot{a}_{61:4}}$ =1-$\frac{1.951}{3.722}$ = .47582

$_3V_{61:4}$=1 - $\frac{\ddot{a}_{64:1}}{\ddot{a}_{61:4}}$ =1-$\frac{1}{3.722}$ = .0.73133

Multiple Decrement Table

Multiple Decrement Table			
X	$q_{[x[}^{d}$ =$(aq)_{[x[}^{d}$	$q_{[x[}^{s}$	$(aq)_{[x[}^{s}$ = $q_{[x[}^{s}$ (1-$(aq)_{[x[}^{d}$)
61	0.006433	0.05	0.04968
62	0.009696	0.05	0.04952
63	0.011344	0.05	0.04943
64	0.012716		
T	$(ap)_{[61]+t-1[}$	$_{t-1}(ap)_{[61]}$	
1	0.943887	1.00000	
2	0.940784	0.94389	
3	0.939226	0.88799	
4	0.987284	0.83403	

Year	Premium	Expense	Openeing Reserve	Interest	Death Claim	Surrender Claim	Maturity Claim	Closing Reserve	Profit
1	117250.7681	800	0	5822.538404	3,216.50	5,825.02		109,679.67	3,552.12
2	117250.7681	3,031.27	116200	11520.97494	4,848.00	11,612.52		223,821.92	1,658.04
3	117250.7681	3,031.27	237910	17606.47494	5,672.00	17,387.12		343,442.08	3,234.78
4	117250.7681	3,031.27	365665	23994.22494	6,358.00		493,642.00	-	3,878.72

Year	Profit Signature	Discount Fa	NPV of Profit Signature
1	3,552.12	0.9259259	3,289.00
2	1,565.00	0.8573388	1,341.74
3	2,872.45	0.7938322	2,280.25
4	3,234.97	0.7350299	2,377.80
NPV of Profit Signature			9,288.79

Year	Premium	$_{t-1}V_{[61]}$	Discount F	NPV Premium
1	117250.7681	1.00000	1	117,250.77
2	117250.7681	0.94389	0.925926	102,473.91
3	117250.7681	0.88799	0.857339	89,263.98
4	117250.7681	0.83403	0.793832	77,629.38
		NPV of Premium		386,618.04

Profit Margin = 2.40% NPV profit signature/NPV premium

[10 Marks]

Solution 7:

For a unit-linked life assurance contract, we have:
the unit fund that belongs to the policyholder. This fund keeps track of the premiums allocated to units and benefits payable from this fund to policyholders are denominated in these units. This fund is normally subject to unit fund charges.

The non-unit fund that belongs to the company. This fund keeps track of the premiums paid by the policyholder which are not allocated to units together with unit fund charges from the unit-fund. Company expenses will be charged to this fund together with any non-unit benefits payable to policyholders.

[4 Marks]

Solution 8:

i) Constant force of a mortality is a method that enables us to calculate survival probabilities over non-integer time periods and from non-integer ages. It assumes that the force of mortality takes a constant value between consecutive integer ages. [2]

ii) $P55 = e^{-\mu}$

$\mu = -\log(1-q_{55})$

= -log (1-0.004469)

= 0.004479016 [2]

[4 Marks]

Solution 9:

i) Assuming claim is payable only at the end

Premium = P $\ddot{a}^{(12)}_{40:\overline{20|}}$

$=P\left[\ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1-\frac{D_{60}}{D_{40}}\right)\right]$

$=P\left[13.927 - \frac{11}{24}\left(1-\frac{882.85}{2052.96}\right)\right]$

=13.66576724 P

Claim Cost

75,00,000 $A'_{40:\overline{20|}}$ + 50,00,000 $A'_{[40]:\overline{20|}}$

75,00,000 $\left[A'_{40:20} - \frac{D_{60}}{D_{40}}\right] + 50,00,000\left[A'_{[40]:\overline{20|}} - \frac{D_{60}}{D_{[40]}}\right]$

$75,00,000\left[0.46433-\frac{882.85}{2052.96}\right]+50,00,000\left[0.46423-\frac{882.85}{2052.54}\right]$

=2,57,193.968202- 1,70,521.965824

=4,27,714.965824

Alternate solution (if claim is paid immediately)

=4,27,714.965824* 1.04^0.5

=4,36,185.3914

Claim Investigation expense

=0.03 x 427714.965824

=12,831. 448975

Alternate solution (if claim is paid immediately)

=0.03 x 43,6185.3914

=13,085.56174

Commission Expense

$0.3P\ P\ \ddot{a}^{(12)}_{40:\overline{1}|} + 0.05\ P\ \ddot{a}^{(12)}_{40:\overline{20}|}$

$=0.3\ P\left[\ddot{a}_{40:1} - \frac{11}{24}\left(1-\frac{D_{41}}{D_{40}}\right)\right] + 0.05P \times 13.66576724$

$=0.3P\left[\frac{1}{1.04}\ x\ (1-0.000937) - \frac{11}{24}\ x\ \left(1-\frac{1972.15}{2052.96}\right)\right] + 0.683288362\ P$

=0.3P x 0.942596 + 0.683288362 =0.966067 P

Management Expense

$=0.1P \times \ddot{a}^{(12)}_{40:\overline{20}|} \times 1.04^{0.5}$

$=0.1P \times 13.66576724 \times 1.04^{0.5}$

=1.39364 P

Underwriting Expense

= 500

Calculation of Monthly Premium

13.66576724 P = 4,27,714.965824 +12,831.448975 +0.966067P +1.39364P +500

11.30607 P = 4,41,046.4148

P = 39009.7

Alternate solution (if claim is paid immediately)

13.66576724 P = 4,36,185.3914 +13,085.56174 +0.966067P +1.39364P +500

11.30607 P = 4,49,770.95

P = 39,781.37

Monthly premium =39,009.7/12 = 3,250.8

Alternate solution (if claim is paid immediately)

Monthly premium = 39,781.37/12 = 3,315.1

[9]

ii) Calculation of Reserve

I. Policies in which only accidental claim has been made

$=75{,}00{,}000\, A'_{45:\overline{15}|} - 39{,}009.7\, \ddot{a}^{(12)}_{45:\overline{15}|}$

$=75{,}00{,}000 \left[A_{45:15} - \frac{D_{60}}{D_{45}}\right]$

$-39{,}009.7 \left[\left[\ddot{a}_{45:\overline{15}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{45}}\right)\right]\right]$

$=75{,}00{,}000 \left[0.56206 - \frac{882.85}{1677.97}\right] - 39{,}009.7 \left[11.386 - \frac{11}{24}\left(1 - \frac{882.85}{1677.97}\right)\right]$

=75,00,000 x 0.035918293 – 39,009.7x 11.14485172

=2,69,387.1975- 4,34,750.6352

=-1,65,363.4377

Alternate solution (if claim is paid immediately)

$75{,}00{,}000\, A'_{45:\overline{15}|} - 39{,}781.37\, \ddot{a}_{45:\overline{15}|}$

$=75{,}00{,}000 \left[A_{45:15} - \frac{D_{60}}{D_{45}}\right]$

$-39{,}781.37\left[\left[\ddot{a}_{45:\overline{15}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{45}}\right)\right]\right]$

$=75{,}00{,}000 \left[0.56206 - \frac{882.85}{1677.97}\right] - 39{,}781.37\left[11.386 - \frac{11}{24}\left(1 - \frac{882.85}{1677.97}\right)\right]$

=75,00,000 x 0.035918293 – 39,781.37 x 11.14485172

=2,69,387.1975- 4,43,357.4699

=-1,73,970.2724

II. Policies in which only illness claim

(12)

=50,00,000 $A'_{45:15}$ - 39,009.7 $\ddot{a}_{45:15}$

=50,00,000 x 0.035918293 – 4,34,750.6352

= -2,55,159.1702

Alternate solution (if claim is paid immediately)

=50,00,000 $A'_{45:15}$ - 39,781.37 $\ddot{a}_{45:15}$

=50,00,000 x 0.035918293 – 4,43,357.4699

= -2,63,766.0049

III. Policies in which no claim has been made

=1,25,00,000 x 0.035918293 – 4,34,750.6352

= 14,228.0273

Alternate solution (if claim is paid immediately)

=1,25,00,000 x 0.035918293 – 4,43,357.4699

= 5,621.1926 [4]

[13 Marks]

Solution 10:

i) Let P be the annual premium

Premium = P $\ddot{a}_{\overline{50^m:50^f}}$

= P $\left[\ddot{a}_{50^m} + \ddot{a}_{50^f} - \ddot{a}_{50^m:50^f}\right]$

= P [18.843+19.539-17.688]

= 20.694 P

Initial expense = 1,000

Recurring expense = 0.05 P x $\ddot{a}_{\overline{50^m:50^f}}$

= 0.05P x 20.694

= 1.0347 P

Claim cost = 2,00,000 $\bar{A}_{\overline{50^m:50^f}}$

= 2,00,000 x $1.04^{0.5}$ - $A_{\overline{50^m:50^f}}$

=2,00,000 x $1.04^{0.5}$ x $\left(1 - d\ddot{a}_{\overline{50^m:50^f}}\right)$

=2,00,000 x $1.04^{0.5}$ x $\left(1 - \frac{0.04}{1.04} \text{ x } 20.694\right)$

= 41,623. 68852

=20.694 P = 1000 + 1.0347 P + 41,623.68852

P = 2,168. 02 [7]

ii) Net Premium

$= 2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(\frac{1}{ä_{50^m:50^f}} - d\right)$

$=2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(\frac{1}{20.694} - \frac{0.04}{1.04}\right)$

= 2,011.39

We will require three provisions at the end of 5th policy year.

- **Both lives are alive**

$= 2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(1 - \frac{ä_{\overline{55:55}}}{ä_{\overline{50:50}}}\right)$

$= 2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(1 - \frac{17.364+18.210-16.016}{18.843+19.539-17.688}\right)$

= 11,196.46

- **Only Male alive**

$= 2,00,000 \text{ x } \bar{A}_{55} - 2,011.39\ ä_{55}$

$= 2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(1 - \frac{0.04}{1.04}\ x17.364\right) - 2,011.39 \text{ x } 17.364$

= 32,820.60

- **Only Female alive**

$= 2,00,000 \text{ x } \bar{A}_{55} - 2,011.39\ ä_{55}$

$= 2,00,000 \text{ x } 1.04^{0.5} \text{ x } \left(1 - \frac{0.04}{1.04}\ x\ 18.210\right) - 2,011.39 \text{ x } 18.21$

= 24, 482.39

Mortality Profit/ loss = Expected Death Strain – Actual Death Strain

There are four components

(a) Both lives die during 2018: no claims reported

= (4900 x q_{54^m} x q_{54^f} -0) x (2,00,000 x $1.04^{0.5}$ -11,196.46)
= 849.37

(b) Female alive at the start of the year but dies during the year: one actual claim
=(100 x q_{54^f} -1) x (2,00,000 x $1.04^{0.5}$ -24,482.39)
= -1,63,109.96

(c) Both lives alive at start of the year, only male dies during the year: one actual claim
=(4900 x q_{54^m} x p_{54^f} -1) x (24,482.39 – 11,196.46)
=50,845.17

(d) Both lives alive at start of the year, only female **dies during the year: No actual claims**
=(4900 x p_{54^m} x q_{54^f} -0) x (32,820.611 – 11,196.46)
=96,538.66

Total mortality loss = 849.37 - 1,63,109.96 + 50,845.17+ 96,538.66 **= (14,876.77)**

[13]

[20 Marks]

Solution 11:

i) Net future loss random variable

Let x be the annual annuity payment amount.

The net future loss random variable at the outset for this policy is

L = X $\ddot{a}_{\overline{\max(k_{60^m}, k_{55^f}) + 1}}$ - P

Where,

P is the single premium i.e. 1,00,000

K_{60} is the curtate future lifetime of a male life aged 60

K_{55} is the curtate future lifetime of a female life aged 55

[3]

ii) Annual annuity payment amount

P = x $\ddot{a}_{\overline{60^m:55^f}}$ @ 4%

=1,00,000 = x $\left[\ddot{a}_{60^m} + \ddot{a}_{55^f} - \ddot{a}_{\overline{60^m:55^f}}\right]$

=1,00,000 = x (15,632 + 18. 210 – 14.756) x = 5,239. 442523

Annual annuity payment is 5,239.44

[6]

iii) Standard deviation

The variance of the net future loss random variable is

Var (L) = $5{,}239.44^2 \left[\left\{ 2_{A_{\overline{60^m:55^f}}} - \left(A_{\overline{60^m:55^f}}\right)2\right\} ./\ d \right]$

Using premium conversion and the result of 11 (b) we have,

$A_{\overline{60^m:55^f}}$ = $\left(1 - d\ \ddot{a}_{\overline{\mathbf{60:55}}}\right)$ @ 4%

= 1- $\frac{0.04}{1.04}$ x 19.086

= 0.26592307

And,

$A_{\overline{60^m:55^f}}$ = $\left(1 - d\ \ddot{a}_{\overline{\mathbf{60:55}}}\right)$ @ 8.16%

= 1- $\frac{0.0816}{1.0816}$ x $\ddot{a}_{\overline{\mathbf{60:55}}}$

So, the standard deviation of L is

$$\sqrt{5{,}239.44\text{^}2 + \left[\left\{ 2_{A_{\overline{60^m:55^f}}} - \left(A_{\overline{60^m:55^f}}\right)\text{^}2\right\} ./\ d^2\right]}$$

[6]

[15 Marks]

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