Institute of Actuaries of India

Subject CT6 – Statistical Methods

September 2016 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) We know that loss amount for the individual claim distribution follows an exponential distribution with mean and s.d 1000. Thus $\lambda = 1/1000$

The expected payment per claim made by Expert Re is

$= \int_{5000}^{\infty} (x - 5000)\lambda e^{-\lambda x}\, dx$

On Substituting (x-5000) = y we get,

$= \int_0^{\infty} y\lambda e^{-\lambda(y+5000)}\, dy = e^{-5000\lambda} * \int_0^{\infty} y\lambda e^{-\lambda y}\, dy$

$= e^{-5000\lambda}$ * (Expected Value of Exponential Distribution with parameter λ)

$= e^{-5000\lambda} * 1/\lambda$

$= 1000 * e^{-5000\lambda} = 1000 * e^{-5}$

= 6.73

Given that the total number of expected claims are 200, the expected claim cost over the portfolio of 1000 policies for Expert Re is 200 * 6.73 = 1346

Thus expected total claim amount ceded over the entire portfolio to Expert Re is 1346.

[5]

ii) The annual premium quoted by Expert Re is 1600. The expected claim cost to be ceded to Expert re is 1346. Thus, the premium exceeds the expected claim cost by 18.87%.

Since the premium is higher than the claim cost ABC may not want to reinsure

However the option of reinsuring or not would depend on lot of other factors such as:

- ABC's risk appetite and its need for reinsurance.
- Premium quoted by other reinsurers in the market.

[2]

[7 Marks]

Solution 2:

i) The pdf of the lognormal distribution is $f(x) = \frac{1}{x\sqrt{2\pi}\sigma} * e^{-\{(\log x - \mu)^2/2\sigma^2\}}$, x >0

Hence the log likelihood function is

$\ln L = -1/2 * \sum_{i=1}^{12} (\ln xi - \mu)^2 / \sigma^2 - 12 \ln \sigma - 12 \ln\sqrt{2\pi} - \sum_{i=1}^{12} \ln xi$

Differentiating w.r.t μ and equating it to zero we get,

$\partial l/\partial \mu = \sum_{i=1}^{12} (\ln xi - \mu)/ \quad \sigma^2 = 0$

μ = 1/12 $\sum_{i=1}^{12} \ln xi$ .... (a)

Differentiating w.r.t σ and equating it to zero we get,

$\partial l/\partial\ \sigma$ =1/σ* $\sum_{i=1}^{12}(\ln xi - \mu)^2\ /\sigma^2 - 12$ / σ = 0

- ⇨ $\sum_{i=1}^{12}(\ln xi - \mu)^2 / \sigma^2 = 12$
- ⇨ $\sum_{i=1}^{12}(\ln xi - \mu)^2 / 12 = \sigma^2$ .... (b)

Using equation (a) in (b) we get

σ^2 =1/12*($\sum_{i=1}^{12} \ln xi^2 + 12\ \mu^2 - 24\mu^2$)

- ⇨ σ^2 =1/12*($\sum_{i=1}^{12} \ln xi^2$) $- \mu^2$

We have, $\sum_{i=1}^{12} \ln xi^2$ = 607.6138 and $\sum_{i=1}^{12} \ln xi$ = 84.5174

Thus μ = 1/12 $\sum_{i=1}^{12} \ln xi$ = 7.0431

And σ^2 = 1/12*($\sum_{i=1}^{12} \ln xi^2$) $- \mu^2$ = 1.0289

[6]

ii)

The 75th percentile is the average of the 9th and 10th observations = ½*(2000+2500) = 2250

The 25th percentile is the average of the 3rd and 4th observations = ½*(790+825) = 807.5

Using the distribution function of the Weibull distribution we have,

For the 25th percentile, 1 – exp (-c*807.5^{γ}) = 0.25

For the 75th percentile, 1 – exp (-c*2250^{γ}) = 0.75

Thus we have,

0.75 = exp (-c*807.5^{γ}) => ln .75 = (-c*807.5^{γ}) .... (a)

0.25 = exp (-c*2250^{γ}) => ln .25 = (-c*2250^{γ}) .... (b)

Dividing a by b we get

- ⇨ .207519 = (807.5^{γ})/ (2250^{γ}) = $.358889^{\gamma}$
- ⇨ γ = 1.5345

Therefore c = ln .75 / 807.5^{γ} = 0.00001

[5]

iii) Under (i) log X follows a normal distribution with mean.

Therefore, P(X>2500) = 1 – Φ ((ln 2500 – 7.0431)/1.01439) = 1 – Φ (0.77) = 1 – 0.7793

= 22.07%

Under (ii) X follows a Weibull distribution, thus P (X>2500) = exp (-c*2500^{γ})

= exp (-0.00001*2500$^{1.5345}$)

= 19.45%

[3]
[14 Marks]

Solution 3:

i) The cumulative claim amount table is given below

Cumulative Claim Amount

Accident Year	0	1	2	3
2012	3000	4850	5570	5920
2013	3750	5950	6875	
2014	4915	7837		
2015	6200			

The cumulative claim number table is

Claim Number

Accident Year	0	1	2	3
2012	170	250	265	290
2013	195	261	292	
2014	249	310		
2015	272			

The cumulative average costs per claim along with the grossing up factors are

Cumulative average cost per claim

Accident Year	0	1	2	3	Ultimate average cost per claim
2012	17.65	19.40	21.02	20.41	20.4138
Grossing up factor	0.8645	0.9503	1.0296	1.0000	
2013	19.23	22.80	23.54		**22.8667**
Grossing up factor	0.8410	0.9969	1.0296		
2014	19.74	25.28			**25.9650**
Grossing up factor	0.7602	0.9736			
2015	22.79				**27.7337**
Grossing up factor	0.8219				

Similarly the claim numbers along with the grossing up factors are

Claim Number

Accident Year	0	1	2	3	Ultimate claim number
2012	170.00	250.00	265.00	290.00	290.0000
Grossing up factor	0.5862	0.8621	0.9138	1.0000	
2013	195.00	261.00	292.00		**319.5472**
Grossing up factor	0.6102	0.8168	0.9138		
2014	249.00	310.00			**369.3005**
Grossing up factor	0.6742	0.8394			
2015	272.00				**436.2019**
Grossing up factor	0.6236				

The total projected loss estimate =

AY	Ultimate cost per claim (1)	Ultimate claim number (2)	Projected Loss estimate (1)*(2)
2012	20.4138	290.0000	5920.0000
2013	22.8667	319.5472	7307.0018
2014	25.9650	369.3005	9588.8923
2015	27.7337	436.2019	12097.5098

Since claims amounting to 15000 have been paid till date the reserves to be kept aside are

(5920 + 7307.0018 + 9588.8923 + 12097.5098) – (15000)

= 19913.4038 **[8]**

ii) The assumptions that underlie our model are

- For each accident year number of claims in each development year is a constant proportion of the total number of claims arising from that accident year
- For each accident year the average claim amount in each development year is a constant proportion of the ultimate average claim amount arising from that accident year.

[1]
[9 Marks]

Solution 4:

i) The likelihood function is

$f(x|\beta) = \prod_{i=1}^{n} \beta e^{-\beta x}$

$\propto \beta^n * e^{-\beta \sum_{i=1}^{n} xi}$

Gamma distribution as a function of β is

$f(\beta) = \lambda^{\alpha}.\beta^{\alpha-1}e^{-\beta\lambda} / \Gamma(\alpha)$

which is a conjugate prior distribution for β.

[2]

ii) Mean and variance of gamma distribution with parameters α and λ are α/λ and α/λ^2 respectively.

Thus we have,

α/λ = 0.450 ……………………..(a)

α/λ^2 = 0.1030 …………………… (b)

Dividing (a) by (b) we get

λ = .450/.1030 = 4.3671

and hence α = 0.450*4.3671 = 1.9652

Now we know that posterior = prior * likelihood

Thus, posterior distribution $\propto \beta^{\alpha-1}e^{-\beta\lambda} * \beta^n * e^{-\beta\sum_{i=1}^{n} xi}$

$$\propto \beta^{\alpha+n-1}\, e^{-\beta(\lambda+\sum_{i=1}^{n} xi)}$$

Which is proportional to the pdf of a gamma distribution with parameters (α+n, $\lambda + \sum_{i=1}^{n} xi$)

The Bayesian estimator of β under quadratic loss function is the mean of the posterior distribution.

Thus Bayesian estimator of β = (α+n) /($\lambda + \sum_{i=1}^{n} xi$) = 26.96/63.67 = 0.4235

[5]

iii) Using the complete data n = 100 and $\sum_{i=1}^{n} xi$ = 2.272*100 = 227.2

Thus Bayesian Estimate under quadratic loss = mean of posterior = (α+n) /($\lambda + \sum_{i=1}^{n} xi$) = 101.96/231.56=0.4403

[2]

[9 Marks]

<u>Solution 5:</u>

i) λ/(α-1) = 10000000 and $\alpha\lambda^2/\{(\alpha-2)(\alpha-1)^2\} = 15000000^2$

Substituting the first equation in the second and solving for α we get,

1.25α = 4.5

=> α = 3.6 and thus λ = 26000000

The expected amount paid by the reinsurer on a single calamity is

$\int_{30000000}^{\infty}(x - 30000000)f(x)dx$

= $\int_{30000000}^{\infty} x.\alpha\lambda^{\alpha}/(\lambda + x)^{\alpha+1}dx$ – 3*10^7 * P(X>30000000)

= 3*10^7 * $(\frac{26000000}{56000000})^{3.6}$ + $26000000^{3.6}/2.6$ * $1/(56000000)^{(3.6-1)}$ -3*10^7* $(\frac{26000000}{56000000})^{3.6}$

= 1360322.353

Expected amount payable by the insurer without reinsurance = 10000000

Thus expected amount payable by the insurer on a single calamity after reinsurance = 10000000 – 1360322.353 = 8639677.647 **[6]**

ii) Premium charged by the reinsurer under R1 = 1360322.353 * 1.10 = 1496354.588
Premium charged by the reinsurer under R2 = 2 * premium under R1
= 2*1496354.588 = 2992709.177

Insurer's net cost under any scenario in the decision matrix is equal to

Insurer's net cost = gross claim amount ***less*** claim paid by reinsurer(if any) ***less*** premium received by insurer ***plus*** premium payable to the reinsurer (if any)

Thus we have

	A_0 (no calamity)	A_1 (1 calamity)	A_2 (2 calamity)	A_3 (3 calamity)
No reinsurance	= 0	=10000000	=20000000	=30000000
R1	=1496354.588	=8639677.647 +1496354.588	=8639677.647 +10000000+1496354.588	=8639677.647+20000000+1496354.588
R2	=2992709.177	=8639677.647 +2992709.177	=8639677.647*2 +2992709.177	=8639677.647*2+10000000+2992709.177

Therefore the decision matrix of the insurer's net cost is:

	A_0 (no calamity)	A_1 (1 calamity)	A_2 (2 calamity)	A_3 (3 calamity)
No reinsurance	0	10000000	20000000	30000000
R1	1496354.588	10136032.24	20136032.24	30136032.24
R2	2992709.177	11632386.82	20272064.47	30272064.47

[6]
[12 Marks]

Solution 6:

i) The linear predictor is given by:

$\eta = \alpha + \beta x$

The value η may not necessarily take a value in the internal (0, 1) and it might be any value.

We can use the below link function

$$g(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$$

And setting this equal to the linear predictor η , we get:

$$g(\mu) = \eta = \log\left(\frac{\mu}{1-\mu}\right)$$

$$\therefore \mu = \frac{e^{\eta}}{1+e^{\eta}} = \left(1+e^{-\eta}\right)^{-1}$$

Therefore, the above μ will be in the range (0, 1) and so can be used for probability of claim.

Now, we can use maximum likelihood estimator to arrive at the parameters.

We have to calculate likelihood function for Binomial (n,μ)

Now, $f_y(y;\theta,\phi) = \exp\left[n\left(y\log\left(\frac{\mu}{1-\mu}\right) + \log(1-\mu)\right) + \log\binom{n}{ny}\right]$

Likelihood function $f(\alpha,\beta) = \prod_{i=1}^{n} \exp\left[n\left(y_i \log\left(\frac{\mu_i}{1-\mu_i}\right) + \log(1-\mu_i)\right) + \log\binom{n}{ny_i}\right]$

Taking log we get; $\ln f(\alpha,\beta) = \sum_{i=1}^{n}\left[n\left(y_i \log\left(\frac{\mu_i}{1-\mu_i}\right) + \log(1-\mu_i)\right) + \log\binom{n}{ny_i}\right]$

Now, $g(\mu_i) = \log\left(\frac{\mu_i}{1-\mu_i}\right) = \eta_i = \alpha_i + \beta x_i$

Now, $1-\mu_i = \frac{1}{1+e^{\eta_i}} = \left(1+e^{\eta_i}\right)^{-1}$

Or, $\ln f(\alpha,\beta) = \sum_{i=1}^{n}\left[n\left(y_i\eta_i - \log\left(1+e^{\eta_i}\right)\right) + \log\binom{n}{ny_i}\right]$

Or, $\ln f(\alpha,\beta) = \sum_{i=1}^{n}\left[n\left(y_i(\alpha+\beta x_i) - \log\left(1+\exp(\alpha+\beta x_i)\right)\right) + \log\binom{n}{ny_i}\right]$

Now, taking partial derivation with respect to α and β and equating with zero we get:

$$\frac{\partial}{\partial\alpha}\ln f(\alpha,\beta)=\sum_{i=1}^{n}\left[y_i-\frac{1*\exp(\alpha+\beta x_i)}{1+\exp(\alpha+\beta x_i)}\right]=0$$

$$\frac{\partial}{\partial\beta}l_n f(\alpha,\beta)=\sum_{i=1}^{n}\left[y_i x_i-\frac{1*\exp(\alpha+\beta x_i)x_i}{1+\exp(\alpha+\beta x_i)}\right]=0$$

Now putting values of response (y – probability of claim) and covariate (x – age of voyage) variables we get,

$$\frac{e^{\alpha+4\beta}}{1+e^{\alpha+4\beta}}+\frac{e^{\alpha+8\beta}}{1+e^{\alpha+8\beta}}=0.65$$

$$4*\frac{e^{\alpha+4\beta}}{1+e^{\alpha+4\beta}}+8*\frac{e^{\alpha+8\beta}}{1+e^{\alpha+8\beta}}=4.2$$

Now, taking $e^{-(\alpha+4\beta)}=a$ and $e^{-4\beta}=b$

Now we get, $\frac{1}{1+a}+\frac{1}{1+ab}=0.65$ ----(1)

And $\frac{4}{1+a}+\frac{8}{1+ab}=4.2$ ------------(2)

From equation (1), we get $\frac{1}{1+a}=\left(0.65-\frac{1}{1+a}\right)$

And from equation (2), $\frac{4}{1+a}+8*\left(0.65-\frac{1}{1+a}\right)=4.2$

Or, $8*0.65-\frac{4}{1+a}=4.2$ or, a = 3

Now, $\frac{1}{1+3}+\frac{1}{1+3b}=0.65$ Or, b=0.5

Now, $e^{-4\beta}=b$, therefore β=0.17329

And $e^{-(\alpha+4\beta)}=a$, therefore α=-1.791759

$\therefore \eta_i=-1.791759+0.17329x_i$

[9]

ii) The minimum and maximum probability can be attained at age zero and max age at 10.

Now at age zero, $\eta_0=-1.79176$

And, $\mu_0 = \left(1+e^{-\eta_0}\right)^{-1} = 14.29\%$

Similarly at age 10, $\eta_{10} = -0.05889$

And hence, $\mu_{10} = \left(1+e^{-\eta_{10}}\right)^{-1} = 48.53\%$

Hence the minimum and maximum probability which can be attained by the above GLM model is given by 14.29% and 48.53% respectively.

[2]
[11 Marks]

Solution 7:

i) p=1 , q =4 hence it will follow ARMA(1,4)

[1]

ii) Non-linear non stationary time series models includes:-

Bilinear models are those that exhibit "bursty" behaviour:
$X_n - \alpha(X_{n-1}-\mu) = \mu + e_n + \beta e_{n-1} + b(X_{n-1}-\mu)e_{n-1}$

Threshold autoregressive models are used to model "cyclical" behaviour:
$$X_n = \mu + \begin{cases} \alpha_1(X_{n-1}-\mu) + e_{n,} if X_{n-1} \le d, \\ \alpha_2(X_{n-1}-\mu) + e_{n,} if X_{n-1} > d, \end{cases}$$

Random coefficient, autoregressive models is a sequence of independent random variables:
$X_t = \mu + \alpha_t(X_{t-1}-\mu) + e_t, where\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is a sequence of independent random variables

Autoregressive with conditional heteroscedasticity (ARCH) models are used to model asset prices, where we require the volatility to depend on the size of the previous value:

$$X_t = \mu + e_t\sqrt{\alpha_0 + \sum_{k=1}^{p}\alpha_k(X_{t-k}-\mu)^2}$$

..Only 4 names (0.25*4=1.0) + Definitions (4*0.5=2)

[3]

iii) X_t follows MA(1), hence we can write

$X_t = e_t + \beta e_{t-1}$, Where $e_t \sim (0, \sigma^2)$

Now, $\text{var}(X_t) = \text{var}(e_t + \beta e_{t-1})$

$= \text{var}(e_t) + \beta^2\,\text{var}(e_{t-1})$

$= (1+\beta^2)\sigma^2$ ....(1)

Now, $\Delta Y_t = (0.6 + 0.3t + X_t) - [0.6 + 0.3(t-1) + X_{t-1}]$

$= 0.3 + (X_t - X_{t-1})$

Hence $\text{var}(\Delta Y_t) = [\text{cov}(X_t - X_{t-1}, X_t - X_{t-1})]$

$= [2\gamma_X(0) - \gamma_X(-1) - \gamma_X(1)]$ ....(2)

$\therefore Now, \gamma_x(0) = (1+\beta^2)\sigma^2$

And, $\gamma_x(1) = \gamma_x(-1) = \text{cov}(e_t + \beta e_{t-1}, e_t + \beta e_{t-1}) = \beta\sigma^2$

Therefore, from (2) we get,

$\text{var}(\Delta Y_t) = [2(1+\beta^2)\sigma^2 - 2\beta\sigma^2] = 2[1-\beta+\beta^2]\sigma^2$
..(0.5)

Now, $\text{var}(\Delta Y_t) - \text{var}(X_t)$

$= [2 - 2\beta + 2\beta^2]\sigma^2 - (1+\beta^2)\sigma^2$

$= [1 - 2\beta + \beta^2]\sigma^2$

$= (1-\beta^2)\sigma^2 > 0$

Hence the standard deviation of first difference of Y_t is higher than that of X_t

[6]
[10 Marks]

Solution 8:

i) All the figures calculated are in lacs.

Expected claim is given by:-

E(x) = 60 * 0.4 + 100 * 0.35 + 164 * 0.25 = 100

And $E(x^2) = 60^2 * 0.4 + 100^2 * 0.35 + 164^2 * 0.25 = 11,664$

We can consider the inter-occurrence time of a poison process with parameter λ follow the exponential distribution with mean $\frac{1}{\lambda}$

Hence time to second claim (T_2) will follow the Gamma distribution with scale parameter λ and shape parameter 2.

Let E_1 and E_2 are the first and second claim amount. Now the surplus at T_2 can be written as,

$150 + 1.30 * 0.75 * 0.30\, T_2 * 100 - E_1 - E_2$

Hence probability of ruin on second claim is:

= P[150 + 0.975 * 0.30 T_2 * 100 – 120 < 0] * P[E_1 = 60] * P[E_2 = 60]

+ P[150 + 0.975 * 0.30 T_2 * 100 – 160 < 0] * P[E_1 = 60] * P[E_2 = 100]

+ P[150 + 0.975 * 0.30 T_2 * 100 – 224 < 0] * P[E_1 = 60] * P[E_2 = 164]

+ P[150 + 0.975 * 0.30 T_2 * 100 – 200 < 0] * P[E_1 = 100] * P[E_2 = 100]

+ P[150 + 0.975 * 0.30 T_2 * 100 – 264 < 0] * P[E_1 = 100] * P[E_2 = 164]

+ P[150 + 0.975 * 0.30 T_2 * 100 – 328 < 0] * P[E_1 = 164] * P[E_2 = 164]

= 0

+ P[(0.3T_2) < 10/97.5] * 0.4 * 0.35

+ P[(0.3T_2) < 74/97.5] * 0.4 * 0.25

+ P[(0.3T_2) < 50/97.5] * 0.35 * 0.35

+ P[(0.3T_2) < 114/97.5] * 0.35 * 0.25

+ P[(0.3T_2) < 178/97.5] * 0.25 * 0.25

= 0

+ [1 – $1.102564e^{-0.102564}$] *0.4 * 0.35

+ [1 – $1.758974e^{-0.758974}$] *0.4 * 0.25

+ [1 – $1.512821e^{-0.512821}$] *0.35 * 0.35

+ [1 – $2.169231e^{-1.169231}$] *0.35 * 0.25

+ [1 – $2.825641e^{-1.825641}$] *0.25 * 0.25

= 0. 092463

Hence probability of ruin on second claim is 9.25%.

[6]

ii)

Let the aggregate claim at time t is S(t)

S(t) = $\sum_{i=1}^{N_t} X_i$

Where, N_t is the number of claim till time t

Therefore E[S(t)] = E(N_t) * E(X) = λt *100 = 30t

Var[S(t)] = E(N_t) * E(X^2) = λt * 11,664 = 3499.2t

Therefore the probability of ruin at the end of the time t is given by:

P[150 + 1.3*0.75 * E[S(t)] – S(t) < 0]

Now, E[S(2)] = 60

Var[S(2)] = 6998.4

Hence the probability of not ruin within second year is given by:

P[150 + 1.3*0.75 * E[S(2)] – S(2) > 0]

$$= P\left[\frac{S(2)-E[S(2)]}{\sqrt{Var[S(2)]}} < \frac{150+[1.3*0.75-1]E[S(2)]}{\sqrt{Var[S(2)]}}\right]$$

Using normal approximation we get the probability equal to:

$$= \Phi\left[\frac{150-1.5}{83.65644}\right] = \Phi(1.775117) = 96.2061\%$$

Hence the probability of not ruin within second year is 96.21%.

[5]

iii) Let the minimum loading factor is θ

Now, E[S(4)] = 120

Var[S(4)] = 13,996.8

Hence the required ruin probability at the end of the year 4 less than 10% is given by:

P[150 + (1+θ)*0.75 * E[S(4)] – S(4) < 0] ≤ 0.1

$$\text{Or,}\quad P\left[\frac{S(4)-E[S(4)]}{\sqrt{Var[S(4)]}} < \frac{150+[(1+\theta)*0.75-1]E[S(4)]}{\sqrt{Var[S(4)]}}\right] \le 0.1$$

Using normal approximation we get,

$$\Phi\left[\frac{150+[(1+\theta)*0.75-1]*120}{118.3081}\right] \ge 0.9$$

Or, (1+θ)*0.75 – 1 $\ge \frac{1.281552*118.30181-150}{120}$

Or θ ≥0.35131

Hence the minimum loading factor is 35.13%

[4]
[15 Marks]

Solution 9:

Let N-d = r, hence we have to calculate

P(N-d = r|d)

=P(N=d + r) * $\frac{\text{P('d' claims of size 300 out of (d + r) claims)}}{\text{P('d' claims of size 300)}}$

Now, P('d' claims of size 300)

= $\sum_{i=0}^{\alpha} e^{-\lambda} \frac{\lambda^{d+i}}{(d+i)!} \frac{(d+i)!}{d!i!} (0.3p)^d (1-0.3p)^i$

$= \frac{\lambda^d (0.3p)^d}{d!} \sum_{i=0}^{\alpha} e^{-\lambda} \frac{\lambda^i}{i!} (1-0.3p)^i$

$= \frac{\lambda^d}{d!} (0.3p)^d e^{-\lambda} e^{\lambda(1-0.3p)}$

Hence, P(N-d = r|d)

= $\frac{e^{-\lambda} \frac{\lambda^{d+r}}{(d+r)!} \frac{(d+r)!}{d!r!} (0.3p)^d (1-0.3p)^r}{e^{-\lambda} e^{\lambda(1-0.3p)} (0.3p)^d \frac{\lambda^d}{d!}}$

$= e^{\lambda(1-0.3p)} \frac{\lambda^r (1-0.3p)^r}{r!}$

Which is a probability from Poisson distribution with parameter λ (1-0.3p)

Therefore the conditional mean of P(N-d|d) is λ (1-0.3p).

[6 Marks]

Solution 10:

i) Let U_1, U_2, …….U_n be n random samples from U(0,1). Then monte-carlo simulation for η can be written as:-

$$\hat{\eta} = \frac{1}{n} \sum_{i=1}^{n} \left[U_i \left(e^{u_i} \right) - 1 \right]$$
$$= \frac{1}{n} \sum_{i=1}^{n} U_i e^{u_i} - 1$$

[1]

ii) Now we need to find variance of the function g(u) = Ue^u-1 where U~U(0,1)

$$\therefore E[g(u)] = \int_0^1 \left(xe^x - 1 \right) dx$$

$$= \left[xe^x - e^x - x \right]_0^1$$
$$= 0$$

$$E\left[g(u)^2\right] = \int_0^1 \left(xe^x - 1\right)^2 dx$$

$$= \int_0^1 \left(x^2 e^{2x} - 2xe^x + 1\right)\ dx$$

$$= \left[x^2 \frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \left(\frac{e^{2x}}{2} 2xdx\right)\ - \left[2\left(xe_x - e^x\right)\right]_0^1 + \left[x\right]_0^1$$

$$= \frac{e^2}{2} + 1 - 2[+1] - \left[x\frac{e^{2x}}{2} - \frac{1}{2}.\frac{1}{2}e^{2x}\right]_0^1$$

$$= \frac{e^2}{2} - 1 - \left[\frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4}\right]$$

$$= \frac{e^2 - 5}{4}$$

$$\therefore \text{var}\left[g(u)\right] = E\left[g(u)^2\right] - \left\{E\left[g(u)\right]\right\}^2$$

=0.59726

$$\therefore \text{var}(\hat{\eta}) = \frac{0.59726}{n}$$

Now, n should satisfy

n $\geq \frac{z_\alpha^2}{0.05^2} * 0.59726$

Here, α=5%, Z_α = 1.96

n $\geq \frac{1.96^2}{0.2^2} * 0.59726$ = 57.36

Therefore, required minimum number of random numbers is 58.

[6]

[7 Marks]

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