

# **Institute of Actuaries of India**

## **Subject ST8 – General Insurance: Pricing**

### **September 2016 Examination**

## **INDICATIVE SOLUTIONS**

#### **Introduction**

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

**Solution 1:**

$$i) \quad E[X] = \int_0^{50} x * f(x) dx = 0.008 \int_0^{50} (50x - x^2) dx$$

$$= 0.008 (25x^2 - x^3 / 3) \Big|_0^{50}$$

$$= 16.67$$

$$E[X^2] = \int_0^{50} x^2 * f(x) dx = 0.008 \int_0^{50} (50x^2 - x^3) dx$$

$$= 416.67$$

$$1 + CV^2 = E[X^2] / E^2[X] = 416.67 / 16.67^2 = 1.5$$

The standard for Full Credibility for the pure premiums for  $k=2.5\%$  is, therefore,  $n_s = n_N (1 + CV_X^2) = 5,000 * 1.5 = 7,500$ .

For  $k=9\%$  we need to multiply by  $(2.5/9)^2$  since the full credibility standard is inversely proportional to  $k^2$ .  $7,500(2.5/9)^2 = 579$ .

**[4]**

ii)

If we limit the size of claims, then the coefficient of variation is smaller. Therefore, the criterion for full credibility for basic limits losses is less than that for total losses. (1 mark) If claims amounts are limited then there will be more certainty attached to their values. Also, by capping large losses, the claim amounts below the cap limit can be grouped with the attritional (*i.e.* small) claims and modelled together.

Claim distributions are typically skewed towards right. When the claim amount is capped, it leads to a reduction in both mean and standard deviation. But the proportion of decrease in standard variation is greater than the proportion of decrease in mean, thereby reducing the coefficient of variation.

**[4]****[8 Marks]****Solution 2:**

i) From tables, pdf of geometric dist:  $p(N=n) = p * q^n$  (put  $k=1$  in type 2 negative binomial)

Using recursive formula:  $pr = (a+b/r)p(r-1)$  we get  $pr = (1-p) * p(r-1)$

$$\Rightarrow A=1-p \text{ and } b=0$$

For  $S=0$ , use  $p(N=0) = p$

Substituting the value of  $a$  and  $b$  in Panjer recursive formula we get the desired result.

**[3]**

ii) Ideally, with unlimited computing capacity, one would like to keep the intervals of discrete distribution as small as possible to have higher accuracy. E.g. 1 paise each. But, one needs to also look at the available computational resources and time required in performing the analysis.

Having fewer numbers of strips would reduce the required time and computational resources. At the optimal number of strips, adding more number of strips would not produce significant gains in the accuracy.

Thus, number of discrete intervals should be based on balancing the purpose of work, accuracy of results required, time constraint and computational resources.

[2]

$$\text{iii) } E(X) = \lambda / (\alpha - 1) = 15 / 3 = 5$$

$$\text{Var}(X) = (\alpha \lambda^2) / ((\alpha - 1)^2 * (\alpha - 2)) = 50$$

$$E(N) = q/p = .99 / .01 = 99$$

$$\text{Var}(N) = .99 / (.01)^2 = 9900$$

$$E(S) = 99 * 5 = 495$$

$$\text{Var}(S) = E(N) * \text{Var}(X) + E(X)^2 * \text{Var}(N) = 252450$$

$$(S - E(S)) / \text{Sqrt}(\text{Var}(S)) \sim N(0, 1)$$

$$(S - 495) / (502.44) \sim N(0, 1)$$

$$P(S > 1500) = P((S - 495) / 502.44 > (1500 - 495) / 502.44) = P(N(0, 1) > 2) = (1 - .97725) = 0.02275$$

[4]

[9 Marks]

### Solution 3:

i) Inflation adjusted limits:  $240 / (1 + 20\%)$  and  $300 / (1 + 20\%) = 200$  and  $250$

$$\text{Loss Cost} = 50 * 1.2 * (2.19 - 1.8) = 23.4$$

[2]

ii) Providing (re)insurance at higher layers results in lesser diversification and greater uncertainty. To compensate for the greater expected variation, risk load is generally applied.

Other components include expense loading for ALAE and ULAE.

[2]

iii) The portfolio on which market curves are based could be very different from the portfolio to be covered under the discussed reinsurance contract. Eg. Nature of PI covered (doctor, senior mgmt., etc), possibility of deductible, policy terms etc.

Need to check the historical years considered for the market curves and if they are outdated. If not, the curves need to be adjusted for inflation, frequency trends, etc.

Changes in risk environment etc. would also reduce the appropriateness of the ILFs. For example, in Bodily Injury component of Motor Liability, the introduction of air bags led to reducing or eliminating several BI claims.

Modeled Loss cost can be extremely sensitive to the selected ILF curve, especially for higher layers. Judgment plays a vital role and hence total reliance can't be placed on the selected ILF curve.

Clarity needs to be present in terms of items considered in the market curves: Risk load/ALAE/ULAE etc. If these are not allowed in the curve, they need to be explicitly added. If these are already present, the loading needs to be verified.

Also, current portfolio might have inuring reinsurance/stacked limit as other reinsurance may inure to the benefit of contract which would impact the ILF curve.

[5]

[9 Marks]

#### Solution 4:

i)

	Severity as a % of full Value (end of year)-A	Freq (during the year)-B	Average Severity as a % of full value (During the year)-C	Risk Premium as a % of full value (during the year)-D	Risk Premium during the Year (in INR Crores)
Year 1	15%	5%	7.5%	0.3750%	3.75
Year 2	60%	3%	37.5%	1.1250%	11.25
Year 3	100%	1%	80%	0.8000%	80.00

Let P denote the upfront premium to be charged.

Present Value (P.V) of Premium = P

(no discounting since premium is collected upfront for all three years)

P.V of Acquisition Cost =  $0.1 * P$

(no discounting since this expense is incurred upfront)

P.V. of Expected Claims (assuming that the expected claims cost for each year could be attributed to the middle point of that year)

$$= 3.75 * \{(1+0.05)^{-0.5}\} + 11.25 * \{(1+0.05)^{-1.5}\} + 80.00 * \{(1+0.05)^{-2.5}\}$$

$$= 3.66 + 10.46 + 7.08 = 21.20$$

P.V. of General administration Expenses (assuming that the general administration expenses for each year is equal and that they could be attributed to the middle point of that year)

$$= 0.033P * \{(1+0.05)^{-0.5}\} + 0.033P * \{(1+0.05)^{-1.5}\} + 0.033P * \{(1+0.05)^{-2.5}\}$$

$$= 0.092 * P$$

P.V. of Income = P.V of Outgo, implies that:

$$P = 0.1P + 21.20 + 0.092P$$

$$0.808P = 21.20$$

$$P = 26.24 \text{ INR Crores}$$

[5]

ii) All risks, big or small, are ceded in same proportion in quota share. Diversification is possible as with same available capital insurer can write more number of risks. Thus, spread of risk helps to improve solvency to an extent. But for engineering line of business, large claims are more likely to impact the solvency of insurer. These are not avoided completely by QS (only part of the claim is shared).

XoL usually provides better protection from large claims (depending on limits) as claims above the attachment point of the treaty are paid by the reinsurer. For the reinsurer, it leads to unpredictability and potentially high losses.

For each unit of losses net of expected RI recovery, the amount of uncertainty is lower after an XoL treaty when compared to QS treaty. Therefore, XoL treaty is expected to provide greater protection from insolvency for the insurer.

Since the unpredictability for each unit of expected RI recovery is greater for the reinsurer in an XoL treaty compared to a QS treaty, the Reinsurer would likely charge more risk loading or profit margin per unit of expected RI recovery. Therefore, expected profits for the insurer would be greater under QS treaty when compared to XoL treaty.

[4]

[9 Marks]

**Solution 5:**

i) Cyber risk insurance may include one or more of the following types of coverage:

- Liability for security or privacy breaches. This would include loss of confidential information by allowing, or failing to prevent, unauthorized access to computer systems.
- The costs associated with a privacy breach, such as consumer notification, customer support and costs of providing credit monitoring services to affected consumers.
- The costs associated with restoring, updating or replacing business assets stored electronically.
- Business interruption and extra expense related to a security or privacy breach.
- Liability associated with libel, slander, copyright infringement, product disparagement or reputational damage to others when the allegations involve a business website, social media or print media.
- Expenses related to cyber extortion or cyber terrorism.

[3]

ii) Typically, an underwriting analysis will include a review of the following:

- General risk exposure of the industry and business activities,
- General risk exposure of the size of the company,
- Loss History,
- Years in business,
- Financial condition
- Extent of use of outsourced network security services
- Dependency on third parties networks
- In depth analysis of network security pursuant to various international standards such as ISO

[4]

iii) The following exclusions can be considered:

- Bodily injury or property damage
- Loss or destruction of tangible property, other than Third Party Data
- Any contractual liability accepted by the Insured
- Any act, error or omission by the Insured which are of criminal, dishonest or fraudulent nature
- Any willful disregard or non-compliance with a ruling
- Any infringement of patents / trade secrets due to unauthorized disclosures
- Any form of war, terrorism or riot
- The unlawful or unauthorized collection of Third Party Data by the Insured

[3]

[10 Marks]

**Solution 6:**

i) ALAE are claims handling expenses that are directly attributable to a particular claim.

ULAE are claims handling expenses that are not directly attributable to a particular claim. They may have resulted from an array of claims or from some other general process involved in handling claims.

[2]

ii) Allocated loss adjustment expenses occur when the insurance company pays for an investigator to survey claims made on a specific policy. In the case of a third-party review of any damaged vehicle, the cost associated with hiring that professional is an allocated loss adjustment expense.

Other examples can be - cost of obtaining police reports, or the cost required to evaluate whether a driver filing a claim is really injured.

Unallocated expenses could be those related to the salaries of office staffs who take down claim information and process requests.

Other unallocated expenses can involve maintenance costs of the fleet of vehicles used by in-house investigators.

[2]

iii) In case of proportional allocation of expenses, rate differentials widens between classes of insureds with, for example young drivers - generally paying higher premiums. Since expenses were proportional to premiums, these drivers need to pay a larger part of insurance expenses, although many of these expenses did not vary directly with claim amount. This can bring up the affordability problem.

A proportional expense loading that does not accurately reflect total costs may result in poor pricing strategy. The insurer using a proportional expense loading would have too high an expense load on high premium policies and too low an expense load on low premium policies. It would lose market share among the high premium policies and attract more risks among the low premium policies.

But the low expense loading on the latter policies would make the overall premium inadequate, leading to reduced profits.

[2]

iv) Alternate way could be expense flattening procedures to allocate insurance expenses to policyholder classifications. High risk insureds with large expected claim cost, do not necessarily cause higher "general administration" expenses. Flattening the expense loading reduces total premium for high loss cost policyholders.

[1]

v) In reality, many high risk policyholders have poor persistency. Administration expenses are incurred primarily in issuing the policy, not in renewing it, particularly for direct writing carriers. For example, Know Your Customer (KYC) norms might be required in future for new customers but not needed if the same customer is renewing.

Expenses incurred are a higher than average percentage of premium for many high-risk low-persistency insureds.

Usually the persistency rates for young male drivers are lower than the other drivers in personal motor insurance. The average expense incurred for insuring young male drivers is a larger percentage of premium than that incurred for insuring the average driver.

[3]

[10 Marks]

**Solution 7:**

i)

a) The insurance company needs to estimate the technical premium of the business suggested by the agent using the risk premium model company may have. This would require all details about the rating factors used in the risk premium model from the agent.

The risk premium needs to be then loaded for acquisition cost and all expenses, to arrive at the technical premium. If this amount comes to be equal or lesser than the premium suggested by the agent (excluding service tax), the portfolio is expected to be profitable at the desired level.

[2]

b) The company might not have a technical premium each vehicle model/segment or it may be outdated. E.g. for a small or new insurance company not enough claims experience would be available. It would be very hard for the company to regularly update the model.

The rating factors of the model may not be defined at granular level to access the actual profitability. E.g. Model might be at city level, while the experience may vary by pin code.

There may be certain factors, like particular location of the agent or particular vehicle segment, for which not enough credible experience may be present. E.g. A new car model.

Factors for future expenses, claims inflation, change in claim frequency need to be accounted. Actual management expenses on the agent business are very hard to estimate as usually expense allocation is done at a high level and then pro-rated.

The impact of recent/upcoming changes in legislation, regulation and market needs to be studied. E.g. how the ban on diesel cars would impact the profitability of other segments.

[3]

ii)

a) Even if the premiums brought by the agent are broadly in line with the technical premiums, there could be something inherently wrong with the business. So monitor actual claims experience to check if the loss ratios exceed the target loss ratios due to any of the following reasons:

- The agent may be bringing business in segments where the risk premiums used in the technical premiums are considerably lower than the actual expected claims.
- The business from the agent, no matter deliberate or not, may be more prone than usual to fraud or moral hazard.

[2]



b) Since it has just been a year, the earning would be limited. If the portfolio is assumed to be uniformly spread, only half of it would be earned in a year. So, the portfolio needs to be sufficiently large in size to take decision on profitability.

Claims experience tends to be volatility over the years even for a bigger portfolio. So, a year's experience may not mean that the agent is giving profitable or loss making business. There could have been some major claims (like total loss) or cat losses in the business provided by the agent which distorted the claims experience.

OD claims are generally short tailed and may not have much influence of IBNR, but TP claims are long tailed. Estimating profitability with allocated IBNR may not give the correct picture if the claims development pattern of business of agent is considerably different from the portfolio studied for IBNR. Estimating IBNR at agent's portfolio level may not be statistically significant.

The unearned business mix may be quite different from earned business mix, and so profitability may be influenced when the unearned business gets earned.

The insurer may have certain reinsurance arrangements which operate on total portfolio of the insurer like, aggregate XoL or stop loss. It would be difficult to estimate the recoveries in the business provided by the agent.

[3]

[10 Marks]

### **Solution 8:**

i)

a) Loss sensitive premiums are a form of experience rating. These are premiums that depend, at least in part, on the actual claims experience of that risk in the period covered. It is also known as swing rated premium.

[2]

b) There are two forms of swing rate:

In the first version, sometimes called "minimum plus", the final premium that the cedant pays consists of a minimum (paid upfront) plus a factor times the actual losses to the layer, subject to an overall maximum amount.

In the second version, the cedant pays an upfront premium and then the final premium is determined as a factor times the losses subject to a minimum and maximum. Usually the deposit is between the minimum and maximum.

In the second form, the cedant might end up with a refund of some premium.

[3]

ii)

a) If the reinstatement provision is pro rata as to amount, then we only need to consider the fraction of the annual premium corresponding to the proportion of the loss amount to the occurrence limit, multiplied by the percentage in the reinstatement provision.

$$\begin{aligned} \text{Reinstatement premium} &= (42,000,000/60,000,000) * 3,500,000 * 120\% \\ &= 2,940,000 \text{ (INR)} \end{aligned}$$

[2]

b) If the reinstatement provision is pro rata as to amount and to time, then a further reduction of the reinstatement premium is needed to account for the time during which the new coverage will be effective - i.e., from October 1, 2017, until the end of 2017, or  $3/12 = 1/4$  years.

$$\text{Reinstatement premium} = 2,940,000 * (1/4) = 735,000 \text{ (INR)}$$

[2]

c) Most reinstatement premiums for catastrophe excess-of-loss treaties are not pro rata as to time because many catastrophes, such as hurricanes, occur seasonally, so a pro rata approach to time does not take into account the actual exposure to risk during the remainder of the treaty period.

[2]

[11 Marks]

### **Solution 9:**

i)

a) XYZ is likely to use one of two methods to estimate the maximum exposure and, therefore, determine whether they have sufficient capacity to accept the risk. These two methods are explained below:

#### **Estimate the full value sums insured**

- A single loss can often trigger different covers, so the aggregation of the sums insured should be considered in order for the true maximum exposure to be calculated.
- The risk should not be seen simply as the total cost of rebuilding the warehouse if, for example, it were destroyed by fire.
- In the case of a fire, both the material damage costs and the business interruption costs must be considered.

#### **Estimated maximum loss (EML)**

- The second method used to measure exposure in such property insurance business is to assess the EML at this location, which is the worst possible financial impact of the maximum foreseeable loss.

[2]

b) If the sum insured year is used as exposure measure, the following complications can arise:

- The amounts of stocks held may vary considerably over the period of the insurance. Hence the stock may be covered on a declaration basis, determined retrospectively with an adjustment premium.
- There is no standard way of allowing for inflation in the policy. Hence policies with different types of inflation treatment need to be considered separately to determine the exposure.

In case of EML,

- The correct EML can be difficult to establish and would require a detailed site survey report where the surveyor must consider fire protections, construction material, contents of the warehouse, the use of the building, proximity to any other high-risk buildings, standard of risk management etc.
- An overestimate of the EML could mean the risk is not covered if it falls above XYZ's maximum capacity, or the unnecessary purchase of reinsurance. Underestimation could lead to the risk being accepted even though it is over XYZ's maximum capacity limits and potentially leading to a situation where a loss is paid for where insufficient provision has been made, resulting in poor profitability.

[3]

ii)

- In addition, there will be more competition for the better risks and xyz's underwriting policy may enable the underwriting and acceptance considerations to be more aggressive, or more relaxed in terms of cover restrictions and pricing, to ensure they are competitive.
- In a soft market, it is also more likely to have more capacity. This will lift the maximum capacity for this particular risk, meaning XYZ will have a greater appetite to write this risk and potentially other risks as it will have more capital to write business.
- Terms offered by the reinsurers in the current soft market may also encourage the underwriter to write the business in less profitable term.
- If the brand of the insurer is well known in the market, the underwriter may just be able to charge slightly more than the market premium and still able to attract the client.

[3]

iii)

### **Withdrawing from Property line of business**

Merits:

- XYZ Insurance Company may find it difficult to re-enter If the overhead expenses can also be reduced, this strategy can help in eliminates unprofitable business provided the property line is potentially unprofitable in current soft market scenario
- It can be seen as a very decisive move by shareholders and the stock market reaction can be positive

Demerits:

- Unless the reduction in Property line can be compensated by the growth in other lines, this strategy will possibly reduce overall market share which may eventually lead to lower business in other lines too
- market if desired later
- It can also potentially reduce diversification

**Continue writing same business but reduce exposure**

Merits:

- It will reduce loss in soft market without having the disadvantages associated with withdrawing from the market
- If the relationship is good with reinsurer, the company may want to reduce the line size and get more reinsurance (may be at a lower rate) – so reinsurer will share part of the problem

Demerits:

- This may not be a helpful move, if the business is still marginally profitable because the overheads are still the same
- If not analysed properly, the company may even fail to cover the fixed expenses
- The clients may not be happy and they can get better term from other insurers – business will be lost

[4]

[12 Marks]

**Solution 10:**

i)

Model 1: Excluding Sum Insured

Model 2: Original model

$$D1-D2=261981-249719=12262$$

$df1-df2$ =Number of parameters in model 2 – number of parameters in model 2

$$= 40 - (40-5+1) = 4$$

$$D2/df2=249719/(100-40)=4161.98$$

$$(D1-D2)/((df1-df2)*(D2/df2))=.7365$$

The upper 5% of  $F_{df1-df2,df2}=F_{4,60}=2.525$

Since the test statistic is lower than the F value, we do not reject Model 1 over Model 2. That is, the model excluding the Sum Insured is preferred.

[3]

ii)

	Fire		Theft		Relativity	
	Frequency	Severity	Frequency	Severity	Fire Risk Premium	Theft Risk Premium
Base values	1%	200,000	3%	15,000		
Rating factors & Relativities						
<b>Age of building</b>						
<5	1	1	1	1	1	1
>=5	1.1	0.9	1.3	1.2	0.99	1.56
<b>Sum Insured</b>						
Low	1	1	1	1	1	1
Medium	0.8	1.6	0.9	1.2	1.28	1.08
High	0.7	2	0.8	1.5	1.4	1.2
<b>Sprinklers Installed</b>						
Yes		0.8			0.8	
No		1			1	

Earning will happen over 2017 and 2018. Average accident date can be assumed at end of 2017. Current data is middle of 2015. So, inflation and frequency trend is assumed for 2.5 years.

$$\text{Inflation factor-Claims (A)} = (1+8\%)^{2.5}$$

$$\text{Frequency Factor (B)} = (1-2\%)^{2.5}$$

$$\text{Fire Risk Premium} = \text{Base Fire Freq} * \text{Base Fire Severity} * A * B * (1+\text{IBNR Load})=2535.40$$

$$\text{Theft Risk Premium} = \text{Base Theft Freq} * \text{Base Theft Severity} * A * B * (1+\text{IBNR Load})=570.47$$

[4]

iii)

Individual trends are easy to predict, validate and explain as compare to the compounded trend of frequency and severity combined. In interpreting trends of rating factors in total claims cost model, it is usually difficult to get into core reason.

Total claims cost model would be a slightly simplified approach in terms of models (one model vs two models). Also, additional step of model combining is reduced. Thus, modeling time is expected to be less in total claims cost model.

Having said the above, one might end up spending more time building the total claims cost model as the trends would be complex (mix of frequency and severity) and hence difficult to comprehend and simplify. Modeling severity is less time consuming.

No correlation is assumed when Frequency and severity models are combined and both models are assumed to be independent, but in burning cost implicit correlation is assumed.

[3]

iv)

- Selecting a dataset that most accurately reflects the likely future mix of business
- Calculate an expected claims cost by claim type for each record in the data
- Combine these fitted values, for each record, to derive the expected cost of claims (according to the individual GLMs) for each record
- Fit a further GLM to this total expected cost of claims, with this final GLM containing the union of all factors (and interactions) in all of the underlying models.

[2]

[12 Marks]

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