INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st December 2018

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Attempt all questions, beginning your answer to each question on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer booklet and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) **i)** Define the following types of Stochastic Process:

a) Compound Poisson Process (1.5)
b) Simple Random Walk (1.5)

ii) For each of the processes in **(i)**, state whether it operates in continuous or discrete time and whether it has a continuous or discrete state space. (2)

iii) For each of the processes in **(i)**, describe one practical situation in which an actuary could use such process to model a real world phenomenon. (2)

iv) Define white noise and show that a white noise process with a discrete state space does not have independent increments, but is a Markov process. (4)

[11]

Q. 2) Let N1(t) and N2(t) be two independent Poisson processes with rates α1=1 and α2=2, respectively. Let N(t) be a merged process with N(t) = N1(t) + N2(t).

i) Find the probability that N(1)=2 and N(2)=5 (2)

ii) Given that N(1)=2, find the probability that N1(1)=1 (3)

[5]

Q. 3) **i)** Define a Jump chain process. (1)

ii) How does Jump chain differ from Standard Markov Chain? (1)

iii) Give an example of Jump chain process. (1)

iv) A fair coin is tossed repeatedly and independently. Find the expected number of tosses till the pattern HTH appears. (6)

[9]

Q. 4) **i)** Show that Markov Chain with Transition matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0 & 1 \end{bmatrix}$$

has more than one stationary distributions. (4)

ii) Differentiate between time homogeneous and time inhomogeneous Markov jump process, with example. (2)

[6]

Q. 5) **i)** State the age ranges over which Gompertz Law is an appropriate model for human mortality. (1)

ii) Show that, under Gompertz Law, the probability of survival from age x to age x + t is equal to:

$${}_{t}P_{x} = (g)^{C\text{^}x\,(C\text{^}t\,-1)}$$

Where g is defined as log g = -B/log c (3)

[4]

Q. 6) People in the country of Indania use ticket vending machine to purchase ticket during their journey. The machine has a tendency to breakdown, at which point it must be repaired. The time until breakdown and time required to repair both follow exponential distribution.

Let P1i (t), i = 0, 1, be the probability that at time t (t >0) there are i ticket machines in working condition, given that it is working at time t = 0.

i) Derive the Kolmogorov forward differential equations for P_{1i} (t), i = 0,1 in terms of:

- μ, where 1/μ is the meantime to breakdown for a machine; and
- α, where 1/α is the meantime to repair a machine (3)

ii) Show that $P_{10}(t) = (\mu/(\mu+\alpha))\,(1 - e^{-(\mu+\alpha)t})$ and deduce the value of $P_{11}(t)$ (4)

[7]

Q. 7) The likelihood function is given by $L = Ce^{\beta} / ((e^{\beta} + 2)\,(e^{\beta} + 3))$ where C is constant. Calculate the maximum partial likelihood estimate of β. **[4]**

Q. 8) **i)** In a certain population, qx = 0.3, calculate the value of mx (weighted average of force of mortality) assuming:

a) that deaths are uniformly distributed between the ages of x and x+1

b) a constant force of mortality between ages of x and x+1 (2)

ii) Comment on the above result by considering the force of mortality over the year of age x to x+1 in each case. (2)

[4]

Q. 9) **i)** List the conventions of the Kaplan-Meier estimator. (3)

A study of the mortality of 12 laboratory-bred insects was undertaken. The insects were observed from birth until either they died or the period of study ended, at which point those insects still alive were treated as censored.

The following table shows the Product-Limit estimate of the survival function, based on data from the 12 insects.

t (weeks) S(t)

0 *t* < 1 1.0000

1 *t* < 3 0.9167

3 *t* < 6 0.7130

6 *t* 0.4278

ii) Calculate the number of insects dying at durations 3 and 6 weeks. (6)

iii) Calculate the number of insects whose history was censored. (1)

[10]

Q. 10) A motor insurer's No Claims Discount system uses the following levels of discount {0%, 25%, 40%, 50%}. Following a claim free year a policyholder moves up one discount level (or remains on 50% discount). If the policyholder makes one (or more) claims in a year they move down one level (or remain at 0% discount). The insurer estimates that the probability of making at least one claim in a year is 0.1 if the policyholder made no claims the previous year, and 0.25 if they made a claim the previous year.
New policyholders should be ignored.

i) Explain why the system with state space {0%, 25%, 40%, 50%} does not form a Markov chain. (2)

ii) a) Show how a Markov chain can be constructed by the introduction of additional states. (3)
b) Write down the transition matrix for this expanded system and draw its transition diagram. (2)

iii) Comment on the appropriateness of the current No Claims Discount system. (2)
[9]

Q. 11) A school provides children with musical instrument classes. The authority is concerned about the number of children giving up playing their instrument and is testing a new schooling method with a proportion of the children which it hopes will improve persistency rates. Data have been collected and a Cox proportional hazards model has been fitted for the hazard of giving up playing the instrument. Symmetric 95% confidence intervals (based upon standard errors) for the regression parameters are shown below.

	Covariate	Confidence Interval
Instrument		
	Piano	0
	Violin	[-0.05, 0.19]
	Trumpet	[0.07, 0.21]
Schooling method		
	Traditional	0
	New	[-0.15, 0.05]
Sex		
	Male	[-0.08, 0.12]
	Female	0

i) State the regression parameters for the fitted model. (2)

ii) Calculate, using the results from the model, the probability that a boy will still be playing the piano after 4 years if provided with the new tuition method, given that the probability that a girl will still be playing the trumpet after 4 years following the traditional method is 0.7. (5)

iii) Comment on the utility of the Cox model in general practice. (2)

iv) List one disadvantage of this model. (1)
[10]

Q. 12) **i)** Complete the table below:

Definition of x	Rate interval	$\hat{q}$ estimates	$\hat{\mu}$ estimates
Age last birthday			
Age nearest birthday			
Age next birthday			

(3)

In a mortality investigation the total number of deaths at age x during the period of investigation is θ_x. Age x is defined as:

x = [age next birthday at the start of the policy] + [curtate duration at the date of death]

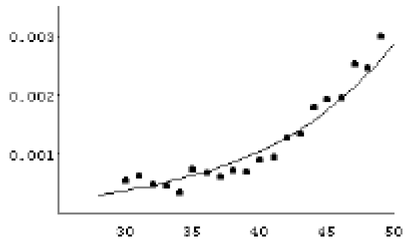
ii) State the rate interval implied by this definition. (1)

iii) Give the age, $x + f$, for which the estimate $\left(\hat{\mu}_{(x)} = \frac{\theta_x}{E_x^c}\right)$ estimates μ_{x+f}. State and explain any assumptions you make in determining f. (2)

[6]

Q. 13) **i)** State the 3 aims of graduation. (2)

ii) Define the type of graduation in the figure below:



(1)

iii) How do you test smoothness while performing graduation? (3)

iv) State the disadvantages of the Chi-Square test as a test of a mortality experience. (3)

v) List one test that does not require making any assumption while testing for mortality experience. (1)

[10]

Q. 14) In a portfolio of 1,000 insurance policies, 900 individuals have one policy, 35 individuals have two policies and 10 individuals have three policies. Calculate the factor by which the variance of the number of claims must be increased compared with the situation in which the 1,000 policies are all held by different individuals. You can make the following assumptions:

- Number of claims is distributed as a binomial random variable
- Independence of deaths

[5]

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