INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st December 2018

Subject CT8 – Financial Economics

Time allowed: Three Hours (15.00 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) **i)** Define Isoelastic utility function? Give an example. (3)

ii) You have plotted a power utility function with $\gamma = 1$. Your friend has a $\gamma = 0.5$

a) Who is more risk averse? Support your answer with calculations. (2)

b) Your friend has initial wealth of 100 and is offered the opportunity to buy Investment X for 100 which offers an equal chance of payout of 110 or 92. Will he choose to buy the investment? (3)

iii) Outline the difference between Utility theory and Prospect theory? (2)

iv) Suppose a factor model is appropriate to describe the returns on a stock. The current expected return on the stock is 10.5 %. Information about those factors is presented in the following chart:

Factor	β	Expected Value	Actual Value
Growth in GDP	2.04	3.5%	4.8%
Inflation	-1.9	7.1%	7.8%

a) What is the systematic risk of the stock return? (2)

b) The firm announced that its market share had unexpectedly increased from 23 percent to 27 percent. Investors know from past experience that the stock return will increase by 0.36 percent for every 1 percent increase in its market share. What is the unsystematic risk of the stock? (2)

c) What is the total return on this stock? (2)

[16]

Q. 2) **i)** Define VaR? What are the shortfalls of this measure? (3)

ii) A portfolio of 100 cr is invested in an asset where the return is normally distributed with mean 5% and standard deviation 6%. Calculate the 95% VaR over 1 year? (4)

iii) If the return of an asset X is modelled as follows:

X	Probability
-7	0.04
5.5	0.96

Calculate the 95% Tail VaR over 1 year of the portfolio consisting of INR 10 cr. (4)

[11]

Q. 3) **i)** You are to price options on a stock. The movements of the stock price are modelled by a binomial tree. You are given the following information:

- Each period is 6 months.
- u/d = 4/3, where u is one plus the rate of gain on the stock price if it goes up, and d is one plus the rate of loss if it goes down.
- The risk-neutral probability of an up movement is 1/3.

- The initial stock price is 80.
- The continuously compounded risk-free interest rate is 5%.
- Option is priced with volatility 20%

Let CI be the price of a 1-year 85-strike European call option on the stock, and CII be the price of an otherwise identical American call option.
Determine CII – CI and explain the result. (6)

ii) How will you check whether the tree has been calibrated appropriately at Time period 1 using both the tree and theoretically? (4)

[10]

Q. 4) **i)** Define Markov process? (1)

ii) What form of market efficiency is the Markov property of stock price consistent with? Explain. (3)

iii) Let (Xn : n ≥ 1) be a sequence of iid mean-zero random variables with finite variance σ^2
Let Sn = X1 + · · · + Xn and let
$M_n = S^2_n - n\sigma^2$
Prove that Mn is a martingale (6)

[10]

Q. 5) The Garman-Kohlhagen formula for the price V (assuming a risk-free force of interest r) of a European call option maturing T years from now with strike price K on a stock that pays dividends at force q whose current spot price is S is:

$$V = S\, e^{-qT}\Phi(d_1) - K\, e^{-rT}\Phi(d_2)$$

where $d_1, d_2 = \dfrac{\ln\left(\frac{S}{K}\right)+(r-q\pm 1/2\sigma^2)T}{\sigma\sqrt{T}}$

i) Show that the hedge ratio $\Delta = \frac{\partial V}{\partial S}$ is given by $\Delta = e^{-qT}\Phi(d_1)$ (6)

ii) Hence find a formula for $\Gamma = \frac{\partial^2 V}{\partial S^2}$ (2)

iii) The "speed" of a derivative is defined as its third order sensitivity to movements in the price of the underlying, i.e. $\frac{\partial \Gamma}{\partial S} = \frac{\partial^3 V}{\partial S^3}$, where S is price of the underlying asset, Γ is the gamma of the option and V is the option price.

show that the "speed" of the option is given by:
$\frac{\partial \Gamma}{\partial S} = -\left(\frac{\Gamma}{S}\right)\left[1 + \frac{d1}{\sigma\sqrt{T}}\right]$ (3)

iv) Sketch graphs for this European Call option on a non dividend paying share at time T from expiry, showing how the following vary with the price of the underlying asset:

a) Price (1)

b) Delta (1)

c) Gamma (1)

d) Speed (1)
[15]

Q. 6) A non-dividend-paying stock has a current price of 200p. Over each of the next two three-month periods its price will either go up by 20p or down by 20p. Price movements for each period are independent of each other. An investment in a cash account returns 2% per quarter, with quarterly compounding. A European call option on the stock pays out in six months based on a strike price of 190p. The price of the stock is to be modelled using a binomial tree approach with three-month time steps.

i) Calculate the value of the call option today using a risk-neutral pricing approach. (3)

Assume that the real world probability of the stock price moving up in each of the next three month periods is 0.8

ii) **a)** Calculate the values of the state price deflator after six months. (2)

b) Calculate the value of the call option today using your answers to **part (ii)(a).** (2)

c) Compare this to your answer to **part (i)**. (1)

Assume that the real world probability has now dropped from 0.8 to 0.7

iii) **a)** Explain, without performing any further calculations, how the state price deflator would change in value. (1)

b) Comment on the impact that this would have on the option price. (1)
[**10**]

Q. 7) **i)** Define the following terms:

a) previsible process (1)

b) self financing (1)

c) replicating portfolio (1)

d) complete market (1)

Let S_t denotes the price of a non- dividend paying equity and B_t denotes the accumulated value of cash at time t of an initial investment of one unit of cash. S_t and B_t follow these processes as given below:

$dS_t = \mu\, S_t dt + \sigma S_t dwt$

and $dB_t = rB_t dt$

where w_t is a standard Brownian motion, and μ, σ and r are constants.

ii) Find the value of μ that makes $\frac{B_t}{S_t}$ a martingale under a suitable risk neutral probability measure. (3)

iii) Further, consider a portfolio consisting of an amount φ_t of the equity and ψ_t of the cash, such that $\varphi t = St$. Initially, the portfolio is entirely invested in the equity.

Derive a formula for ψ_t such that the portfolio is self-financing. (4)
[11]

Q. 8) i) Describe the features of Hull-White (HW) model and compare it with Vasicek model. (3)

ii) Following dynamics have been provided for short rate model of interest rate:

Vasicek: $dr(t) = (\theta - ar(t))dt + \sigma\, d\hat{W}(t)$

Hull-White: $dr(t) = (\theta(t) - ar(t))dt + \sigma\, d\hat{W}(t)$

For HW model calculate the expression for $d(e^{at}r(t))$ (2)

iii) Solve the SDE for r(t) under HW model (3)
[8]

Q. 9) i) Define Credit event. (2)

ii) You have been assigned the portfolio of managing credit risk by the Chief Risk Officer (CRO) of your company.

Your subordinate approaches you to peer review his work in credit risk modelling. A two – state Markov- chain (two states being survival and default) has been used to model the credit risk of a company's zero- coupon bonds. Following assumptions have been used in his analysis:

- Bond recovery rate: 65%
- Current value of a 6 months zero- coupon bond with face value of Rs 100 is Rs 88.50
- The annual continuously compounded risk- free rate is 2%

Calculate the implied risk- neutral default rate based on the assumptions above. (3)

iii) You want to model the term structure of default probability. Your CRO has suggested to use a Merton- style approach with the following assumptions:

- The current Market value of company's asset is Rs 100 Lakhs.
- The company's default point is Rs 75 Lakhs.
- The asset volatility is 30%
- The annual continuously compounded risk- free rate is 2%
- The annual continuously compounded expected return of the company's assets is 7%

Calculate:

a) The real cumulative default probability from time 0 to time 2 (2)

b) The risk- neutral cumulative default probability from time 0 to time 2 (2)
[9]

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