Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

September 2016 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i. Six real factors which impact the mortality and morbidity rates apart from age, sex and smoker status are:

a. occupation
b. nutrition
c. housing
d. climate
e. education
f. genetics

[3]

ii.

How genetics could be a source of difference:

Some of the diseases are hereditary in nature and there are increased chances of such diseases to be passed on from earlier generations to lower generations. Hence the family history and the linked genetics could be a factor leading to different mortality and morbidity rates.

However, genetics is a relatively new area of study for the medical profession, and only in the case of a few specific diseases is there as yet any indication that genetic information provides firm predictive evidence as to the chances of sickness or death relative to a person of average health.

How It could be used for better underwriting:
Genetics may give information about the likelihood of a person contracting particular diseases, and therefore may provide improved information about the chances of sickness or death.

Such information may be used in isolation for the particular life in question or, more usefully, by combining it with the life histories of the current and past generations of the family.

[3]
[6 Marks]

Solution 2:

Expected Present Value = $100,000 * A_x + 100,000 * {}_{n|}A_x$ @ 6%

$= 100,000 * A_{40} + 100,000 * {}_{20|}A_{40}$ @ 6%
$= 100,000 * \{ A_{40} + 1.06^{(-20)} * (l_{60}/ l_{40}) * A_{60} \}$
$= 100,000*\{ 0.12313 + 1.06^{(-20)} * (9287.2164/9856.2863) * 0.32692\}$

EPV= Rs. 21,918

Variance of Present value

= (100,000^2) * (${}^2A_x$ – v^40*(l_{x+20}/l_x)* ${}^2A_{x+20}$) + (200,000^2)* (v^40*(l_{x+20}/l_x)* ${}^2A_{x+20}$)

- (EPV)^2 @ 6%

= (100,000^2) * (${}^2A_{40}$ – v^40*(l_{60}/l_{40})* ${}^2A_{60}$) + (200,000^2)* (v^40*(l_{60}/l_{40})* ${}^2A_{60}$)

- (EPV)^2 @ 6%

= (100,000^2) *(0.02707- (9287.2164/9856.2863)*(1.06^(-40))*0.14098)

+ (200,000^2)* (9287.2164/9856.2863)*(1.06^(-40))*0.14098

- 21,918^2

= 177,751,936

Standard deviation = sqrt (variance)

= sqrt (177,751,936)

= Rs. 13,332 **[8 Marks]**

Solution 3:

Let Monthly Premium be P

EPV of Premiums = 12 * P * $\ddot{a}^{(12)}_{50:10\urcorner}$ at 6%

Now: $\ddot{a}^{(12)}_{50:10\urcorner}$ = $\ddot{a}_{50:10\urcorner}$ - (11/24)*(1 – ${}_{10}p_{50}$ * v^{10})

Now $\ddot{a}_{50:10\urcorner}$ = 7.694 solving it we get

So: $\ddot{a}^{(12)}_{50:10\urcorner}$ = 7.694 - (11/24)*(1 – 1.06^(-10) * (9287.2164/9712.0728))

= 7.694 – (11/24)* (1- 0.533968)

= 7.4804

Hence:

EPV of Premiums = 12 * P * 7.4804

= 89.7648 * P

PV of death benefit = 1,000,000 * $A^1_{50:15\urcorner}$ @ 6%

= 1,000,000 * (A_{50} - 1.06^(-15) * (l_{65}/l_{50}) * A_{65})

= 1,000,000 * (0.20508 - 1.06^(-15) * (8821.2612/9712.0718) * 0.40177)

= Rs. 52,812

PV of monthly incomes = v^10 * (l60/l50) * 10,000 * 12 * $\ddot{a}^{(12)}_{60:5\urcorner}$

= v^10 * (l60/l50) * 10,000 * 12 * ($\ddot{a}_{60:5\urcorner}$ - (11/24)*(1 – ${}_5p_{60}$ * v^5))

= 1.06^(-10) * (9287.2164/9712.0728) * 10,000 * 12 * (4.390 – (11/24) * (1-(8821.2612/9287.2164)*1.06^(-5)))

= Rs. 272,771

PV of maturity benefit = 500,000 * v^(15) * ${}_{15}P_{50}$

= 500,000 * 1.06^(-15) * (l_{65}/l_{50})

= 500,000 * 1.06^(-15) * (8821.2612/9712.0728)

= Rs. 189,496

Now by principle of equivalence

PV of premium = PV of all future benefits

89.7648 * P = 52,812 + 272,771 + 189,496

P (monthly premium) = Rs. 5,738

[8 Marks]

Solution 4:

Policy was sold exactly two years ago

i. Prospective reserves at t = 2 is given by
(PV of future benefits – PV of future Premiums) based on reserving basis

Now

PV of future Premiums = P * $\ddot{a}_{47:18\overline{7}|}$ @ 6%

= 6,300 * 11.206

= Rs. 70,598

PV of future benefits = 100,000 * $A^{1}_{47:18\overline{7}|}$ + 250,000 * v^18 *(l_{65}/l_{47})

= 100,000 * (A_{47} - v^18 *(l_{65}/l_{47}) * A_{65}) + 250,000 * v^18 *(l_{65}/l_{47})

= 100,000 * (0.17651-1.06^(-18)*(8821.2612/9771.0789)*0.40177)
+250,000*1.06^(-18)*(8821.2612/9771.0789)

= Rs. 84,015

Hence Prospective reserve = 84,015 – 70,598

= Rs. 13,418

Retrospective Reserves = Accumulation of historic cashflows at actual returns

= P * (1.075^2 + 1.075)

= 6,300 * (1.075^2 + 1.075)

= Rs. 14,053

[4]

ii. Retrospective and Prospective reserves calculated above are not equal since the experience that has emerged during the two year period and hence used in calculating the retrospective reserves is different from what was expected according to the reserving and pricing assumptions.

For example interest rate as per assumption was 6% P.A whereas actual experience has been 7.5%. Similarly as per assumptions, Mortality rate as per AM92 should have been applicable during the two years of experience, but actually zero mortality has been seen since the policyholder survived during the two year period.

[2]

iii. Since the surrender value paid to surrendering policyholders is 75% of the prospective reserves, it would lead to profit for the company at time of surrender to the extent of 25% of reserves.

Hence if the company allows for surrender rates in calculation of prospective reserves it would lead to higher projected profits in the reserving cashflows and hence lower reserves at the valuation date.

[2]

iv. Purpose of having a prudent assumption is to have higher reserves, so that company could meet the liability outgoes even in stressed situation and hence leading to protection of policyholders interest.

In this scenario, if the realistic assumption for surrender rate is 10% then a prudent assumption for surrender rate for calculation of reserves would be a rate lesser than 10% (say 5% or 8%). This is so because surrenders will lead to surrender profits and hence having more than realistic rate would lead to higher profits and hence lower reserves and rates lower than realistic rates would lead to lower surrender profits and hence higher reserves.

[2]

[10 Marks]

Solution 5:

Let the single premium be P

Also A is male life currently aged x =65

And B is female life currently aged y =60

Annual Payment = Rs. 100,000

Single premium P = PV of benefits = A – B

A = 100,000 * ($\ddot{a}_y$ - $\ddot{a}_{xy}$ - v^n * ${}_np_{xy}$ ($\ddot{a}_{y+n}$ - $\ddot{a}_{x+n:y+n}$))

@ 4% using PA92C20 mortality and n = 5 years

= 100,000 * ($\ddot{a}_{60}$ - $\ddot{a}_{65:60}$ - 1.04^(-5) * ${}_5p_{65:60}$ ($\ddot{a}_{65}$ - $\ddot{a}_{70:65}$))

= 100,000*(16.652-12.682
-1.04^(5)*(9238.134/9647.797)*(9703.708/9848.431)*(14.871-10.494))

= 57,581

B =
= 100,000 * v^15 * ${}_5q_x$ * ${}_{15}p_y$ * a_{y+15}
= 100,000 * 1.04^(-15) * ${}_5q_{65(m)}$ * ${}_{15}p_{60(f)}$ * $a_{75(f)}$)
= 100,000 * 1.04^(-15) * (1-9238.134/9647.797)*(8784.955/9848.431)*9.933
= 20,900
Therefore, P = 57,581 – 20,891 = 36,690

[6 Marks]

Solution 6:

Age dependant force of mortality = μ_x = 0.0005 + 0.001x
Current Age = 35 years exact

i. Probability to die before age 70 = ${}_{35}q_{35}$
= 1 - ${}_{35}p_{35}$
Now:
${}_tp_x$ = exp (-$\int_x^{x+t} \mu_s$ ds)
= exp (-$\int_x^{x+t} (0.0005 + 0.001\ s)$ ds
= exp (- (0.0005 s + (0.001 s^2)/2)]with limits x to x+t

Hence:
${}_{35}p_{35}$ = exp (0.0005*35 + 0.001 * 35^(2) / 2 - 0.0005*70 - 0.001 * 70^(2) / 2)
= 0.156453

Hence:
${}_{35}q_{35}$ = 1- ${}_{35}p_{35}$ = 1-0.156453
= 0.843547

[3]

ii. Survives till age 70 but dies before 75
= ${}_{35}p_{35}$ * ${}_5q_{70}$
= ${}_{35}p_{35}$ * (1- ${}_5p_{70}$)
From i : ${}_{35}p_{35}$ = 0.156453

Now:

${}_5p_{70}$ = exp (- (0.0005 s + (0.001 s^2)/2)]with limits 70 to 75
= 0.694197

Hence:

Probability(Survives till age 70 but dies before 75)

= 0.156453 * (1-0.694197)

= 0.047844 **[3]**

[6 Marks]

Solution 7:

Given age = 65 years

1 year select mortality

Int rate = 8% P.A

$q_{[65]} = 0.75*q_{65}$

$A_{65} = 0.5412$ and $A_{66} = 0.5591$

SA = Rs. 100,000

Single premium = ?

Now:

Single Premium = $100,000 * A_{[65]}$

$= 100,000 * (v * q_{[65]} + v * p_{[65]} * A_{66})$

$= 100,000 * (v * q_{[65]} + v * (1-q_{[65]}) * A_{66})$

We have to calculate $q_{[65]}$ from given data

Now:

$A_{65} = v * q_{65} + v * p_{65} * A_{66}$

$A_{65} = v * q_{65} + v * (1-q_{65}) * A_{66}$

$0.5412 = 1.08^{(-1)} * q_{65} + 1.08^{(-1)} * (1- q_{65}) * 0.5591$

Solving

$q_{65} = 0.0576$

Now as given:

$q_{[65]} = 0.75 * q_{65}$

$q_{[65]} = 0.75 * 0.0576$

$q_{[65]} = 0.0432$

Hence:

Single Premium = $100,000 * (v * q_{[65]} + v * (1-q_{[65]}) * A_{66})$

$= 100,000 * (1.08^{(-1)} * 0.0432 + 1.08^{(-1)} * (1-0.0432) * 0.5591)$

= Rs. 53,532

[5 Marks]

Solution 8:

i. *F* is called the area comparability factor and is a measure of the crude mortality rate for the standard population divided by what the crude mortality rate is for the region being studied, assuming the mortality rates are the same as for the standard population. **[2]**

ii. If its age/sex profile is such that if it experienced the same age/sex specific mortality rates as the country, then its crude death rate would be 2 times of that of the country, i.e. the region has either a older age structure or a higher male proportion (or both) than the country. **[2]**

iii. Nutrition has direct impact on morbidity and in longer period of time on mortality as well.

Poor quality nutrition increases the risk of falling sick and delays recovery as well.

Excessive or inappropriate (e.g. too much fat, carbs) eating can lead to obesity and an increased risk of associated diseases (e.g. heart disease, hypertension, thyroid) leading to increased morbidity and then mortality.

Inappropriate nutrition may be the result of economic factors e.g. lack of income to buy nutrition rich foods or the result of a lack of education in this respect resulting in poor nutritional choices. Also, social and cultural factors encourage or discourage the eating of certain foods
e.g. alcohol consumption, consumption of non vegetarian food.

[3]

iv.

	Standard death rate	**Location A**		**Location B**	
Age		**Initial Exposed To Risk**	**Expected Number of Deaths**	**Initial Exposed To Risk**	**Expected Number of Deaths**
60	0.01392	100	1.39	200	2.78
61	0.0156	175	2.73	150	2.34
62	0.01749	190	3.32	170	2.97
63	0.01965	210	4.13	100	1.97

Actual number of death in Location A = 9, Actual number of death in Location B = 11

Total expected number of deaths for A = 11.6

Total expected number of deaths for B = 10.1

SMR for A = 9/11.6 = 0.78

SMR for B = 11/10.1= 1.09 **[5]**

[12 Marks]

Solution 9:

i. *The unit price at the start of 1st March is:*

=10*(1.01*1.06)^((31+28)/365) = 10.110882

The value of the policyholder's fund on 1st March is then:

=10.110882*500 = 5055.4412

The value of the fund on end of 30th March is:

= (5055.4412 +1000)*(1.01*1.06)^(30/365)-1000 = 5089.4898

On start of 1st April its value (before premium payment i.e. from 30th to 31st march) is:

= (5089.4898)*(1.01*1.06)^(1/365)

=5090.4411

On 15th April its value is:

= (5090.4411+1000)*(1.01*1.06)^(15/365) = 6107.5398

[5]

ii. The surrender benefit might be different from the value calculated on account of following reasons:

1. Company might be applying surrender penalty or market value reduction on this value

2. Minimum surrender value guaranteed might be higher than this value

3. Company might be paying special surrender value which might be higher than this value due to a component of undistributed surplus or terminal bonus.

[2]

[7 Marks]

Solution 10:

i. Decrement Table:

Age	Death Rate	Surrender Rate	Dependent Death rate	Dependent surrender rate	Lx at start	No of deaths	No of surrenders	Lx at end
45	0.0010	0.1200	0.0010	0.1199	1.0000	0.0010	0.1199	0.8791
46	0.0011	0.0600	0.0011	0.0599	0.8791	0.0010	0.0527	0.8255
47	0.0012	-	0.0012	-	0.8255	0.0010	-	0.8245

Unit Fund (Per Policy):

Year	Unit Value at start	Allocated Premium	Bid Offer Spread	Policy Administration Charge	Interest	Management Charge	Unit Value at end
1	0	9,500.00	475	500	767.25	125.45	9,166.80
2	9,166.80	10,000.00	500	550	1,630.51	266.59	19,480.72
3	19,480.73	10,000.00	500	605	2,553.82	417.55	30,512.00

Non Unit Fund:

Year	Unallocated Premium	Pol Admin Charge	Bid Offer Spread	Expenses + Commission	Interest	Management Charge	Surrender Penalty	End of year Cashflow	Inforce Cashflows	Discount Factor	Inforce Cashflows present value	NBM
1	500	500	475	2,500	(61.50)	125.45	299.70	(661.35)	(661.35)	0.8929	(590.50)	
2	0	550	500	710	20.40	266.59	89.90	716.89	630.23	0.7972	502.42	
3	0	605	500	720.2	23.09	417.55	0	825.44	681.37	0.7118	484.98	**3.97%**
											396.90	

[14]

ii.

Maturity Outgo	Revised Cashflow	Revised If Cashflow	Discount Factor	Inforce Cashflows present value	NBM
0	(661.35)	(661.35)	0.8929	(590.50)	**0.38%**
0	716.89	630.23	0.7972	502.42	
610.24	215.20	177.64	0.7118	126.44	
				38.36	

[2]

iii. Revised figures (Changes highlighted in bold)

Decrement Table:

Age	Death Rate	Surrender Rate	Disability rate	Dependent Death rate	Dependent surrender rate	Dependent disability rate	Lx at start	No of deaths	No of surrenders	No of disability	Lx at end
45	0.0010	0.1200	0.0200	0.0010	0.1199	0.0176	1.0000	0.0010	0.1199	0.0176	0.8615
46	0.0011	0.0600	0.0200	0.0011	0.0599	0.0188	0.8615	0.0009	0.0516	0.0162	0.7928
47	0.0012	-	0.0200	0.0012	-	0.0200	0.7928	0.0010	-	0.0158	0.7760

Unit Fund (Per Policy):

Year	Unit Value at start	Allocated Premium	Bid Offer Spread	Policy Administration Charge	Disability Charge	Interest	Management Charge	Unit Value at end
1	0	9,500.00	475	500	600	713.25	116.62	8,521.63
2	8,521.63	10,000.00	500	550	315	1,544.10	252.46	18,448.27
3	18,448.27	10,000.00	500	605	0	2,460.89	402.36	29,401.81

Non Unit Fund:

Year	Unallocated Premium	Pol Admin Charge	Bid Offer Spread	Expenses + Commission	Interest	Management Charge	Surrender Penalty	Diability Charge	Disability Outgo	End of year Cashflow	Inforce Cashflows	Discount Factor	Inforce Cashflows present value
1	500	500	475	3,000	(91.50)	116.62	299.70	600.00	351.60	(951.78)	(951.78)	0.8929	(849.81)
2	0	550	500	710	20.40	252.46	89.90	315.00	187.78	829.98	715.06	0.7972	570.04
3	0	605	500	720.2	23.09	402.36	0	-	0	810.24	642.34	0.7118	457.21
													177.4

NBM = 177.4/10000*100 = **1.77%**

[6]

iv. NBM has decreased. The decrease is on account of additional underwriting cost in 1st year. Due to the underwriting cost, NBM has decreased despite charge being more than outgo for disability. **[1]**

[23 Marks]

Solution 11:

(i) Different groups or classes of policyholders may have higher or lower lapse rates for all major risk factors (age, duration, gender etc.) than other classes. An example would be where a class of policyholders buying through a particular individual agent might show higher lapse rates as compared to those who buy online.

[2]

(ii) Lapse rates may vary by policy duration. At shorter durations, lapse rates may be the result of mis-selling purchase by policyholder whereas at longer durations the policy has become more stable.

[2]

(iii) Lapse rates vary with calendar time for all major risk factors, e.g. economic environment varies over time and this results in a similar variation in lapse rates. Similarly, surrenders might increase at the time where surrender penalty becomes very low or 0.

[2]

[6 Marks]

Solution 12:

EPV of no claim bonus is as below

$= 1000 * e^{-\int_{45}^{65}(0.02+0.03+0.04)dx}$

$= 1000 * e^{(-0.09*20)}$

$= 1000 * e^{-1.8}$

$= 1000 * 0.1653$

=Rs. 165.30

[3 Marks]

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